ICSV14
Cairns • Australia
9-12 July, 2007

14th International Congress on Sound & Vibration

LIMIT-CYCLE OSCILLATION OF SHAPE MEMORY ALLOY HYBRID COMPOSITE PLATES AT ELEVATED TEMPERATURES

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Abstract

A traditional composite plate impregnated with pre-strained shape memory alloy (SMA) fibers and subject to combined thermal and aerodynamic loads is investigated, to demonstrate the effectiveness of using the SMA fiber embeddings in improving the dynamic response of composite plates. The problem investigated is the nonlinear flutter limit-cycle and chaotic oscillations at elevated temperatures. A nonlinear finite element model based on the first-order shear deformable plate theory is derived. von Karman strain displacement relations are utilized to account for geometric nonlinearity. Aerodynamic pressure is modeled using the quasi-steady first-order piston theory. The governing equations are obtained using the principle of virtual work based on thermal strain being a cumulative physical quantity. Newton-Raphson iteration is employed to obtain the dynamic response at each time step of the Newmark numerical integration scheme. A time domain method along with modal transformation is applied to numerically investigate periodic, non-periodic, and chaotic limit-cycle oscillations. The results show that the amplitude of the limit-cycle oscillation is highly decreased by using SMA fiber embeddings.

1. INTRODUCTION

Panel flutter is a phenomenon that is usually accompanied by temperature elevation on the outer skin of high-speed air vehicles. Panel flutter is a self-excited oscillation of a plate or shell in supersonic flow on one side. Because of aerodynamic pressure forces on the panel, two Eigen modes of the structure merge and lead to this dynamic instability. A common remedy to the flutter problem is to stiffen those panels in danger of flutter, a method that usually introduces additional weight to the design.

A vast amount of literature exists on panel flutter using different aerodynamic theories to model the aerodynamic pressure. Mei et *al*. [1] presented a review on the various analytical methods and experimental results of supersonic and hypersonic panel flutter. Liaw [2] studied the geometrically nonlinear supersonic flutter of laminated composite thin plate structures subjected to thermal loads. Abdel-Motagaly et *al*. [3] investigated the effect of arbitrary flow direction on the large amplitude supersonic flutter of composite panels. Dixon and Mei [4] presented a nonlinear flutter analysis of thin composite panels using the finite element method. Xue and Mei [5] presented an innovative finite element frequency domain solution for the nonlinear panel flutter at elevated temperatures.

Shape memory alloys (SMAs) have a unique ability to completely recover large pre-strains (up to 10%) when heated above certain characteristic temperature called the austenite finish temperature. The austenite start temperature for Nitinol can be any where between -50 ºC and 170 ºC by varying the nickel content. During the shape recovery process, a large tensile recovery stress occurs if the SMA is restrained. Cross et al. [6] measured the recovery stress of Nitinol at various pre-strain values and Young's modulus versus temperatures. Both the recovery stresses and Young's modules of SMA exhibit nonlinear temperature-dependent properties.

Birman [7] presented a comprehensive review on the literature concerning SMA up to 1997. Jia and Rogers [8] formulated a mechanical model for composites with embedded shape memory alloy fibers using the micromechanical behavior of the highly nonlinear shape memory alloy, and adopting the classical lamination plate theory. Park et al. [9] investigated the nonlinear vibration behavior of thermally buckled composite plates embedded with shape memory alloy fibers using the first order shear deformable plate theory. An incremental method was adopted to account for the temperature dependent material properties. Guo [10] offered an efficient finite element method to predict the thermal buckling of thin shape memory alloy hybrid composite plates. Tawfik et al. [11] proposed a novel concept in enhancing the thermal buckling and aeroelastic behavior of plates through embedding SMA fibers in it. Guo et. al. [12] developed a finite element procedure to predict the nonlinear flutter response of thin Shape Memory Alloy hybrid composite plates at an arbitrary yawed angle and an elevated temperature.

In this paper, the nonlinear flutter limit-cycle oscillations of shape memory alloy hybrid composite plate panels under the combined effect of thermal and aerodynamic loading are investigated using a nonlinear finite element method. The nonlinear governing equations for a moderately thick rectangular plate are obtained using the first-order shear-deformable plate theory (FSDT), von Karman strain-displacement relations, and the principle of virtual work. The approach is based on the thermal strain being an integral quantity of the thermal expansion coefficient with respect to temperature, whereas the stress is evaluated with the instant elastic modulus at a certain temperature in the thermoelastic stress-strain relations [10]. Therefore, the method does not need the use of many small temperature increments as in the incremental method [11], and hence, it is suitable for any nonlinear temperature-dependent material properties. Numerical results are provided to show the effect of the thermal field and pre-strained SMA fiber embeddings on the nonlinear flutter characteristics of a clamped traditional composite plate panel.

2. FINITE ELEMENT FORMULATION

The equations of motion with the consideration of large deflection and temperature dependent (TD) material properties are derived for a shape memory alloy hybrid composite plate panel subject to aerodynamic and thermal loadings. To account for temperature dependence of the material properties, cumulative thermal strain is adopted for the calculation of the thermal deflections and stresses in the plate.

The nodal degrees of freedom vector of a rectangular plate element can be written as:

$$\{\delta\} = \{\{w_b\},\{\phi_x,\phi_y\},\{u,v\}\}^T = \begin{Bmatrix} \{w_b\} \\ \{w_\phi\} \\ \{w_m\} \end{Bmatrix} \tag{1}$$

where w_b is the transverse displacement of the middle plane, ϕ_x and ϕ_y are rotations of the transverse normal about the x and y axes respectively, u and v are the membrane displacements in the x and y directions respectively, $\{w_\phi\}$ is the nodal rotation of the transverse normal vector, and $\{w_m\}$ is the nodal membrane displacement vector.

The displacement-nodal displacement relation can be presented in terms of interpolation function matrices $[N_w]$, $[N_{\phi x}]$, $[N_{\phi y}]$, $[N_u]$ and $[N_v]$ as:

$$\begin{gathered} w = [N_w]\{w_b\},\ \phi_x = \lfloor N_{\phi x}\rfloor\{w_\phi\},\ \phi_y = \lfloor N_{\phi y}\rfloor\{w_\phi\}, \\ u = \lfloor N_u\rfloor\{w_m\} \text{ and } v = \lfloor N_v\rfloor\{w_m\} \end{gathered} \tag{2}$$

The inplane strains and curvatures, based on von Karman's large deflection and first-order shear deformable plate theory, are given by [14]:

$$\begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{Bmatrix} = \begin{Bmatrix} \dfrac{\partial u}{\partial x} \\ \dfrac{\partial v}{\partial y} \\ \dfrac{\partial u}{\partial y} + \dfrac{\partial v}{\partial x} \end{Bmatrix} + \begin{Bmatrix} \dfrac{1}{2}\left(\dfrac{\partial w}{\partial x}\right)^2 \\ \dfrac{1}{2}\left(\dfrac{\partial w}{\partial y}\right)^2 \\ \dfrac{\partial w}{\partial x}\dfrac{\partial w}{\partial y} \end{Bmatrix} + z\begin{Bmatrix} \dfrac{\partial \phi_y}{\partial x} \\ \dfrac{\partial \phi_x}{\partial y} \\ \dfrac{\partial \phi_y}{\partial y} + \dfrac{\partial \phi_x}{\partial x} \end{Bmatrix} \tag{3}$$

Or in compact form

$$\{\varepsilon\} = \{\varepsilon_{lin}\} + \{\varepsilon_\theta\} + z\{\kappa\} \tag{4}$$

where ε_{lin}, ε_θ, and $z\kappa$ are the membrane linear strain vector, the membrane nonlinear strain vector, and the bending strain vector, respectively.

The transverse shear strain vector can be expressed as [14]

$$\begin{Bmatrix} \gamma_{yz} \\ \gamma_{xz} \end{Bmatrix} = \begin{Bmatrix} \phi_x \\ \phi_y \end{Bmatrix} + \begin{Bmatrix} \dfrac{\partial w}{\partial y} \\ \dfrac{\partial w}{\partial x} \end{Bmatrix} \tag{5}$$

The constitutive equations (6) and (7) of a traditional composite plate impregnated with shape memory alloy fibers are derived assuming that every layer of the composite matrix has an arbitrary orientation angle θ and principal material directions 1, 2 and 3. The SMA fiber is embedded in the 1-direction, and assumed uniformly distributed within each layer.

$$\begin{Bmatrix} \{N\} \\ \{M\} \end{Bmatrix} = \begin{bmatrix} [A] & [B] \\ [B] & [D] \end{bmatrix} \begin{Bmatrix} \{\varepsilon_m\} \\ \{\kappa\} \end{Bmatrix} - \begin{Bmatrix} \{N^T\} \\ \{M^T\} \end{Bmatrix} + \begin{Bmatrix} \{N_r\} \\ \{M_r\} \end{Bmatrix} \tag{6}$$

$$\{R\} = \begin{Bmatrix} R_{yz} \\ R_{xz} \end{Bmatrix} = \begin{bmatrix} A_{44} & A_{45} \\ A_{45} & A_{55} \end{bmatrix} \begin{Bmatrix} \gamma_{yz} \\ \gamma_{xz} \end{Bmatrix} = [A]^s \{\gamma\} \tag{7}$$

where

$$\{\varepsilon_m\} = \{\varepsilon_{lin}\} + \{\varepsilon_\theta\}$$

$$(\{N^T\},\{M^T\}) = \int_{-h/2}^{h/2} \left[Q(T) \left(\int_{T_{ref}}^{T} \alpha(\tau) d\tau \right) \right] (1, z) dz$$

$$(\{N_r\},\{M_r\}) = \int_{-h/2}^{h/2} \{\sigma_r\} V_s \, (1, z) dz$$

where $[A]$, $[A]^s$, $[B]$ and $[D]$ are the laminate membrane, shear, coupling and bending stiffness matrices, respectively. $\{N\}$, $\{M\}$ and $\{R\}$ are the resultant vectors of the inplane, moment and transverse shear forces. In addition, $\{N^T\}$ and $\{M^T\}$ are the inplane thermal load and thermal bending moment vectors, respectively, while $\{N_r\}$ and $\{M_r\}$ are the inplane SMA recovery load and SMA recovery bending moment vectors. V_s is the volume fraction of the SMA fibers T denotes the temperature rise, while a constant temperature distribution in the x, y and z directions are assumed.

By using the principle of virtual work the governing equation of the nonlinear flutter limit-cycle oscillation of a plate embedded with SMA fibers can be derived as follows

$$\delta W = \delta W_{\text{int}} - \delta W_{ext} = 0 \tag{8}$$

The internal virtual work δW_{int} is given as [11]

$$\begin{aligned} \delta W_{\text{int}} &= \int_A \left(\{\delta\varepsilon_m\}^T \{N\} + \{\delta\kappa\}^t \{M\} + \beta \{\delta\gamma\}^T \{R\} \right) dA \\ &= \{\delta w\}^T \left([k] - [k_T] + [k_r] + \frac{1}{2}[n1] + \frac{1}{3}[n2] \right) \{w\} - \{\delta w\}^T (\{p_T\} - \{p_r\}) \end{aligned} \tag{9}$$

where $\{w\}$ is the nodal displacement vector of the element; β is a shear correction coefficient; $[k]$, $[k_T]$ and $[k_r]$ are the linear, thermal and recovery stress stiffness matrices; $[n1]$ and $[n2]$ are the first- and second-order nonlinear stiffness matrices, respectively. In addition, $\{p_T\}$ and $\{p_r\}$ are the thermal load vector and the recovery stress load vector, respectively.
On the other hand, the external virtual work δW_{ext} is given as [9]

$$\begin{aligned} \delta W_{ext} &= \int_A \begin{pmatrix} - I_o \left(\{\delta u\}^T \{\ddot{u}\} + \{\delta v\}^T \{\ddot{v}\} + \{\delta w\}^T \{\ddot{w}_b\} \right) \\ - I_2 \left(\{\delta\phi_x\}^T \{\ddot{\phi}_x\} + \{\delta\phi_y\}^T \{\ddot{\phi}_y\} \right) + \{\delta w_b\}^T P_a \end{pmatrix} dA \\ &= -\{\delta w\}^T [m] \{\ddot{w}\} - \{\delta w_b\}^T [g] \{\dot{w}_b\} - \{\delta w_b\}^T \lambda \, [a_a] \{w_b\} \end{aligned} \tag{10}$$

where $(I_o, I_2) = \int_{-h/2}^{h/2} \rho \, (1, z^2) dz$ with h denoting the plate thickness, P_a is the aerodynamic pressure, $[m]$ is the mass matrix, $[g]$ is the aerodynamic damping matrix, $[a_a]$ is the aerodynamic influence matrix, and λ is the non-dimensional dynamic pressure.

By substituting equations (9) and (10) into (8), the governing equations for a shape memory alloy hybrid composite plate under the combined action of aerodynamic and thermal loads, can be written as

$$\begin{bmatrix} M_B & 0 \\ 0 & 0 \end{bmatrix}\begin{Bmatrix} \ddot{W}_B \\ \ddot{W}_m \end{Bmatrix} + \begin{bmatrix} G_B & 0 \\ 0 & 0 \end{bmatrix}\begin{Bmatrix} \dot{W}_B \\ \dot{W}_m \end{Bmatrix} + \begin{pmatrix} \lambda\begin{bmatrix} A_{aB} & 0 \\ 0 & 0 \end{bmatrix} + \begin{bmatrix} K_B & 0 \\ 0 & K_m \end{bmatrix} - \begin{bmatrix} K_{TB} & 0 \\ 0 & 0 \end{bmatrix} \\ + \begin{bmatrix} K_{rB} & 0 \\ 0 & 0 \end{bmatrix} + \frac{1}{2}\begin{bmatrix} N1_{NmB} & N1_{Bm} \\ N1_{mB} & 0 \end{bmatrix} \\ + \frac{1}{3}\begin{bmatrix} N2_B & 0 \\ 0 & 0 \end{bmatrix} \end{pmatrix}\begin{Bmatrix} W_B \\ W_m \end{Bmatrix} \tag{11}$$

$$= \begin{Bmatrix} 0 \\ P_{mT} \end{Bmatrix} - \begin{Bmatrix} 0 \\ P_{mr} \end{Bmatrix} = \begin{Bmatrix} 0 \\ P_m \end{Bmatrix}$$

where

$$\{W_B\} = \begin{Bmatrix} W_b \\ W_\phi \end{Bmatrix}$$

Note that neglecting the in-plane and shear inertia terms will not bring significant error, because their natural frequencies are usually 2 to 3 orders of magnitude higher than those of bending [10].

Separating the membrane and transverse displacement equations in equation (11) [13]:

$$\begin{aligned} &[M_B]\{\ddot{W}_B\} + [G_B]\{\dot{W}_B\} + \begin{pmatrix} \lambda[A_{aB}] + [K_B] - [K_{TB}] + \\ [K_{rB}] + [N1_{NmB}(\{W_m\}_o)] \end{pmatrix}\{W_B\} \\ &+ \left(\frac{1}{3}[N2_B] - \frac{1}{2}[N1_{NmB}(\{W_m\}_2)] - \frac{1}{4}[N1_{Bm}][K_m]^{-1}[N1_{mB}]\right)\{W_B\} = 0 \end{aligned} \tag{12}$$

Equation (12) can be numerically integrated in the structural nodal degrees of freedom. But this approach turns to be computationally expensive. Therefore, an alternative and effective solution procedure is to transform equation (12) into modal coordinates using reduced system aeroelastic modes by expressing the system bending displacement $\{W_B\}$ as a linear combination of a finite number of aeroelastic mode shapes as [12]:

$$\{W_B\} \approx \sum_{r=1}^{n} q_r\{\phi_r\} = [\Phi]\{q\} \tag{13}$$

where the r[th] aeroelastic mode $\{\phi_r\}$ and the corresponding natural frequency ω_r are obtained from the linear vibration of the system as:

$$\omega_r^2[M_B]\{\phi_r\} = ([K_B] + \lambda_o[A_{aB}])\{\phi_r\} \tag{14}$$

where λ_o is a certain dynamic pressure value selected to be smaller than the critical value, λ_{cr}. Based on the aeroelastic modes evaluated in equation (14), all the matrices in equation (12) are transformed into modal coordinates, using the concept of right and left eigenvectors [15]. Accordingly, equation (12) can be written in modal coordinates as:

$$\left[\overline{M}_B\right]\{\ddot{q}\}+\left[\overline{G}_B\right]\{\dot{q}\}+2\left[\zeta_r f_r\right]\left[\overline{M}_B\right]\{\dot{q}\}+\left(\left[\overline{K}\right]+\left[\overline{K}_{qq}\right]\right)\{q\}=0 \tag{15}$$

A modal structural damping matrix 2 $[\zeta_r\ f_r]$ $[M_B]$ has been added to equation (15) to account for the structural damping effect on the system. The coefficient ζ_r is the modal damping ratio of the rth mode, while f_r is the rth natural frequency in Hz.

3. Numerical Results and Discussions

The nonlinear flutter limit-cycle oscillation of a clamped laminated composite plate panel with and without SMA is performed using the finite element method. A uniform 6 x 6 finite element mesh of nine-noded rectangular elements is found adequate. The dimensions of the plate are 0.381 x 0.305 x 0.0013m and the stacking sequence is $[0/-45/45/90]_s$. Table (1) presents the material properties of the composite matrix and SMA fibers [10]. Uniform temperature change is applied to the plate, and the reference temperature is assumed 21ºC.

Table 1 Material properties of composite matrix and SMA fiber [10]

Nitinol	Graphite-epoxy
See ref. [10] for Young's modulus and recovery stresses.	**E1** 155 (1-6.35x10^{-4} ΔT) GPa **E2** 8.07 (1-7.69x10^{-4} ΔT) GPa
G 25.6 GPa	**G12** 4.55(1-1.09x10^{-3}ΔT)GPa
ρ 6450 Kg/m^3	**ρ** 1550 Kg/m^3
ν 0.3	**ν** 0.22
α 10.26 x 10^{-6} / ºC	**α1** -0.07x10^{-6}(1-0.69x10^{-3}ΔT) / ºC **α2** 30.6x10^{-6}(1+0.28x10^{-4}ΔT) / ºC

Limit-cycle oscillation (LCO) can be categorized into four types: nearly harmonic LCO, periodic LCO, non-periodic oscillation, and chaotic oscillation. A traditional clamped composite plate and a SMAHC plate with 5% volume fraction and 3% pre-strain are studied and compared. The response time histories at the maximum deflection are obtained for various combinations of the temperature rise ΔT and λ. A proportional damping ratio is used with a fundamental modal damping coefficient ζ_1 equal to 0.02. The aerodynamic damping coefficient C_a is set to 0.1. Newmark implicit numerical integration scheme [16] is utilized to solve the system differential equations. Six aeroelastic modes were found adequate.

Figure (1) shows the LCO of both plates at room temperature and λ = 650. It is seen that the SMAHC plate vibrates with amplitude equal to 0.8, which is higher than the 0.7 vibration amplitude of the composite plate, because the SMAHC plate has slightly higher weight than the traditional composite plate, and the recovery stresses is not activated yet at room temperature. Figure (2) shows the LCO of both plates at ΔT= 50 ºC and λ = 500. As temperature rises above room temperature, the recovery stress starts to build up resulting in a stiffer plate compared to the traditional composite plate. It is seen that the SMAHC plate vibrates with nearly harmonic oscillation with amplitude equal to 0.7, which is much lower than the 1.75 vibration amplitude of the composite plate periodic oscillation. At ΔT= 110 ºC and λ = 280, the SMAHC plate periodically oscillates with amplitude equal to 1.3 while the composite plate experiences chaotic oscillation as shown in figure (3). Therefore, it is concluded that SMA fiber embeddings can effectively decrease or even suppress limit-cycle oscillation amplitudes at higher temperatures, resulting in a larger flat and dynamically stable region.

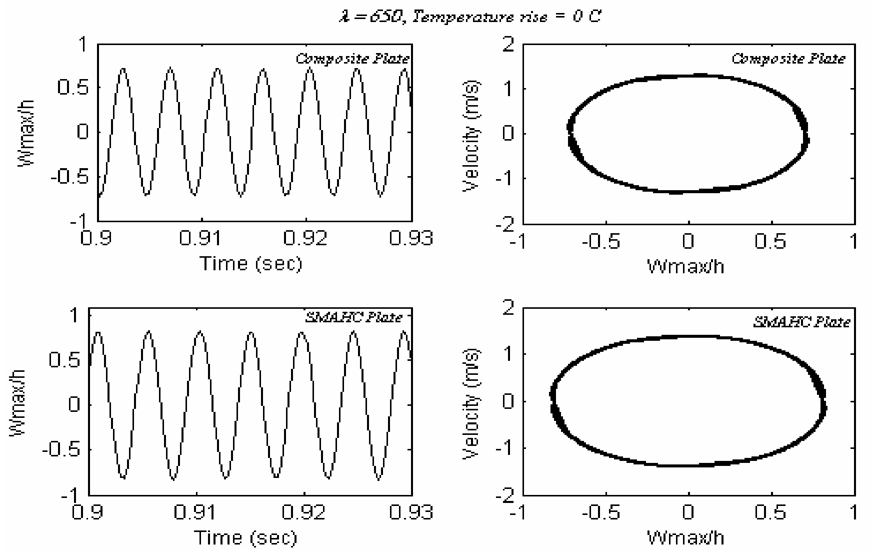

Figure 1. Comparison between LCO of composite and SMAHC plates at λ = 650 and ΔT = 0 ºC

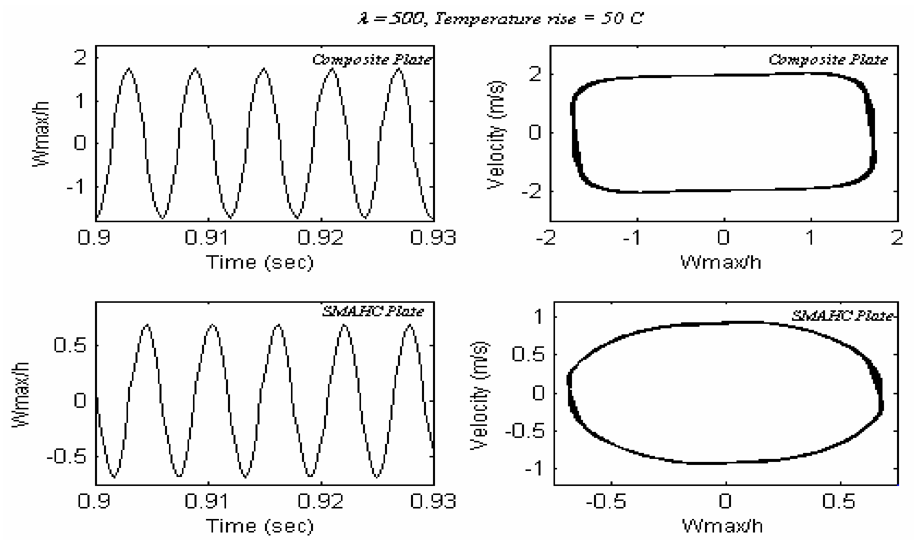

Figure 2. Comparison between LCO of composite and SMAHC plates at λ = 500 and ΔT = 50 ºC

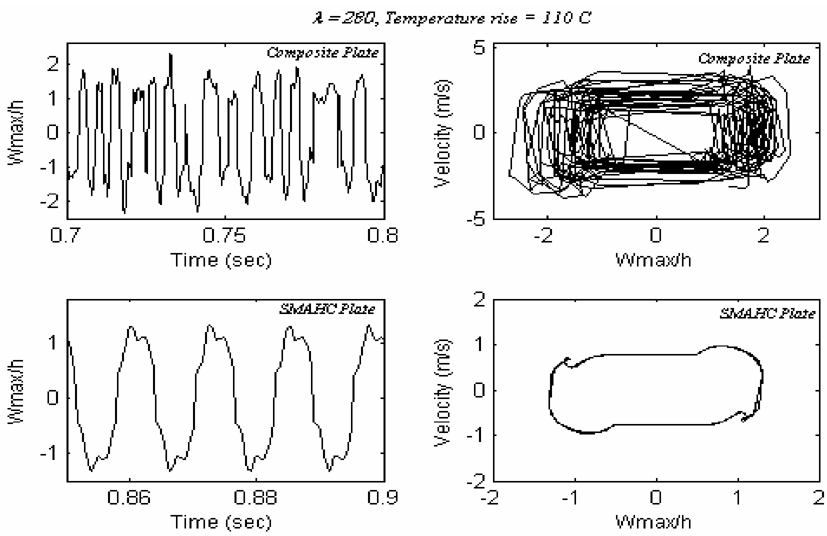

Figure 3. Comparison between LCO of composite and SMAHC plates at λ = 280 and ΔT = 110 ºC

4. CONCLUSIONS

In this work, a traditional composite plate impregnated with pre-strained shape memory alloy fibers and subject to combined thermal and aerodynamic loads, is investigated, to demonstrate the effectiveness of using SMA fiber embeddings in improving the flutter response of composite plates. A new nonlinear finite element model for moderately thick plates based on the first-order shear deformable plate theory is presented using the cumulative thermal strain method adopted by Guo [12]. The governing equations are obtained using the principle of virtual work. The nonlinear temperature dependence of material properties for the composite matrix and SMA fibers is considered in the formulation. The time domain method is applied to numerically investigate limit-cycle oscillations. The finite element modal formulation and solution procedures are developed for the time domain method. Results showed that SMA fiber embeddings can be very useful in flutter control through decreasing the flutter limit-cycle amplitude.

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