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UNDERWATER ACOUSTIC PROPAGATION MODEL TO SIMULATE SEISMIC OCEANOGRAPHY EXPERIMENTS

Jean Kormann[1], Pedro Cobo[1] and Carlos Ranz[1]

[1]Insituto de Acústica, CSIC Madrid, C/ Serrano144, 28006 Madrid, Spain
jkormann@hotmail.com

Abstract

Seismic reflection is a technique used for decades to profile the earth layering beneath the ocean with a high lateral and vertical resolution. In the other hand, oceanographers use probes to obtain the properties of the water layer with a rather lesser lateral resolution. Seismic oceanography focuses the powerful tools of seismics into the water layer to reveal its fine structure with a high lateral resolution. The re-processing of the in-water reflected waves in seismic data has allowed to image eddies, termohaline intrusions and internal waves. Synthetic models of such seismic experiments in water permit to foresee the effects of oceanographic gradients in the propagated waves. This paper describes a synthetic model of such seismic oceanography experiments. The model uses Berkhout convolution operators to extrapolate the wave field from the seismic source to the hydrophones taking into account the sound velocity structure of the water layer. Due to the small amplitudes of the scattered field, special attention is paid to the absorbing properties of the artificial boundaries of the medium.

1. INTRODUCTION

Seismic reflection is a well known method to image the Earth's subsurface. The method processes the waves reflected in their interfaces to obtain a high resolution image of the explored medium. In marine seismics, acoustic waves propagate first in the sea (acoustic waves) and then penetrate into the sedimentary layers of the subbottom (elastic waves). Since they are looking for the structural and lithologic properties of the subsurface layers, they focus the processing window in the elastic part of the seismic traces, disregarding the acoustic part.

On the other hand, oceanographers use probes to explore the properties of the sea layer with low lateral resolution. Recently, Holbrook proposed that marine seismic database can be re-processed to obtain high resolution images of the water masses covered by the corresponding experiments [1]. This re-processing allowed them to get high resolution images of fine-scale structure in the ocean, such as thermohaline intrusion [1], internal waves [2], or mesoscale eddies [3]. The technique that processes marine seismic data to achieve oceanographic properties of the water masses is named seismic oceanography.

In this paper we describe an underwater acoustic propagation model to simulate seismic experiments in the sea. The model propagates the acoustic waves from a seismic source to a streamer of hydrophones, through a layered ocean. The main difficulty of this modelling consists on outlining sea reflections that are about the order of 10^{-4}. Section 2 describes the propagation algorithm, based on Berkhout spatial convolution operators [4], and the finite-difference scheme. Section 3 deals with the absorbing boundary condition included in our propagation model, namely the Perfectly Matched Layer (PML) condition [5-6]. Section 4 shows simulated numerical results for a layered ocean with a realistic sound velocity profile.

2. PROPAGATION ALGORITHM

Let's start with the acoustic wave equation

$$\rho\nabla\cdot\left(\frac{1}{\rho}\nabla p\right)-\frac{1}{c^2}\frac{\partial^2 p}{\partial t^2}=-s \quad , \tag{1}$$

where $p=0$ for $t<0$, and $p(x,y,t)$ is the sound pressure, c the sound speed, and $s(x,y,t)$ the source function. Assuming a constant density, Eq. (1) can be written

$$\partial_t^2 p = c^2\left[\left(d_2(x)+d_2(y)+d_2(z)\right)* p + s\right] \tag{2}$$

where d_2 denotes second spatial derivative and * denotes convolution. Adding the forward and backward Taylor's expansions, Eqs. (3a) and (3b), we get the Eq. (3c)

$$\begin{aligned} p(t+\Delta t) &= p(t)+\frac{\Delta t}{1!}\partial_t p(t)+\frac{\Delta t^2}{2!}\partial_t^2 p(t)+\ldots && (a)\\ p(t-\Delta t) &= p(t)-\frac{\Delta t}{1!}\partial_t p(t)+\frac{\Delta t^2}{2!}\partial_t^2 p(t)+\ldots && (b)\\ p(t+\Delta t) &= -p(t-\Delta t)+2p(t)+2\frac{\Delta t^2}{2!}\partial_t^2 p(t)\ldots \\ &\quad \ldots+2\frac{\Delta t^4}{4!}\partial_t^4 p(t)+\ldots && (c) \end{aligned} \quad , \tag{3}$$

Introducing Eq. (2) into Eq. (3c), the following $2K^{th}$ order algorithm is obtained

$$p(t+\Delta t) = -p(t-\Delta t)+2p(t)+2\sum_{k=1}^{K}\frac{1}{(2k)!}q_k(t)+O\left\{\left(\frac{c\Delta t}{\lambda}\right)^{2K+2}\right\}, \tag{4}$$

where $q_k=(c\Delta t)^2\left[d_2(x,y,z)*q_{k-1}+\Delta t^{2k-2}s_k(x,y,z,t)\right]$, $q_0=p$ and $s_k=\partial_t^{2k-2}s$. If we choose the spatial increment as $\Delta<\frac{\lambda}{10}$ (where λ is the wavelength), and assuming that $c\Delta t<\Delta$, the second order approximation ($K=1$) of Eq. (4) becomes

$$p(t+\Delta t) = -p(t-\Delta t)+2p(t)+(c\Delta t)^2\left[d_2 * p + s\right]. \tag{5}$$

Finally, the second order finite-difference time-domain scheme for acoustic waves propagation is

$$
\begin{aligned}
&p(n,m,k+1) = -p(n,m,k-1) + 2\left[1 - 2\frac{(c\Delta t)^2}{\Delta^2}\right]p(n,mk) + \left(c\Delta t^2\right)s(n,m,k) + \\
&\frac{(c\Delta t)^2}{\Delta^2}\left[p(n-1,m,k) + p(n,m-1,k) + p(n+1,m,k) + p(n,m+1,k)\right]
\end{aligned} \quad . \qquad (6)
$$

Similar reasoning allows to obtain higher order finite-difference schemes. Figure 2 shows a snapshot of the propagated wavefield in a medium with the sound velocity structure of Figure 1, using the numerical scheme of Eq. (6). Events marked 1 and 2 in Figure 2 correspond to reflections at the interfaces 1 and 2 in Figure 1.

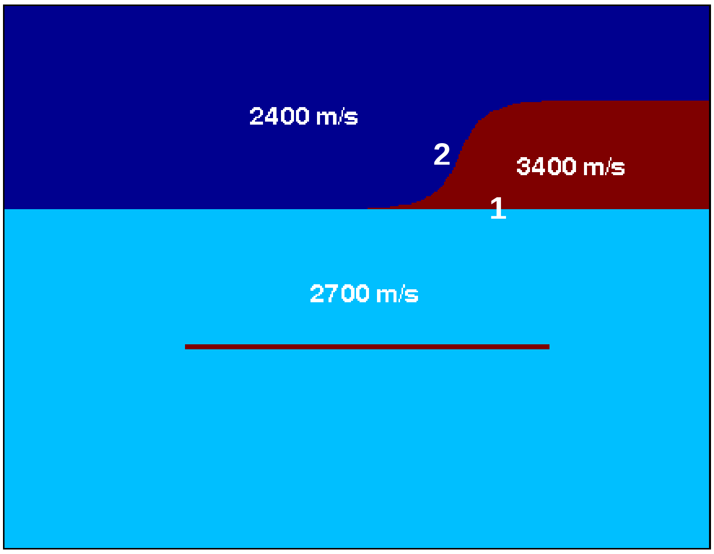

Figure 1. 2D velocity structure model.

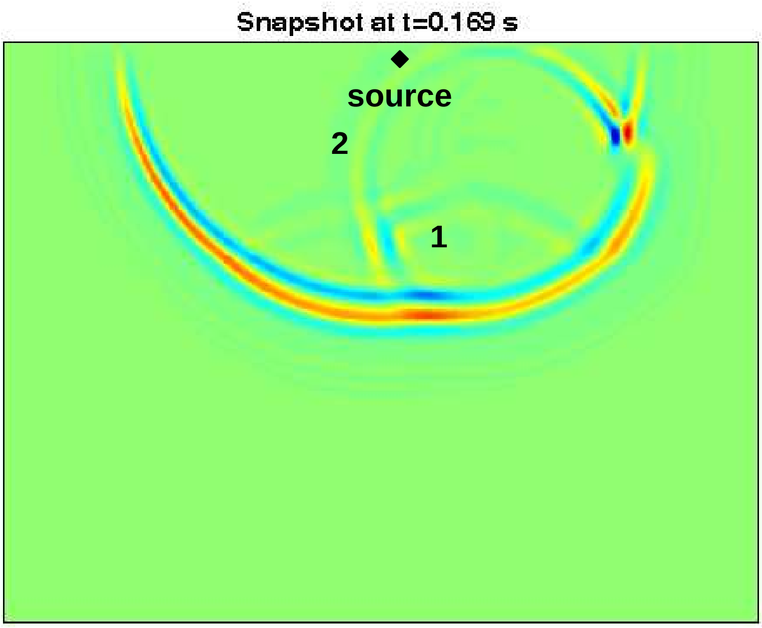

Figure 2. Wavefield snapshot at time t=0.169 s for the velocity structure of Figure 1.

3. PERFECTLY MATCHED LAYER

Absorbing boundaries are used to simulate infinite media in wave propagation models. Perfectly Matched Layers (PML) were proposed firstly by Berenger in 1995 [5] to simulate an infinite medium for electomagnetic waves. Then, the PML theory has been extended to acoustic and elastic problems. The first order PML implementation suggested by Liu [6] starts from

$$\begin{cases} \rho \dfrac{\partial \hat{v}}{\partial t} = -\hat{\nabla} p \\ \dfrac{\partial p}{\partial t} + \gamma c^2 p = -\rho c^2 \hat{\nabla} \cdot \hat{v} + f_s \end{cases}, \tag{7}$$

where p is the sound pressure, v the particle velocity, and f_s the source function ($s = \partial_t f_s$). Introducing the complex variable defined by

$$x \to x + \frac{i}{\omega} \int_{x_0}^{x} \sigma_x dx' \Rightarrow \frac{\partial}{\partial x} \to \frac{1}{1 + i\dfrac{\sigma_x}{\omega}} \frac{\partial}{\partial x}, \tag{8}$$

and Fourier transforming Eq. (7) to the frequency domain, it can be rewritten as

$$\begin{cases} -i\omega\rho\hat{v} = -\sum\limits_{\eta=x,z} \hat{\eta} \dfrac{1}{1 + i\dfrac{\sigma_\eta}{\omega}} \dfrac{\partial p}{\partial \eta} \\ -i\omega p + \gamma c^2 p = -\rho c^2 \sum\limits_{\eta=x,z} \dfrac{1}{1 + i\dfrac{\sigma_\eta}{\omega}} \dfrac{\partial v_\eta}{\partial \eta} + f_s \end{cases}. \tag{9}$$

Splitting of variables, and Fourier transforming back to the time-space domain, provides the first order equation system

$$\begin{cases} \rho \dfrac{\partial v_\eta}{\partial t} + \rho\sigma_\eta v_\eta = -\dfrac{\partial p}{\partial \eta} \\ \dfrac{\partial p^{(\eta)}}{\partial t} + (\gamma c^2 + \sigma_\eta) p^{(\eta)} + \sigma_\eta \gamma c^2 \int_{-\infty}^{t} p^{(\eta)} dt' = -\rho c^2 \dfrac{\partial v_\eta}{\partial \eta} \ldots, \\ \ldots + f_s^{(\eta)} + \sigma_\eta \int_{-\infty}^{t} f_s^{(\eta)} dt' \end{cases} \tag{10}$$

where $p^{(\eta)}$ and $f_s^{(\eta)}$ are defined by

$$\begin{cases} p = \sum\limits_{\eta=x,z} p^{(\eta)} \\ f_s = \sum\limits_{\eta=x,z} f_s^{(\eta)} \end{cases}, \tag{11}$$

and the superscript index η defines the x or z direction.

Figure 3 shows a snapshot of the propagated wavefield through a homogeneous medium with sound velocity equal to 1500 m/s, and a PML boundary condition on the left side. Note as waves incident at this boundary are almost fully absorbed, except for very small spurious reflections. The key point of our propagation model is how small are these reflections as compared to the scattering at the low contrast interfaces of a seismic oceanography experiment.

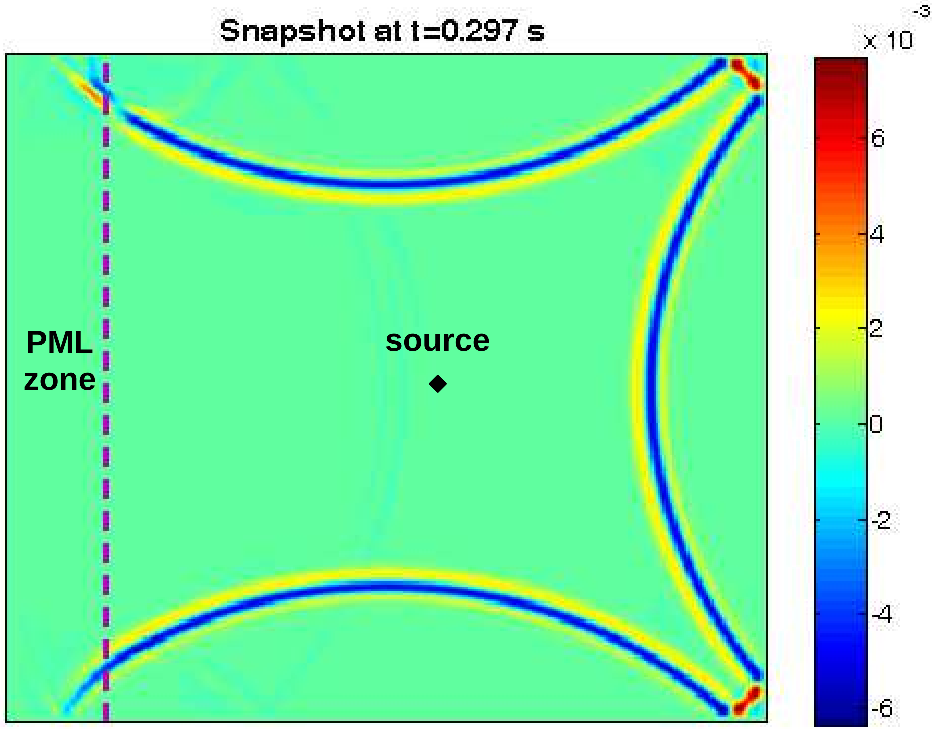

Figure 3. Wavefield snapshot at t= 0.297 s, with totally reflecting boundaries except for a PML zone on the left side.

4. NUMERICAL RESULTS

In this section we will present the propagation of acoustic waves through a realistic oceanic environment. The acoustic source is a Ricker wavelet with a central frequency of 60 Hz. The propagation algorithm is of fourth order in time and second order in space with a three point scheme. A 1D stratified medium is considered with the sound velocity profile shown in Figure 4. This velocity profile is characteristic of the Cadiz Gulf (Spain) when a mass of warm Mediterranean water penetrates into the cold Atlantic water. As mentioned above, density is constant in the medium. The medium is surrounded on left and right sides by PML zones. Pressure-release conditions are assumed for the upper and lower interfaces.

Figure 5 shows a snapshot of the pressure wavefield. As it can be seen, the acoustic wave is fully absorbed by the left PML zone, so that there are not spurious reflections from this interface. On the right part, however, small reflections can be noted due to the PML zone behaves as a not totally absorbing boundary (see the small reflections inside the ellipse in Figure 5.).

Figure 6 illustrates the seismogram that would be recorded by a streamer of hydrophones at depth of 20 m in the Cadiz Gulf area. The intrusion of Mediterranean water into the Atlantic water arose reflection coefficients of the order of 10^{-4}. Notice as reflections on the water interfaces are clearly seen in the seismogram, unlike spurious boundary reflection, which are small enough to be undistinguishable.

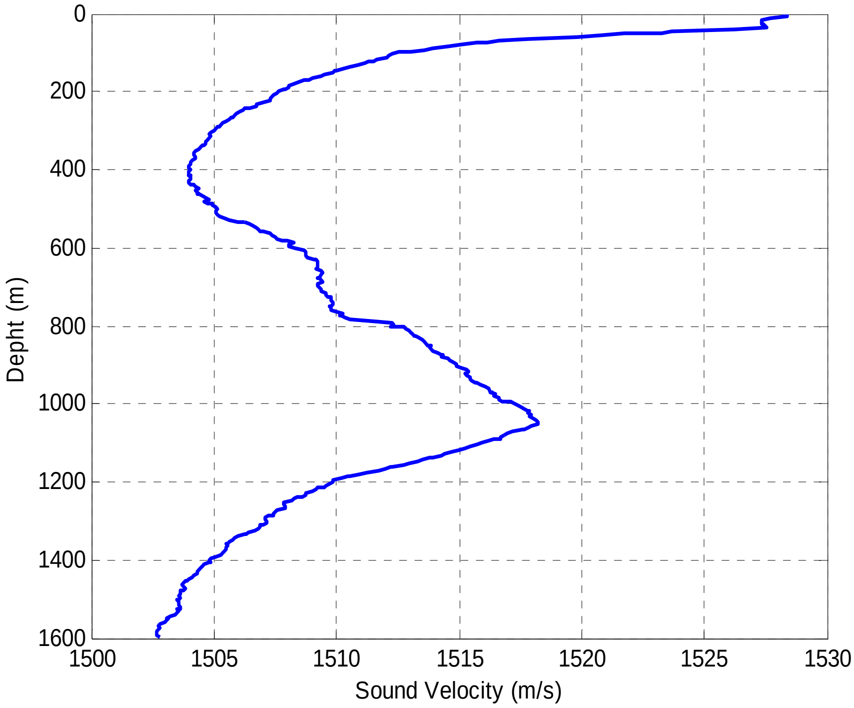

Figure 4. Sound speed profile of the studied ocean.

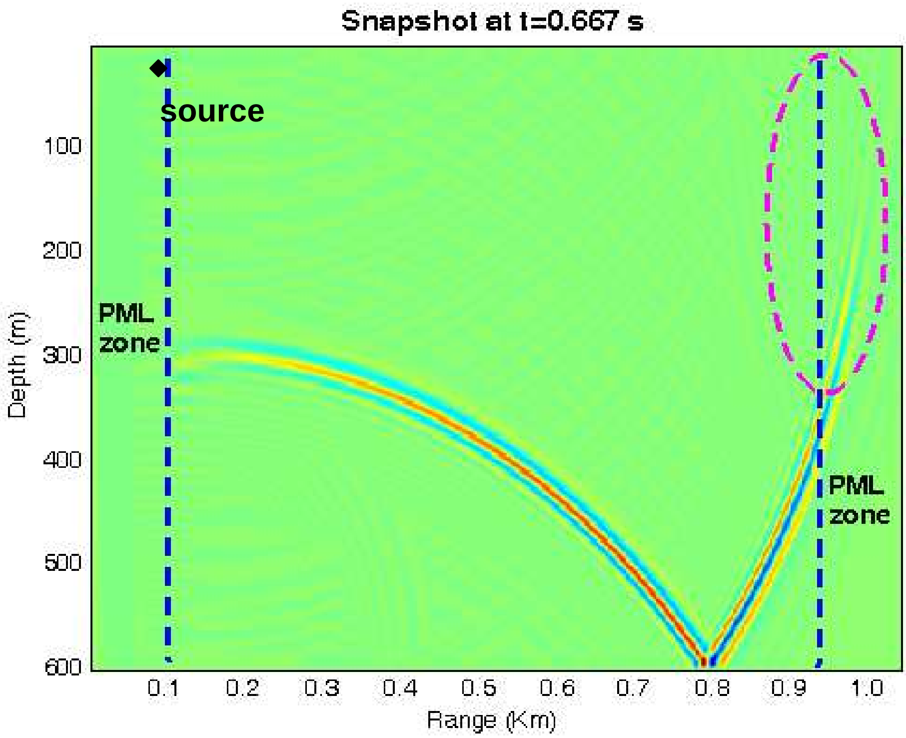

Figure 5. Wavefield snapshot in the ocean with velocity profile of Figure 4.

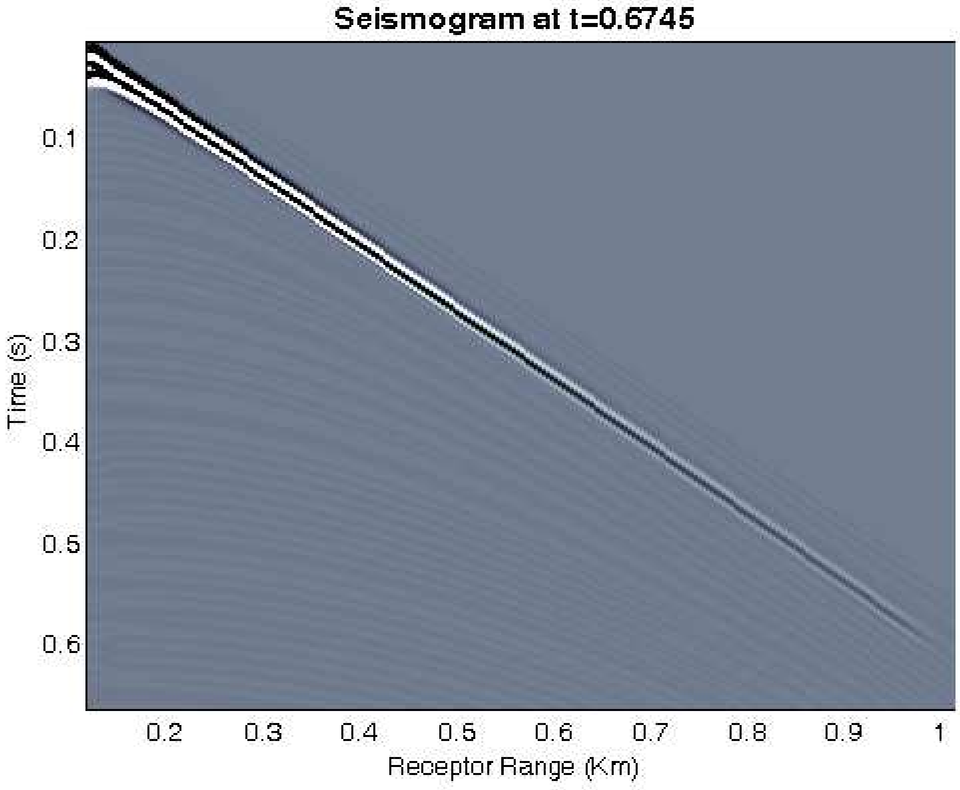

Figure 6. Seismogram at 20 m depth for the medium with velocity profile of Figure 4.

5. CONCLUSIONS

In this paper we analysed the efficiency of the PML boundary condition in a propagation model to simulate seismic oceanography experiments. A first order PML combined with fourth order finite-difference scheme is able to image reflections in the low contrast interfaces in the water column. Some spurious reflections at the boundaries are still observed in the simulated wavefields, but they are smaller than the reflections in the water column interfaces. Our current research tries to reduce further these remaining spurious reflections by using spherical coordinates.

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