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THE ACOUSTIC WAVE PROPAGATION EQUATION IN A TURBULENT COMBUSTING FLOW

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Abstract

Sound generation by turbulent flames originates from the fluctuating heat release in the flame. The description of this fluctuating heat release and its effect on acoustics in turbulent flames is complicated due to the interaction of chemical reactions with turbulence, mixing and pressure fluctuations. In a turbulent flame the instantaneous density, velocity, pressure, temperature and species concentrations are determined by the transport equations for mass, momentum, enthalpy and species and by the equation of state. In the proposed paper an equation is formulated that describes the propagation of acoustic pressure fluctuations, and that determines the source terms. Subsequently events are ordered on basis of their typical time scale. That way source terms can be evaluated for the situation where the combustion is described with the use of time averaged chemical reaction progress variables and a mixture fraction variable. Subsequently the consequences of Reynolds and Favre averaging on these source terms and conservation of acoustical variables in a domain with turbulent flow are discussed. The use of the reaction progress variables, as a basis for acoustic propagation prediction, in a Reynolds averaged Navier-Stokes flow calculation, is demonstrated on a premixed turbulent natural gas flame with finite combustion kinetics.

1. INTRODUCTION

Acoustic effects can dramatically influence the lifetime and combustion performance in a gas turbine combustor. An important side effect of a turbulent combustion process is the radiation of a sound field from the reaction zone. This is called combustion roar and occurs in all turbulent combustion processes. The flame is a source of sound and thus influences the acoustic behaviour of the combustor and its related components. Apart from radiating sound, a flame also acts as an amplifier of sound ([1],[2]). Roar spontaneously generated by the interaction of turbulent flow oscillations and combustion processes shows a broad band noise spectrum. Fortunately only a small part of the thermal power of a flame is converted into acoustic energy and efficiencies are between 10^{-3} and 10^{-8} for the conversion of chemical

energy input to acoustic energy [3]. But as the thermal power is of the scale of MW, the acoustic power generated can still be extremely high. Noise modeling efforts for non-premixed turbulent combustion have been undertaken by Klein and Kok [1]. Their results show good agreement with experimental data. Boineau and Gervais [4] modeled noise generation of turbulent non-premixed flames in a similar way and also obtained good results. For turbulent premixed combustion no recent work on noise generation was found. In this paper the acoustic source term due to combustion will be derived with application on turbulent fluctuations.

2. THE ACOUSTIC WAVE PROPAGATION EQUATION IN A NON-QUIESCENT FLUID

Sound generation by turbulent flames originates from the fluctuating heat release in the flame. The description of this fluctuating heat release in turbulent flames is complicated due to the interaction of turbulence, mixing, combustion and noise. In a turbulent flame the instantaneous density, velocity, pressure, temperature and species concentrations are determined by the transport equations for mass, momentum, enthalpy and species and by the equation of state. Following the method by Sir James Lighthill [5], the equations for mass and momentum can be combined to one equation in pressure and density. This equation relates instantaneous changes in pressure and density to fluctuations in velocity:

$$\frac{\partial^2 \rho}{\partial t^2} - \nabla^2 P = \nabla^2 [(\rho \underline{u}\underline{u}) - \underline{\underline{\tau}}] \tag{1}$$

Defining the excess density:

$$\rho_e = (\rho - \rho_0) - \frac{P - P_0}{c_0^2} \tag{2}$$

The following well known pressure fluctuation propagation equation is obtained:

$$\frac{\partial}{\partial t}\left(\frac{1}{c_0^2}\frac{\partial P}{\partial t}\right) - \nabla^2 P = \nabla^2 [(\rho \underline{u}\underline{u}) - \underline{\underline{\tau}}] - \frac{\partial^2 \rho_e}{\partial t^2} \tag{3}$$

It can be observed that eq (3) describes the propagation of pressure fluctuations at the wave speed c_0, at reference conditions. This equation contains source terms due to variations of the gradient of momentum transport and fluid stress. The most interesting source term here is the 3rd rhs, which represents the driving effect of density fluctuations, due to for example combustion. This term has to be written in a more explicit way, in order to be able to analyze and compute it. Rewriting with the use of several thermodynamic definitions and transport equations leads to:

$$\frac{\partial}{\partial t}\left(\frac{1}{c^2}\frac{\partial P}{\partial t}\right) - \nabla^2 P =$$

$$\tag{4}$$

$$\nabla^2[(\rho \underline{u}\underline{u}) - \underline{\underline{\tau}}] - \frac{\partial}{\partial t}\left(\frac{\alpha}{c_P}\left(\rho \sum_{i=1}^{n}\left(\left(\frac{\mu_i}{W_i} + y_i\right)\frac{Dy_i}{Dt}\right) + \nabla.\underline{q} + (\tau:\nabla\underline{v})\right)\right) + \frac{\partial}{\partial t}\left((\underline{u}\nabla)\left(\rho - \frac{P}{c^2}\right)\right)$$

It can be observed in eq (4), that here the pressure fluctuations propagate at the *local* speed of sound.

For thermoacoustics only the 1st and the 2nd from the right source terms at the rhs of eq (4) are important. These source terms are explored now systematically in sections 3 and 4.

3. FLAME FRONT OSCILLATION AND ACOUSTICS

A very interesting term in a gaseous system where combustion occurs is the first rhs term, when numbered form right to left. This term is:

$$\frac{\partial}{\partial t}\left(\left(\underline{u}\nabla\right)\left(\rho - \frac{P}{c^2}\right)\right) \tag{5}$$

In a gas the density and pressure are related. This is used to rewrite this source term as follows.

$$\frac{\partial}{\partial t}\left(\left(\underline{u}\nabla\right)\left(\rho - \frac{P}{c^2}\right)\right) = \frac{\partial}{\partial t}\left(\left(\underline{u}\nabla\right)\left(\rho\left(1 - \frac{1}{c^2}\frac{P}{\rho}\right)\right)\right) \tag{6}$$

In a perfect gas the equation of state gives, when combined with the expression for the speed of sound:

$$\frac{P}{\rho} = RT = \frac{c^2}{\gamma} \tag{7}$$

When substituted in the source term this gives:

$$\frac{\partial}{\partial t}\left(\left(\underline{u}\nabla\right)\left(\rho - \frac{P}{c^2}\right)\right) = \frac{\partial}{\partial t}\left(\left(\underline{u}\nabla\right)\left(\rho\left(1 - \frac{1}{c^2}\frac{P}{\rho}\right)\right)\right) = \frac{\partial}{\partial t}\left(\left(\underline{u}\nabla\right)\left(\rho\left(\frac{\gamma-1}{\gamma}\right)\right)\right) \tag{8}$$

Neglecting the change of γ with temperature:

$$\frac{\partial}{\partial t}\left(\left(\underline{u}\nabla\right)\left(\rho - \frac{P}{c^2}\right)\right) = \frac{\partial}{\partial t}\left(\left(\underline{u}\nabla\right)\left(\rho\left(1 - \frac{1}{c^2}\frac{P}{\rho}\right)\right)\right) = \frac{\gamma-1}{\gamma}\frac{\partial}{\partial t}\left(\left(\underline{u}\nabla\right)\rho\right) \tag{9}$$

This term is significant in a fluctuating flame. In a flame there is a steep gradient in density over the combustion front. If the flame is fluctuating, and hence the flame front is wobbling, this term will represent a source term in the thermo acoustic propagation equation. The effect of a motion of the flame front with time will hence be felt primarily through the term represented by eq. (9). Investigated will now be the effect of a flow oscillation induced by turbulence or a varying mass flow on the source term in eq (9).

In general in a reaction progress formulation the local density as determined by chemistry and at a given constant background pressure P_0 will depend on the progress variables c_i and the mixture fraction f:

$$\rho_0 = \rho_0(f,c_i) \tag{10}$$

The total differential for the density (using the Einstein convention of summation) is then:

$$\nabla \rho_0 = \left(\frac{\partial \rho_0}{\partial f}\right)_{c_i} \nabla f + \left(\frac{\partial \rho_0}{\partial c_i}\right)_f \nabla c_i \tag{11}$$

This leads finally to the expression for the source term in terms of mixture fraction and progress variable, when the reference density in eq (10) is assumed to be not effected by pressure fluctuations, and second order fluctuation terms are neglected:

$$\frac{\partial}{\partial t}\left((\underline{u}\nabla)\left(\rho - \frac{P}{c^2}\right)\right) = \frac{\gamma-1}{\gamma}\left(\frac{\partial \rho_0}{\partial f}\right)_{c_i} \frac{\partial}{\partial t}\left[(\underline{u}\nabla)f\right] + \frac{\gamma-1}{\gamma}\left(\frac{\partial \rho_0}{\partial c_i}\right)_f \frac{\partial}{\partial t}\left[(\underline{u}\nabla)c_i\right] \tag{12}$$

In case of a premixed flame, the factor $\left(\frac{\partial \rho_0}{\partial c_i}\right)_f$ will vanish in the entire domain, with the exception of the location where the chemical reactions take place, and the progress variable is different from its equilibrium value unity, while mixture fraction is in the ignition range. In case of a diffusion flame, the factor $\left(\frac{\partial \rho_0}{\partial f}\right)_{c_i}$ will vanish in the entire domain, with the exception of the location where the chemical reactions take place, near the stoichiometric point, where mixture fraction is in the ignition range. Hence this term is significant, and only then, at the flame front location. In this area, the gradients in progress variable and mixture fraction will be large too.

4. CHEMICAL REACTION SOURCE TERM IN THE ACOUSTIC PROPAGATION EQUATION

In the equation for the propagation of acoustics the 2nd rhs source term can be recognized to represent the effect of the chemical reactions in a combusting flow:

$$-\frac{\partial}{\partial t}\left(\frac{\alpha}{c_P}\left(\rho\sum_{i=1}^{n}\left(\left(\frac{\mu_i}{W_i} + y_i\right)\frac{Dy_i}{Dt}\right)\right)\right) \tag{13}$$

Following the approach by Kok et al. in [6], this term can be rewritten to a reaction progress variable approach, projecting individual mass fractions on a mixture and progress variable dependence, like eq (10):

$$y_i = y_i(f, c_i) \tag{14}$$

The total differential for the species mass fractions is then in case of n progress variables:

$$dy_i = \left(\frac{\partial y_i}{\partial f}\right)_{c_i} df + \sum_{j=1}^{n} \left(\frac{\partial y_i}{\partial c_j}\right)_f dc_j \tag{15}$$

When substituted into the source term in eq (13) this becomes:

$$-\frac{\partial}{\partial t}\left(\frac{\alpha}{c_P}\left(\rho\sum_{i=1}^{n}\left(\frac{\mu_i}{W_i}+y_i\right)\frac{Dy_i}{Dt}\right)\right)=$$

$$-\frac{\partial}{\partial t}\left(\frac{\alpha}{c_P}\left(\sum_{i=1}^{n}\left(\frac{\mu_i}{W_i}+y_i\right)\left(\left(\frac{\partial y_i}{\partial f}\right)_{c_i}\rho\frac{Df}{Dt}+\sum_{j=1}^{n}\left(\frac{\partial y_i}{\partial c_j}\right)_f\rho\frac{Dc_j}{Dt}\right)\right)\right)$$

(16)

As formulated in [7], a generalized definition for a composed species mass fraction $\hat{y}_i$ for N species can be taken as:

$$\hat{y}_i=\sum_{j=1}^{N}b_{ij}y_j$$

(17)

The progress variable can then be defined as:

$$c_i=\frac{\hat{y}_i-\hat{y}_i^u}{\hat{y}_i^b-\hat{y}_i^u}=\frac{\hat{y}_i-\hat{y}_i^u}{\hat{W}_i}$$

(18)

It follows then that

$$dc_i=\frac{1}{\hat{W}_i}d\hat{y}_i=\sum_{j=1}^{n}\frac{b_{ij}}{\hat{W}_i}dy_j \qquad \text{and:} \qquad \left(\frac{\partial y_i}{\partial c_j}\right)_f=\frac{\hat{W}_i}{b_{ij}}$$

(19)

And the source term becomes:

(20)

$$-\frac{\partial}{\partial t}\left(\frac{\alpha}{c_P}\left(\rho\sum_{i=1}^{n}\left(\frac{\mu_i}{W_i}+y_i\right)\frac{Dy_i}{Dt}\right)\right)=-\frac{\partial}{\partial t}\left(\frac{\alpha}{c_P}\left(\sum_{i=1}^{n}\left(\frac{\mu_i}{W_i}+y_i\right)\left(\left(\frac{\partial y_i}{\partial f}\right)_{c_i}\rho\frac{Df}{Dt}+\sum_{j=1}^{n}\frac{\hat{W}_i}{b_{ij}}\rho\frac{Dc_j}{Dt}\right)\right)\right)$$

For a perfect gas this can be worked out as:

$$-\frac{\partial}{\partial t}\left(\frac{\alpha}{c_P}\left(\rho\sum_{i=1}^{n}\left(\frac{\mu_i}{W_i}+y_i\right)\frac{Dy_i}{Dt}\right)\right)=$$

$$-\frac{\partial}{\partial t}\left(\left(\frac{\gamma-1}{c^2}\sum_{i=1}^{n}\left(\frac{\mu_i}{W_i}+y_i\right)\left(\left(\frac{\partial y_i}{\partial f}\right)_{c_i}\rho\frac{Df}{Dt}+\sum_{j=1}^{n}\frac{\hat{W}_i}{b_{ij}}\rho\frac{Dc_j}{Dt}\right)\right)\right)$$

(21)

In case the rate of heat release is equal to:

$$\dot{Q}=\sum_{i=1}^{n}\frac{\mu_i}{W_i}\left(\left(\frac{\partial y_i}{\partial f}\right)_{c_i}\rho\frac{Df}{Dt}+\sum_{j=1}^{n}\frac{\hat{W}_i}{b_{ij}}\rho\frac{Dc_j}{Dt}\right)$$

(22)

Then the source term for the acoustics can be simplified to:

$$-\frac{\partial}{\partial t}\left(\frac{\alpha}{c_P}\left(\rho\sum_{i=1}^{n}\left(\frac{\mu_i}{W_i}+y_i\right)\frac{Dy_i}{Dt}\right)\right) \approx -\frac{\partial}{\partial t}\left(\left(\frac{\gamma-1}{c^2}\dot{Q}\right)\right) \tag{23}$$

It is assumed here that $\frac{\mu_i}{W_i} >> y_i$. This is correct for most fuels, which are hydrocarbons, but not for hydrogen (!). For hydrogen the Gibbs potential is zero.

In case of a premixed flame, eq (18) can be evaluated more specifically. For a premixed flame, the gas mixture composition can be divided in a type 'unburned' and a type 'burned' mixture, at constant mixture fraction. The source term in eq. (18) is then calculated from the net release of thermal energy, assuming a constant value of the specific heat:

$$\sum_{i=1}^{n}\frac{\mu_i}{b_i W_i} = c_p(T_{burnt} - T_{unburnt}) \tag{24}$$

This gives for the thermo acoustic source term of eq (18):

$$-\frac{\partial}{\partial t}\left(\frac{\gamma-1}{c^2}(T_{burnt} - T_{unburnt})(y_{burnt} - y_{unburnt})\cdot\frac{Dc}{Dt}\right) \tag{25}$$

Application of the TFC combustion model [8] will yield the following term for the source term Dc/Dt:

$$\frac{Dc}{Dt} = \rho_{unburnt}C_f u'^{3/4} S_L^{1/2} \chi^{-1/4} l_t^{1/4} |\nabla c| \tag{26}$$

In this term C_f is a constant equal to 0.52, u' is the mean turbulent velocity fluctuation, S_L is the laminar flame speed depending on the relevant combustion conditions, χ is the thermal diffusivity of the mixture, l_t is the turbulent length scale.

5. NUMERICAL SIMULATIONS

Results in progress of the modelling work on the thermo acoustic in the previous sections are presented here. The results are obtained using CFX 5.6 with the TFC combustion model [10] for perfectly premixed mixtures. With user defined subroutines all data necessary with respect to eqs.(20-21) were retrieved from the steady state simulation solution. Subsequently the acoustic source terms in the combustor were computed in a post processing code. The geometry for this study is a generic swirl combustor in a 100 mm x100 mm square cross section and a combustor length of 1800 mm, used in an experimental setup at our laboratory, see [9], for a detailed description of the setup. The combustor is operated with air preheated to 300ºC and mixed prior to combustor entry with natural gas. The air factor was 1.8. The pressure is atmospheric and the thermal power is 100 kW. The Reynolds number of this setup is around 45,000. The CFD calculations were set up using the k-ε turbulence model, taking into account a quarter of the burner meshed with 527,018 elements, using periodic boundary conditions.

The predicted velocity field is presented in Fig. 1 by a vector plot. Clearly it can be observed that the premixed gas/air flow emerging from the burner at the top rhs of the figure, develops a central recirculation area and a minor area of recirculation in the corner of the combustor. In Fig. 2 the predicted field of the reaction progress variable c is presented. The reaction progress variable increases from zero at the burner inlet to a transition zone of about 0.5 to 0.7 at the flame front at the edge of the recirculation area. Subsequently the reaction

variable progresses to a burnt situation with c equal to unity. The predicted field of the source term of the reaction progress variable as presented in equation (26) is depicted in Fig. 3. The chemical source term can be observed to be zero in the burner inlet flow and to be large at the two combustion interfaces, at the side in contact with the inner and at the side in contact with the outer recirculation area.
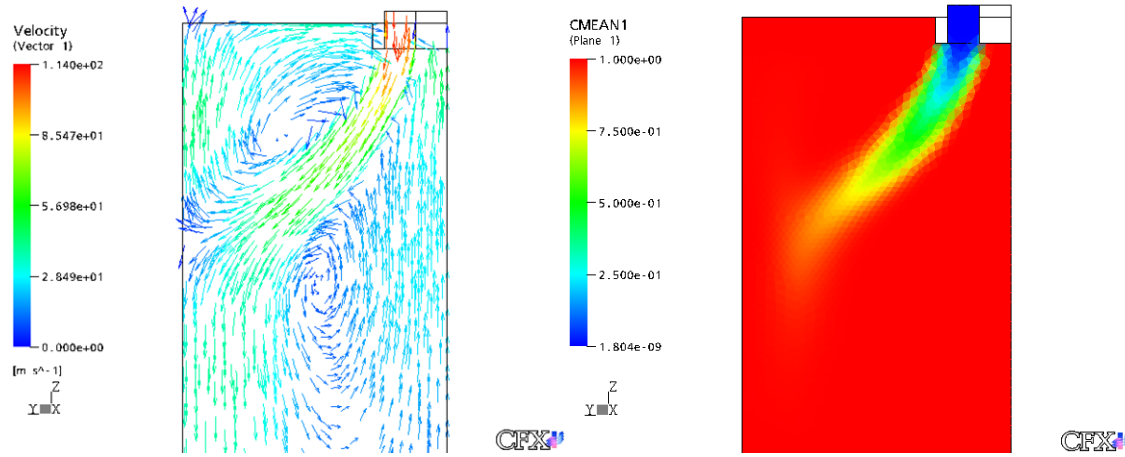

Fig. 1 left: Vector plot of the predicted velocity field.

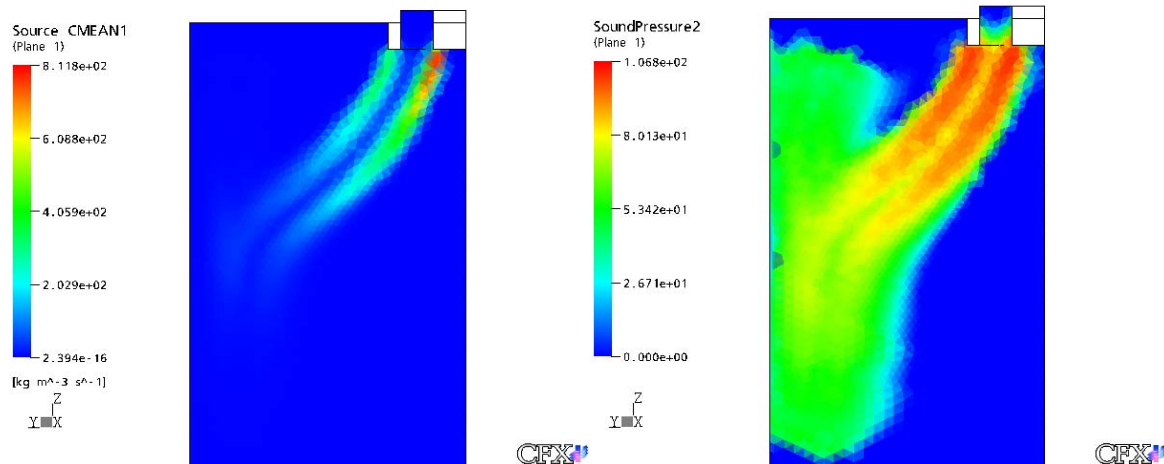Fig. 2 right: The predicted field of the reaction progress variable

Fig. 3 left: The predicted field of the source term of the reaction progress variable.

Fig. 4 right: The field of the predicted sound pressure specific source term..

The predicted field of the volume specific acoustic source pressure on basis of eq. (25) is shown in Fig. 4. The source term can be observed to be very high at the areas where combustion occurs, and is also at a considerable level in a large area downstream the flame front, where the reaction progress variable develops slowly to the burnt equilibrium situation. Here the material derivative of the progress variable, and hence the acoustic source term is about an order of magnitude smaller than in the flame front, but present over a large area. In view of this the flame shows two significant contributions: one compact at the front and one distributed downstream the front. This leads to the conclusion that the flame front oscillations,

that were neglected here, might have a significant contribution too, as described in eq. (12). This is being pursued in continuing research.

6. CONCLUSION

The acoustic source terms induced by turbulent combustion were derived. On basis of a background pressure and density, the driving effect of chemical reaction was related to temporal and spatial changes of the mixture fraction and the progress variables, at the time scale of the acoustic pressure fluctuations. Two terms were identified, one due to flame front oscillations, and one due to chemical reaction. In a simulation of a premixed flame the chemical source term and the affiliated acoustic source term were determined. Clearly a compact and a distributed source domain could be observed. The large gradients in reaction progress at the flame front hint at a possible significant contribution of the flame front oscillation source term.

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REFERENCES

[1] Kok, J.B.W., Klein, S.A., 1999, "Sound Generation by turbulent non-premixed flames", Proc. Int. Congress on Sound and Vibration, **99**, 3385.

[2] Klein, S.A., 2000, "On the acoustics of turbulent non-premixed flames", Ph.D.Thesis, University of Twente, Enschede, NL.

[3] Putnam, A., Faulkner L., 1983, "An overview of combustion noise", J. of Energy, Vol. 7, no 6. pp 458-469.

[4] Boineau, P., Gervais, Y., 1998 , "Determination du spectre de puissance acoustique émis par une flamme turbulente", CR.Acad. Sci. Paris, **326**, Série IIb, pp. 747-754.

[5] Crighton, D.G, Dowling, A.P., Ffowcs Williams J.E., Heckl, M., Leppington. F.G., 1992, "Modern methods in analytical acoustics", Lecture notes, Springer-Verlag, London.

[6] Kok , J.B.W., Van den Hoon, C.G.J., in print, "Modeling turbulent premixed CH_4 flames with flow recirculation using reaction progress variables", in "Progress in combustion research", ed. F. Columbus, Nova Science Publishers, Hauppauge, New York, US.

[7] Jager, B. de, Kok, J.B.W., Skevis G., The effects of water addition on pollutant formation from lpp gas turbine combustors, Proc. 31st Int. Symp. on Comb., Heidelberg, 2006.

[8] Zimont, V., Polifke, W., Bettelini, M., Weisenstein, W., 1998, "An efficient computational model for premixed turbulent combustion at high Reynolds numbers based on a turbulent flame speed closure" (97-GT-395). J. Eng. Gas Turb. and Pow., **120**, pp. 526-532.

[9] Van Kampen, J.F., 2006, "Acoustic pressure oscillations induced by confined turbulent premixed natural gas flames", PhD Thesis, University of Twente, Enschede, NL.