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A GEOMETRIC APPROACH TO THE OPTIMAL DESIGN OF REMOTELY LOCATED BROADBAND VIBRATION CONTROL SYSTEMS

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Abstract

Over the past three decades, a wide variety of active control methods have been proposed for controlling problematic vibration. The vast majority of approaches make the implicit assumption that sensors or actuators can be located in the region where vibration attenuation is required. However, for many large scale structures or where the system environment is harsh, this is either not feasible or prohibitively expensive. As a result, the optimal control of local vibration may lead to enhancement at remote locations. Motivated by such problems in marine system environments, a geometric methodology that provides an approach for defining the design freedom available for reducing vibration both at local and remote locations has been previously been proposed by the authors. In an earlier paper, the fundamental results were used to develop design procedures for discrete frequency control; in the current paper, however, the focus is on design procedures for broadband control. A systematic approach is developed that provides an additional design constraint to the geometric methodology to ensure that the resulting compensator provides closed loop stability. The design procedure is illustrated using a test rig that has been built to replicate the problems associated with the control of rotor blade vibration.

1. INTRODUCTION

Active vibration controller design has reached a high level of maturity following three decades of intensive development and a number of design methods are now well established ([1]-[6], for example). The majority of approaches assume implicitly that sensing and actuation can be located in the region where vibration attenuation is ultimately required, however, for many applications this is not feasible. Although it may be possible to recover the lost sensor information with an observer, for example, direct forces often cannot be applied. Such problems are particularly evident in large scale interconnected structures where either the environmental conditions do not allow a wide distribution of sensors and actuators or it is prohibitively expensive to do so. Design based solely on information local to the actuators can result in increased levels of vibration at remote locations. The implementation of a locally sub-optimal solution is therefore often necessitated for the attainment of a globally optimal one.

The feasibility of controlling remote vibration using only local sensing and actuation has previously been studied by the authors [7]. A geometric design methodology for discrete frequency (or harmonic) control has been proposed, where the design freedom available for providing both local and remote vibration attenuation is parameterised. In this paper, design procedures for broad-band control are presented. Specifically, the problem of disturbance attenuation over an arbitrary frequency band is considered. It is found that an optimal broad-band geometric controller can be designed via a linear matrix inequality (LMI) approach. Remarkably, further design freedom can be defined allowing the designer to tune the performance of the optimal geometric controller, in relation to the performance outside the targeted frequency band. The design methodology is illustrated through the application to a laboratory scale test rig that has been developed to replicate the generic problems associated with the propagation of rotor blade vibration.

2. BROAD-BAND GEOMETRIC CONTROLLER DESIGN

2.1 Preliminaries

It is assumed that the vibrating system can be described by the following frequency response function (FRF):

$$\begin{bmatrix} y(jw) \\ z(jw) \end{bmatrix} = \begin{bmatrix} g_{11}(jw) & g_{12}(jw) \\ g_{21}(jw) & g_{22}(jw) \end{bmatrix} \begin{bmatrix} u(jw) \\ d(jw) \end{bmatrix} \tag{1}$$

Where $y(jw)$, $z(jw)$, $u(jw)$ and $d(jw)$ represent the locally measured vibration, the remote vibration, the control force and the disturbance force respectively; $g_{11}(jw)$, $g_{12}(jw)$, $g_{21}(jw)$, $g_{22}(jw)$ are the corresponding FRFs. The control aim is to achieve reductions in both $y(jw)$ and $z(jw)$ (where possible) over an arbitrary frequency band $[w_1, w_N]$ through the application of the feedback control law:

$$u(jw) = -k(jw)y(jw) \tag{2}$$

Although a measurement of $z(j\omega)$ is not available during implementation, it is assumed that the transfer function matrix:

$$G = \begin{bmatrix} g_{11}(jw) & g_{12}(jw) \\ g_{21}(jw) & g_{22}(jw) \end{bmatrix} \tag{3}$$

can be obtained during a commissioning phase.

2.2 Introduction to Geometric Controller Design

Denote the sensitivity at a discrete frequency $w = w_0$ by $S(jw_0)$. By defining the following equations:

$$\alpha(jw_0) = S(jw_0) - 1 \tag{4}$$

$$\alpha(jw_0) = \beta(jw_0)g(jw_0) \tag{5}$$

Where: $g(jw_0) = \dfrac{g_{11}(jw_0)g_{22}(jw_0)}{g_{12}(jw_0)g_{21}(jw_0)}$.

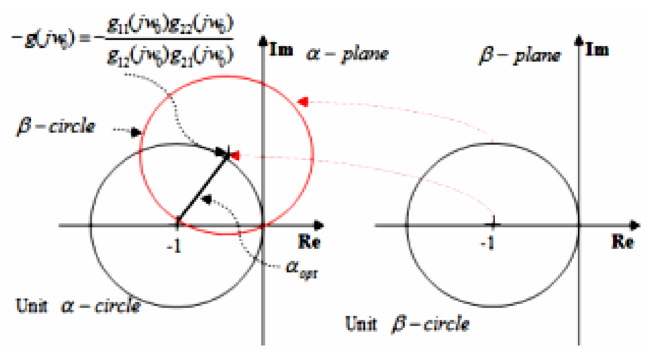
it can be shown [7] that reduction in $y(jw)$ and $z(jw)$ for the discrete frequency $w = w_0$ is equivalent to satisfying the following conditions, respectively:

$$|\alpha(jw)+1| < 1 \tag{6}$$

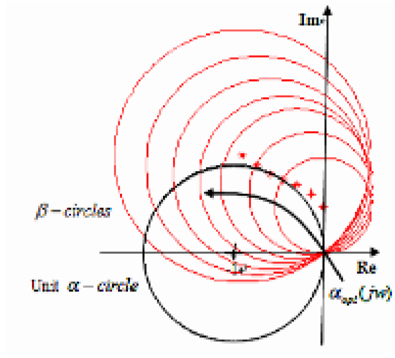
$$|\beta(jw)+1| < 1 \tag{7}$$

From the observation that equation (5) relating $\alpha(jw)$ and $\beta(jw)$ defines a Möbius transformation, it is clear that the mapping of equation (7) on complex $\alpha - plane$ is a circle (and its interior) with centre at $-g(jw_0)$ and radius $|g(jw_0)|$. Hence it can be concluded that simultaneous reduction of $y(jw)$ and $z(jw)$ is achievable for this discrete frequency $w = w_0$ if and only if the mapping of the unit circle (and its interior) $|\beta(jw)+1| < 1$ on the complex $\alpha - plane$ intersects the unit α circle (and its interior) $|\alpha(jw)+1| < 1$, and the corresponding reduction is the scaling with respect to each circle. Finally it is noted that an optimal line jointing (-1, 0) with ($-g(jw_0)$) on the complex $\alpha - plane$ can be defined and desired levels of attenuation can be obtained by choosing an appropriate point on the line, e.g. a choice at point $-g(jw_0)$ provides infinite attenuation in $z(jw_0)$.

The aim of reducing $y(jw)$ and $z(jw)$ over an arbitrary frequency band $[w_1, w_N]$ can be approached on a frequency-by-frequency basis. However, it is noted that the optimal choice α_{opt} is to be varied from one frequency to the next over the desired frequency band $w \in [w_1, w_N]$ and this results in an optimal trajectory on the complex $\alpha - plane$. These observations are illustrated in Fig. 1.



(a)



(b)

Figure 1(a): Mapping of $|\beta(jw)+1| < 1$ on the complex $\alpha - plane$ (b): Optimal solution $\alpha_{opt}(jw)$ on the complex $\alpha - plane$ for broadband control.

However it is not true to state that a compensator then exists that can provide simultaneous reduction across an arbitrarily wide range of frequencies, since closed loop stability is not considered. It turns out (Proposition 5 in [7]) that *internal stability will be guaranteed if $\alpha_{opt}(jw)$ is a mapping of a stable function*. As a result the optimal broad-band geometric controller can be obtained from $k(s) = -\dfrac{\alpha(s)}{[\alpha(s)+1]g_{11}(s)}$, which follows from the definition of the sensitivity. Thus the question that needs to be answered is how to find such a stable transfer function $\alpha(s)$ given the optimal choice α_{opt} over any targeted frequency band $[w_1, w_N]$? This is considered in the following section.

2.3 Obtaining an Optimal Broad-Band Geometric Controller via LMIs

Given the data points resulting from the optimal choice α_{opt} over $[w_1, w_N]$, the problem of fitting a stable transfer function $\alpha(s)$ into these points consists of two sub-problems: one is the existence problem, or the possibility of finding a stable transfer function $\alpha(s)$ that interpolates the data points; the other is the optimization problem, namely finding the best approximation to the given data points in the event of a failure to provide exact interpolation.

The existence problem turns out to be a Nevanlinna-Pick interpolation problem that widely occurs in robust identification, signal processing and circuit theories [8,9]. The answer to it is provided by a modified Pick condition as shown in [10]:

Existence Problem: A stable transfer function $\alpha(s)$ that interpolates the optimal choice α_{opt} over $[w_1, w_N]$ exists if and only if the following Pick matrix P

$$P = \left[\frac{1 - \alpha_k \overline{\alpha_l} / M^2}{j(w_k - w_l) + 2a} \right]_{1 \le k, l \le N} \tag{8}$$

is positive definite.

Where α_i is the optimal choice $\alpha(jw_i)$ for frequency w_i $\forall i \in [1, N]$; a and M are positive real numbers defining the minimal degree of stability and maximum modulus of $\alpha(s)$ on the half plane $\Re(s) \ge a$ (or more accurately $M \ge \sup_{\Re(s) \ge a} |\alpha(s)|$).

Optimization Problem: When the Pick matrix above fails to be positive definite, there does not exist a stable transfer function $\alpha(s)$ that interpolates exactly through the optimal choice $\alpha(jw_i)$ $\forall i \in [1, N]$. The problem of finding the best approximation to the data points proves to be provided by a series of Linear Matrix Inequalities (LMIs). This is obtained in [10] by defining $T_0 = \left[\frac{1}{j(w_k - w_l) + 2a} \right]_{1 \le k, l \le N}$ and $Q = blockdiag\ (\frac{\alpha_1}{M}, \ldots, \frac{\alpha_N}{M})$, and then the Pick Matrix P can be rewritten as:

$$P = T_0 - Q T_0 Q^* \tag{9}$$

Where * is the Hermitian operator. Following from the Schur complement lemma, P is positive definite if and only if

$$\begin{bmatrix} T_0 & Q \\ Q^* & T_0^{-1} \end{bmatrix} > 0 \tag{10}$$

Equation (10) together with those LMIs defining the uncertainty around each data points α_i for frequency w_i, when presented with an LMI solver [11] provides the best approximation to the optimal choice $\alpha(jw_i)$ over the frequency band $[w_1, w_N]$. The desired stable transfer function $\alpha(s)$ can then be obtained from the interpolation data (see [12] for its transfer function or state-space representation). By the definition of sensitivity the broad-band geometric controller is thus $k(s) = -\dfrac{\alpha(s)}{[\alpha(s)+1]g_{11}(s)}$.

3. PERFORMANCE TUNING

For any design methodology it can be very important to provide additional freedom to tackle objectives beyond the main design goal. But it is also important to be able to describe the effects of design freedom *in terms of system performance*. The optimal choice α_i defines directly the levels of attenuation in $y(jw)$ and $z(jw)$. It is also noted that the Pick condition introduces additional design parameters, namely the minimal degree of stability a and the maximum modulus M of $\alpha(s)$ on the half plane $\Re(s) \geq a$. Their introduction arises from the determination of the existence of a stable transfer function $\alpha(s)$ that interpolates exactly through the optimal choice $\alpha(jw_i)$ $\forall i \in [1, N]$. The consequence is that a stable $\alpha(s)$ cannot be found to interpolate $\alpha(jw_i)$ when the Pick matrix fails to be positive definite. Hence an approximation to the optimal choice $\alpha(jw_i)$ has to be made to obtain a stable $\alpha(s)$ (to ensure internal stability). In the following it is shown that *a and M can be used to increase the accuracy of the approximation and hence improve the performance of the broad-band geometric controller $k(s)$ within the targeted frequency band $[w_1, w_N]$.*

Denote the *i-th* eigenvalue of P by λ_i, which is a real number following from the fact that P is hermitian, then a fundamental theorem from linear algebra states that the sum of eigenvalues is equal to the matrix trace. That is:

$$\sum_{i=1}^{N} \lambda_i = Trace(P) = \sum_{k=1}^{N} \frac{1 - \frac{|\alpha_k|}{M}}{2a} \tag{11}$$

Thus $\sum_{i=1}^{N} \lambda_i$ is monotonic with respect to a or M. Hence increasing M or decreasing a can increase the "positiveness" of $\sum_{i=1}^{N} \lambda_i$. With a large enough M or small enough a, the Pick matrix can eventually become positive definite and therefore allows an exact interpolation with a stable $\alpha(s)$ to the optimal choice $\alpha(jw_i)$ $\forall i \in [1, N]$. This leads to the following tuning rule:

Tuning Rule: The performance of the broad-band geometric controller $k(s)$ within the targeted frequency band $[w_1, w_N]$ can be refined by decreasing a or increasing M.

However caution is required since allowing the stability margin a to be small, the system becomes more vulnerable to instability and in addition, allowing the maximum modulus M of $\alpha(s)$ on the half plane $\Re(s) \geq a$ to be large, the performance outside the targeted frequency band $[w_1, w_N]$ is sacrificed. Nevertheless, for the case where there is no disturbance force outside the targeted frequency band, it is still useful to redesign a or M to improve the performance of the geometric controller by sacrificing the performance outside that frequency band. The above tuning rule thereby retains its usefulness as a design guide.

4. EXAMPLE: GEOMETRIC CONTROLLER DESIGN FOR A ROTOR BLADE SYSTEM

The design procedures are illustrated for a marine application, specifically the problem of attenuating propeller blade axial vibration using only sensors and actuators located at the thrust bearing end of the drive shaft[1]. This concept is illustrated in Fig. 2(a) where, in the context of equation (1), $y(jw)$ represents the axial vibration of the thrust bearing, $z(jw)$ the axial vibration of the blades, $u(jw)$ the control force applied at the thrust bearing and $d(jw)$ the disturbance forces that excite the blades. This does represent a case where during a commissioning phase all the necessary transfer functions can be measured but where it is not currently practically viable to provide in-service sensing and actuation of the blade.

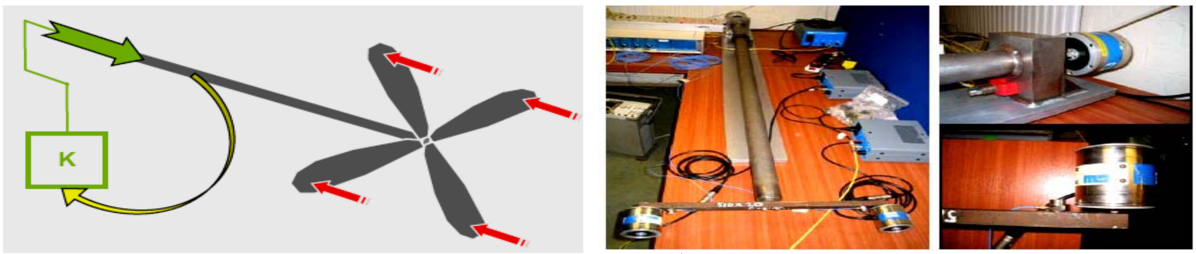


Figure 2: (a) Remote control of rotor blade vibration[1] (b) Blade Vibration Experimental Rig

To replicate this class of problem, a laboratory scale test rig has been constructed as illustrated in Fig. 2 (b). The rig has a flexible beam that represents a rotor blade, which is pinned in the centre to one end of a hollow shaft. The other end of the shaft is fixed to a block that is rigidly connected to the supporting foundation. The beam can be excited by two small 30N Gearing and Watson IV40 inertial shakers and the fixed end of the shaft by a 50N Gearing and Watson IV45 shaker. The vibration in the blade is measured by two accelerometers located close to the shakers and another accelerometer measures the acceleration of the block.

4.1 Design Broad-Band Geometric Controller

Fig. 3(a) shows the frequency response of the sum blade acceleration to excitation of the blade using a common force for both shakers (the plot is therefore of $|g_{22}(jw)|$).

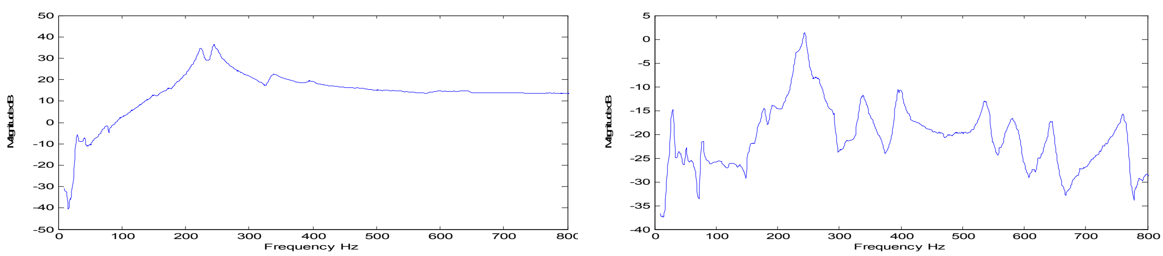
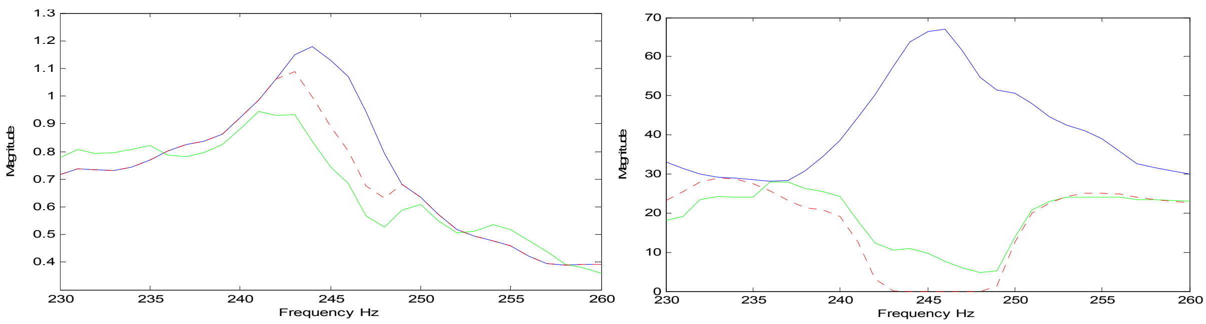


Figure 3: (a) Blade Response to Blade Excitation (b) "Thrust Bearing" Response to Blade Excitation

It can be seen that the first bending mode resonance occurs in the region of 244Hz and that this leads to a peak in the transmission along the shaft, as can be seen in the block response to the

[1] Note that this general concept is the subject of several BAE SYSTEMS Patents. The provision of experimental hardware by BAE Systems and the support of Roger Harrison and John Pearson are gratefully acknowledged.

same forcing ($|g_{12}(jw)|$) in Fig. 3(b). Therefore a control scenario can be considered where the objective is to provide optimum reduction at the blade (z : remote location) without increasing vibration at the thrust bearing (y : local end) over the frequency band $[230,260]Hz$. The optimal choice (the data points over $[230,260]Hz$) can be computed from elementary geometric arguments by considering the relative locations of $\alpha - circle$ and $\beta - circle$ illustrated in Fig.1. With the assumption of $a = 0.6$ and $M = 6$, it is found that the Pick matrix is not positive definite and therefore the Pick condition is violated, and hence the initial optimal choice is infeasible. The application of the above optimization over LMIs, while setting the real and imaginary part of uncertainty for the initial data set to be uniformly bounded within *0.1* results in a new feasible "optimal choice". With this new feasible optimal choice, the closed-loop as well as the open-loop FRFs is illustrated in Fig. 4. Also shown is the closed-loop FRFs that would have resulted from the original optimal choice, had this been feasible. The deviation between the two closed-loop FRFs reveals the accuracy of the approximation and hence the loss of theoretical performance. It is clear that such a deviation in achieving the feasible solution is very small. The design procedure proposed is therefore practically viable.

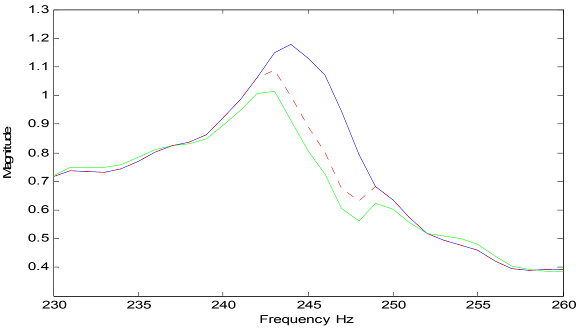


(a) (b)

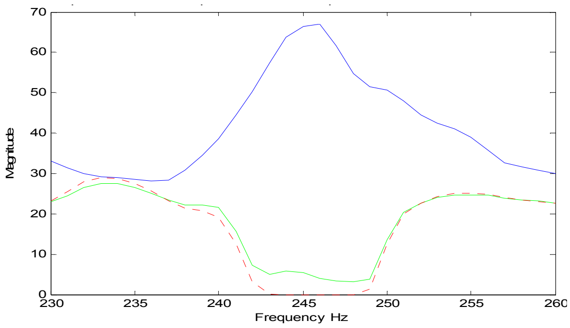
Figure 4: Open-loop (———) and closed-loop FRFs with feasible (———) and infeasible (- - - - -) optimal choice (a) blade (b) thrust bearing

4.2 Tuning Geometric Controller Performance

Now increase the value $M = 6$ to $M = 8$ while still assuming $a = 0.6$ and the uncertainty bound to be *0.1* in the above example, it is found that the Pick condition is still violated, and hence the initial optimal choice is still infeasible. The application of the above optimization over LMIs results in another new set of feasible "optimal choice". The open-loop as well as the two closed-loop FRFs is illustrated in Fig. 5. It is seen that the accuracy of approximation is increased. The loss of theoretical performance becomes smaller, and this validates the results presented in Chapter 3. Finally it is noted that when M *increases* to $M = 9$ or a *decreases* to $a = 0.58$, the Pick matrix becomes positive definite and this results in an exact interpolation and hence no loss of theoretical performance. It is clear that the method has therefore successfully enabled the extension of the geometric design approach to the control of remotely located vibrating systems to the broadband case.



(a)



(b)

Figure 5: Open-loop (——) and closed-loop FRFs with feasible (——) and infeasible (-----) optimal choice (a) blade (b) thrust bearing

5. CONCLUSION

A geometric methodology has been developed for the broadband control of remotely located vibrating systems. This method is particularly targeted at situations where it is required to apply control at a particular point on a structure but sensors and actuators can only be located at some remote location. The approach results in a straightforward design strategy where the design freedom can be directly related to system performance. A calculation of feasible controllers is carried out via convex optimization over LMIs. These theoretical considerations have been validated through their application to the broadband control of vibration in a rotor blade system.

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