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CHARACTERIZATION OF VARIABLE MATERIAL PROPERTIES BY NONLINEAR ACOUSTIC TECHNIQUES

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Abstract

The recent results on the development of the mathematical basis of nonlinear acoustic techniques for nondestructive testing (NDT) of inhomogeneously prestressed nonlinear elastic material, weakly inhomogeneous nonlinear elastic material and nonlinear viscoelastic material are presented. The one-dimensional problems of nonlinear propagation and interaction of longitudinal waves in inhomogeneous materials are solved analytically. The possibility to utilize the derived solutions in NDT is studied in detail in the case of sine-wave propagation. The idea is to extract, besides the wave velocity measurement data, information from nonlinear effects of wave propagation and interaction. The algorithms for four different NDT techniques are proposed.

1. INTRODUCTION

The rapid advancements in computer technology, in electronics and in measurement techniques [1] affords to develop effective and versatile nonlinear acoustic techniques for nondestructive testing (NDT) of materials with continuously variable in space and time properties. The amounts of information needed to be determined in ultrasonic NDT increase essentially in the case of materials (specimens, structural elements) with variable properties. This involves the necessity to extract additional information from the measurement data of wave velocity and wave profile distortion. One possibility to obtain more information is to record and analyse the nonlinear effects of wave propagation and interaction [2, 3].

In this paper relatively simple methods for NDT of inhomogeneous material properties are proposed. The basic idea is to use the wave profile distortion phenomenon that accompanies nonlinear propagation and interaction of waves in the material. To that end the one-dimensional problems of nonlinear propagation and interaction of longitudinal waves in inhomogeneous materials are investigated theoretically. The possibility to utilize the derived solutions in NDT is studied in detail in the case of sine-wave propagation. Two NDT techniques – the through transmission technique and the wave interaction technique are developed for the case when the material inhomogeneity has a polynomial functionality. The following NDT algorithms are constructed on the basis of plots wave characteristics versus variable properties of nonlinear elastic material:

- Through transmission techniques for nondestructive characterization of weak inhomogeneity in material properties and for two-parametric inhomogeneous prestress.
- Wave interaction techniques for qualitative and quantitative characterization of two-parametric prestress in the material on the basis of nonlinear effects of interaction of counterpropagating ultrasonic waves.

2. THEORETICAL BASIS

Attention is confined to two different materials: (i) physically nonlinear elastic material and (ii) viscoelastic material. Small but finite deformations of the nonlinear elastic material are described by the theory of elasticity with quadratic nonlinearity [4]. Constitutive equation of the viscoelastic material corresponds to the model of standard viscoelastic body [5] and it has linear viscous and nonlinear elastic properties. In both cases the geometrical and the physical nonlinearity of the problem is taken into account. Nonlinear deformations of materials are studied in Lagrangian rectangular coordinates.

The aim is to clarify the possibility of nondestructive characterization of (i) inhomogeneous prestress in the sample (structural element) made of nonlinear elastic material or viscoelastic material and (ii) weakly inhomogeneous properties of the nonlinear elastic material. The nonlinear propagation of longitudinal waves in all materials is considered as one-dimensional with the exception of wave propagation in prestressed materials that is considered as quasi one-dimensional (the prestressed state is two-dimensional). The wave motion in elastic material is described by the nonlinear second order hyperbolic differential equation with variable in space coefficients. The coefficients are functions of the physical properties of the material and the parameters of the prestressed state. Viscous properties of the material involve in addition the dependence of coefficients on time. The problem is solved by assumption that the kind of prestressed state and the physical properties of the material are known and the variable coefficients of the equations of motion are known functions and they have a polynomial functionality. It is assumed that the prestressed state of the material (structural element) corresponds to the plane strain. In this case the coefficients of the equation of motion are determined as solutions to a set of two nonlinear second order elliptic differential equations that describe equilibrium of the material in the static prestressed state. The solution to the equations of equilibrium is sought in the form of polynomials making use of the perturbation technique.

All equations of motion are solved by means of perturbation technique. The result is that at our disposal there are analytical solutions that describe wave propagation in considered materials.

3. THROUGH TRANSMISSION TECHNIQUE

This technique is widely used for nondestructive characterization of homogeneous prestress and it is interpreted resorting to the acoustoelastic effect – the dependence of the wave velocity on the value of prestress [7]. Utilization of wave velocity data only may give misleading information about the inhomogeneous prestress [6]. Here, the necessary additional information is proposed to extract from the data of wave profile distortion. The basic physical effects responsible for the distortion of the wave profile are nonlinearity, dispersion, inhomogeneity and dissipation. These effects can act separately or in various combinations. Below, the advantages of exploitation of wave profile distortion data as a source of additional information for nondestructive characterization of variable material properties is demonstrated in special cases.

3.1 Inhomogeneously Prestressed Material

The wave motion in the prestressed material is described on the basis of equation of motion derived by the assumption that the material has three different states [6]. Initially it is in the prestress free natural state. Then it is subjected to external forces and now it is in the prestressed state. At some instant the wave motion is excited in the material. The displacement of the material point in the present state is assumed to be a vectorial sum of displacements caused by external forces and wave motion. Interesting is that the equation of motion for the present state includes the wave – prestress interaction only if the geometrical nonlinearity of the problem is taken into account.

The quasi one-dimensional problem of nonlinear propagation of longitudinal wave with arbitrary smooth initial profile in the specimen is studied theoretically [6]. The specimen of the prestressed material has two parallel boundaries. The longitudinal wave is excited on one boundary in terms of particle velocity and is recorded on the opposite boundary by the assumptions that the deformations and their spatial derivatives caused by the propagating wave are much larger in the propagating direction than in the orthogonal direction with respect to it. This enables to treat the problem as one-dimensional wave propagation in two-dimensional prestressed material. The analytical solution to describe the propagation of longitudinal wave with arbitrary smooth initial profile in physically nonlinear elastic material undergoing two-parametric inhomogeneous prestress is derived making use of the perturbation technique. The solution describes the initial stage of wave propagation and it is valid in some time interval. It is supposed that in this initial stage the distortion of wave profile is weak and the shock wave is not generated.

Propagation of harmonic wave in inhomogeneously prestressed material is studied in detail. It is important that the recorded nonlinear effects of harmonic wave propagation contain maximum information about the inhomogeneous prestress. The amount of this information is dependent on the ratio of strain intensities in the material caused by the propagating wave and the prestress, correspondingly [8]. The assumption is that the strain caused by both affects is small, i. e., the deformations are elastic and not plastic. Consequently, the problem involves two small parameters that characterize displacements of the material points evoked by prestress and wave motion, respectively. The wave motion is described by first three terms in perturbative solution. The first term depicts linear wave propagation in the prestress free linear material. The amount of information about inhomogeneity and nonlinearity in the second and the third terms of solution depend on the ratio of small parameters. This phenomenon is studied for three different cases.

First case: displacements caused by wave motion are much smaller than the corresponding displacements caused by prestress. The second term in solution corrects the amplitude and phase shift of the first harmonic for prestress. The third term regulates the first term once more and describes the evolution of the second harmonic in a prestress free material. This three-term solution is inapplicable for nondestructive characterization of prestressed state of material on the basis of nonlinear effects of wave propagation.

Second case: large displacements of wave motion are superposed on small displacements caused by prestress. Strong nonlinear effects characterize the wave motion. The higher terms in solution describe evolution of the second and the third harmonics in the prestress free material. The third term defines the influence of prestress on the first harmonic only. In this case the strong nonlinear effects are not affected by prestress and the conclusion from the point of view of prestress characterization is the same as in the first case.

Third case: displacements caused by wave motion are of the same order as the displacements caused by prestress. The second term of solution adjusts the amplitude of the first harmonic to prestress and describes the evolution of the second harmonic in a prestress free material. The third term determines the influence of prestress on the second harmonic and traces evolution of the third harmonic in the prestress free material.

The conclusion is that by ultrasonic nondestructive characterization of prestress it is important to pay attention to the intensity of the applied wave. Nonlinear effects of wave motion contain maximum information about prestressed state provided the displacements caused by wave motion are of the same order as the displacements caused by prestress.

The unknown quantities to be determined in nondestructive testing of inhomogeneously prestressed specimen (structural element) are (i) physical properties of the material, (ii) dimensions of the specimen and (ii) parameters of prestress. The nonlinear elastic material is characterized by the density and by five elastic constants (two Lame constants and three third order constants). Viscoelastic material described by the model of standard viscoelastic body [5] is characterized in addition by the relaxation and creep time. From the dimensions of the specimen one-dimensional wave motion can determine only the thickness of specimen. The considered prestress corresponds to the pure bending with compression or tension. The main domain of the 2D prestress is determined by the component $T_{22} = a + b\ X_1$ of the Kirchhoff pseudostress tensor where a and b are constants and direction of the coordinate X_1 is perpendicular to the surface of the specimen. The constant a characterizes the constant part of prestress and the constant b the linearly variable part of it.

The material characterization problem is proposed to solve on the basis of the data obtained from first two wave harmonics. Information may be extracted from the evolution data of the amplitudes and phase shifts of two harmonics. Informative are also variations of wave frequency, phase and group velocity.

Conclusion is that it is not possible to determine the properties and the prestressed state of the material on the basis of the wave propagation data counted above. The problem may be solved provided preliminary information about the properties and the state of the specimen (structural element) is available.

There are many possibilities to use the approach presented above in nondestructive characterization of material. For example, let the physical properties and the geometry of the specimen (structural element) be known. The aim is to evaluate the parameters of the prestressed state of the material. Let the preliminary inspection of the specimen (structural element) and the loading scheme permit to determine that the specimen is undergoing pure bending with compression or tension, i. e., the two-parametric prestressed state described above. The analytical expressions wave characteristics versus material properties and prestress parameters are derived. These nonlinear expressions are too cumbersome for the analytical analyses. The corresponding plots are composed making use of the numerical simulation. After that, utilizing the wave propagation data in real specimen and resorting to these plots the prestress characterization problem may be solved.

In the case of wave propagation in the material undergoing two-parametric prestress the analyses of the results of numerical simulation point out the strong sensitivity of the variation of the first harmonic amplitude to the inhomogeneity of prestress and less sensitivity to the homogeneous part of prestress. The values of the second harmonic amplitude, phase shifts and phase velocities of both harmonics are all strongly sensitive to both parameters of prestress. Interesting is that all these dependences are qualitatively different and close to linear. Dissimilarity in these dependences enables to propose an algorithm to nondestructive evaluation of prestress parameters. Algorithm may be composed making use of the first harmonic amplitude and velocity measurement data. The prestress parameter b is determined with some precision from the plot first harmonic amplitude versus prestress parameters by assumption that the value of the first harmonic amplitude is known for the real specimen. Then, using the known values of the velocity of the first harmonic and parameter b the prestress parameter a is determined resorting to the plot phase velocity of the first harmonic versus parameters of prestress.

In the case of prestressed specimen (structural element) made from viscoelastic material the dependences wave characteristics versus prestress parameters are time dependent. The described above algorithm for nondestructive evaluation of the parameters of two-parametric prestressed state is valid for fixt instant provided the loading time of the specimen is known.

3.2 Weakly Inhomogeneous Material

The nonlinear elastic material is characterized by the density and by the second and third order elastic coefficients [6]. In one-dimensional problem five elastic coefficients form two groups. The elastic properties are determined by the linear elastic coefficient (sum of Lame coefficients) and by the nonlinear elastic coefficient (sum of third order elastic coefficients) [9]. The weak inhomogeneity in material density, linear and nonlinear elasticity is described by polynomials involving the small parameter. The one-dimensional wave motion is excited and recorded in the specimen in the way described above. The analytical expression for boundary excitation contains a small parameter that determines the smallness of strain. The equation of motion of the specimen is solved making use of the perturbation technique.

The material characterization problem is solved on the basis of first three terms in the perturbative solution that describe propagation of first two harmonics in the inhomogeneous material. The physical meaning of terms coincides with description above. The analytical expression of the solution is too cumbersome for direct analysis. That’s why the numerical experiment is posed. The plots wave characteristics versus material properties are computed by additional preliminary information about the material (duralumin). The variation of material properties is considered with respect to the reference homogeneous material.

The analysis of plots related to harmonic wave propagation in the homogeneous material leads to the conclusion that the sensitivity of wave characteristics to different material properties is dissimilar. The value of the first harmonic amplitude is very sensitive to the value of material density and the linear elasticity but less sensitive to the nonlinear elasticity. The phase shift of the first harmonic is sensitive to all three material properties. The value of the second harmonic amplitude is less sensitive to the value of the density. Its sensitivity to the variation of linear and nonlinear elasticity is about of the same order. The different sensitivity of wave characteristics to the material properties permits to propose the algorithm for nondestructive evaluation of the homogeneous material properties [9]. The homogeneity of the nonlinear elastic material is characterized by the phenomenon that the phase shift of the second harmonic is equal to zero.

The properties of the inhomogeneous material are assumed to change into the depth of the material according to the linear law. Harmonic wave is excited in this material. Following to the accepted procedure the plots wave characteristics versus parameters that determine variation of material properties are computed. These plots exhibit the fact that almost all dependences of wave characteristics upon material inhomogeneity parameters are close to linear. Exceptional is the strongly nonlinear relation between the value of the second harmonic amplitude and the inhomogeneity parameters of the material. The evaluation problem of inhomogeneity parameters is recommended to solve resorting to plots first harmonic amplitude and phase shift versus variation of density and linear elasticity and second harmonic amplitude versus variation of density and nonlinear elasticity.

4. WAVE INTERACTION TECHNIQUE

Two longitudinal waves with arbitrary smooth initial profiles are excited simultaneously on opposite parallel boundaries of the prestressed nonlinear elastic material (specimen, structural element). The inhomogeneous two-parametric prestressed state is assumed to be known and it is described above. The nonlinear governing equation for wave propagation is solved making use of the perturbation technique and the Laplace integral transform [10]. The resulting solution describes the initial stage of nonlinear wave propagation, reflection and interaction and is valid in some time interval.

Counter-propagation of harmonic waves with the same amplitude and frequency are excited on opposite boundaries in terms of particle velocity. The distorted wave profiles are recorded on the same boundaries in term of stress. The recorded data are analysed on the basis of the perturbative solution that makes it possible to separate the linear and nonlinear effects of counter-propagation of harmonic waves. The first term in perturbative series describes linear propagation and superposition of waves in the physically linear prestress free material. The second and the subsequent terms correct the solution and take the influence of nonlinearity and inhomogeneity into account. The second term describes the main domain of nonlinear effects including the evolution of the second harmonic, influence of the prestress to the evolution of the first harmonic, nonlinear interaction between two first harmonics and influence of the nonlinear physical properties of the material to the wave propagation.

The main attention is paid to the wave induced oscillations on two parallel boundaries of the material. Due to the complexity of the analytical solution these oscillations are studied on the basis of this solution numerically in duralumin.

Analysis of the results on numerical simulations leads to the conclusion that the amplitude of boundary oscillations is dependent on the physical properties of the material and prestress. In the prestress free material boundary oscillations have constant amplitude in the interval of propagation and in the interval of interaction. The homogeneous prestress modulates these oscillations. The oscillations on both boundaries coincide. The amplitude and the depth of modulation are dependent on the value and the sign (compression or tension) of prestress. The inhomogeneous prestress (bending of the sample, for example) involves disparity in oscillation profiles on different boundaries. In the special case of pure bending the boundary oscillation profiles coincide with phase shift.

Consequently, it is easy to determine qualitatively the presence and the nature of prestress and to distinguish (i) prestress free material, (ii) homogeneously prestressed material, (iii) material undergoing pure bending and (iv) material undergoing pure bending with tension or compression.

With the view to investigate dependence of the value of boundary oscillation amplitude on prestress parameters the plots amplitudes versus parameters are composed for two instants in the wave interaction interval. The result is that the oscillation amplitude is strongly sensitive to the values of both parameters of two-parametric prestress state and these dependences are close to linear. The algorithm for nondestructive evaluation of prestress parameters on the basis of wave interaction data is proposed.

5. CONCLUSIONS

Nonlinear effects of wave propagation are not widely used in nondestructive characterization of materials and their states. The information that contains in nonlinear effects of wave propagation and interaction enables to enhance the efficiency of nondestructive material characterization and makes it possible to propose the relatively simple algorithms for nondestructive testing. This is demonstrated in this paper by model problems for through transmission technique and wave interaction technique. The main conclusion is that the proposed algorithms may be used effectively provided some preliminary information about the problem is available.

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