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TRANSMISSION PATH CHARACTERIZATION FOR PASSIVE VIBRATION CONTROL

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Abstract

Analysis, prediction and reduction of vibration transmission in built-up structures are addressed. The sensibility to vibrations varies spatially within a structure, due to issues as passenger comfort and/or the localization of certain sensitive components or instruments. Traditionally, vibration transmission has been analyzed adopting transfer path analysis (TPA). The contribution to the vibration amplitude or sound pressure level in some particular locations from each transfer path is quantified using the transfer path operational forces and the frequency response function (FRF) of the receiving structure. TPA considering energy based quantities such as supplied power has gained popularity as it provides more stable path contributions and ranking of dominant paths. The supplied power is associated with the far-field contribution to the response, which may be used as an approximation of the complete response in the mid and high frequency range. For low frequencies however, the near-field contribution, which is associated with the reactive power, will be significant. Hence, in that case the supplied power is not well suited to characterize vibration transmission. Furthermore, it is not obvious how to modify a dominant transfer path so that the vibration response or the supplied power attenuates. Generally, the system must be considered as a whole in order to avoid sub-optimization. Examples that stress the statements above are given and an alternative tool for transfer path ranking, suitable for structural optimization is proposed. A scalar vibration exposure function is defined and the transfer paths are ranked based on its gradient with respect to physical parameters associated with each path.

1. INTRODUCTION

When studying the transmission structure-borne noise and vibrations (SBN) in a particular structure, the most straightforward alternative is to analyze the contribution from each transfer path to the vibration amplitude or sound pressure level in some particular locations. This

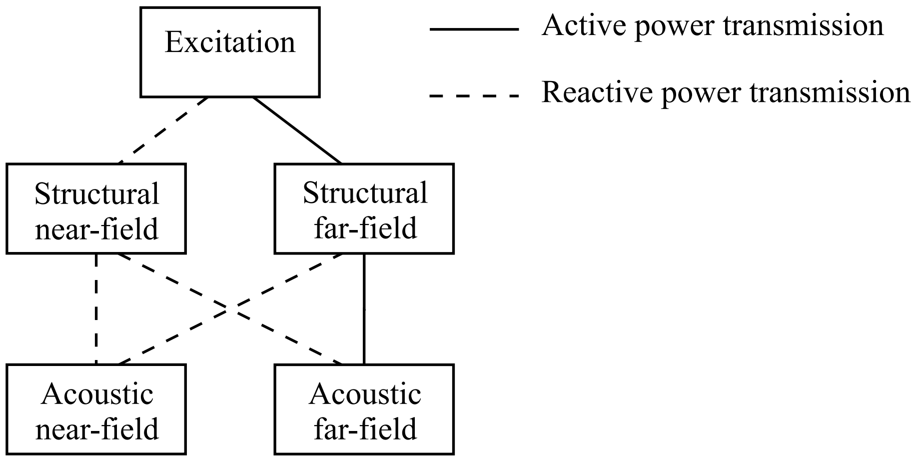

Figure 1. Scheme of the basic vibration energy flow for a structural-acoustic system.

is sometimes referred to as classic transfer path analysis (TPA) and is a fairly well established technique, see Plunt [1]. The method requires that the transfer path operational forces and the frequency response function (FRF) of the receiving structure are available. Determination of the forces is made indirectly, either by using resilient connecting elements as force transducers or by inversion of the FRF matrix. The first alternative requires that the dynamic stiffness of the coupling element is known. The inversion alternative is associated with problems of ill-conditioning, for example near resonances, which has been extensively investigated by Thite and Thompson [2]. Both experimental TPA, utilizing measured forces and FRFs, and analytical TPA, typically based on FE modeling and enabling early design evaluations, may be performed adopting this approach.

For high frequencies, poor repeatability of the FRF over nominally identical structures has been observed, see Plunt [1]. Therefore, TPA considering energy based quantities such as supplied power is proposed as it provides more stable path contributions and ranking of dominant paths. Due to its simplicity, this approach has gained popularity over the years. Basically the energy flow can be computed as a post-processing step of a FE method. The interior noise level is assumed to be proportional to the total supplied power and the paths are ranked with respect to their contribution to the total power. Hence, only interface quantities, in terms of operational forces and vibration response, are required for the analysis. Thereby, the simplicity and user-friendliness follow. This approach is closely related to statistical energy analysis (SEA) which is based on a high modal density assumption. It is assumed that the response in a certain frequency band is governed by a large number of modes, i.e. high modal density and high modal overlap, allowing for ensemble averaging. Thereby the system is conveniently described in terms of the vibration energy of each frequency band rather than by the velocity or displacement field. However, when studying the low frequency range, which in many applications ranges up to 200-300 Hz, SEA and energy/power based TPA methods are inherently infeasible.

Describing the flow of vibration energy from the excitation points on the structure to pressure fluctuations inside the cavity experienced as sound or noise, a scheme according to Fig. 1 may be used. The excitation forces induce structural vibrations and the structural response can be divided into near-field and far-field contributions. From fundamental wave theory we know that the far-field is responsible for the transmission of active power via propagating waves whereas the near-field contribution is basically associated with reactive power. Hence, energy based TPA mainly characterizes the transmission from excitation via structural far-field to cavity far-field. The near-field decays rapidly with the distance to the exciting force and with increasing frequency, see Cremer et al. [3]. However, this is only true for idealized structural members, e.g. (semi-) infinite beams, solids, etc. A real structure is finite and possesses corners, flanges and other inhomogeneities which also generate near-fields. Consequently, the near-field contribution to the structural response can generally not be neglected. Furthermore, on the next level we have radiation from the structure into the air volume of the cavity. The pressure fluctuations of the air can then also be divided into near- and far-field. Again, considering the ideal case of radiation from an infinite panel to a semi-infinite air volume, the far-field is dominant except very close to the panel. For an enclosed cavity the response will however be strongly governed by the acoustical modes (Fahy [4] and Kruntcheva [5]) which in this frequency range are relatively well separated, see Nefske et al. [6]. Nevertheless, power flow analysis of low frequent structural vibrations has been performed and reported; see e.g. Palmer et al. [7], Alfredsson et al. [8], Wilson and Josefson [9] and Lee [10]. Although using this as a tool to propose structural modifications may be dubious and lead to unwanted results.

2. THE VIBRATION EXPOSURE FUNCTION

Here we propose an alternative tool for transfer path characterization and TPA, which is well suited to aid structural optimization with respect to vibration transmission properties. The sensibility to vibrations varies spatially within a structure, due to issues such as passenger comfort and/or the localization of certain sensitive components or instruments. Consequently, it is desired that the vibration or noise in a user-defined set of locations is considered. Thereby, both spatial and frequency domain averaging can be applied on the response to form a scalar vibration exposure function (VEF). Mathematically the VEF is defined as

$$f_{\mathrm{VEF}}(\boldsymbol{\theta}) = \sum_k w_{\mathrm{f}}(\omega_k)^2 \mathbf{v}(\omega_k, \boldsymbol{\theta})^T \mathbf{W}_{\mathrm{s}}(\omega_k) \mathbf{v}(\omega_k, \boldsymbol{\theta}) \tag{1}$$

where $\mathbf{v}(\omega_k, \theta)$ is the response vector at frequency ω_k for the parameter set $\boldsymbol{\theta}$, which contains structural velocities and/or sound pressure levels, $\mathbf{W}_{\mathrm{s}}(\omega_k)$ is a symmetric positive definite weighting matrix for the spatial average and $w_{\mathrm{f}}(\omega_k)$ is a function that describes the weighting of the frequency domain average. The latter is preferably chosen as a standardized comfort filter with respect to human perception, e.g. as described in ISO 226 [11] regarding sound pressure levels.

Provided that the parameters in $\boldsymbol{\theta}$ describe relevant physical properties of the possible paths of vibration transmission, the gradient of the VEF with respect to these parameters may be used to characterize transfer paths. The gradient can be viewed as a measure of sensitivity and thus our proposal is to rank paths based on the magnitude of the gradient components. Furthermore, the information from the VEF gradient may conveniently be utilized to suggest structural modifications that reduce the VEF.

3. SUBSTRUCTURE MODELING

In order to establish the VEF, a model of the complete system is required. The system may be described as an assembly of subsystems which are modeled individually one-by-one. In this case the receiving structure, possibly with an enclosed air volume, the source structure which is subjected to the external excitation and the coupling elements constitute the subsystems.

There are different alternatives to model a subsystem. Setting out from first principles and using a finite element (FE) method the governing equations of motions for a structural system in discretized form may be written

$$\mathbf{M}\ddot{\mathbf{q}} + \mathbf{V}\dot{\mathbf{q}} + \mathbf{K}\mathbf{q} = \mathbf{f} \tag{2}$$

where $\mathbf{q}$ and $\mathbf{f}$ are the n-dimensional vector of structural displacements and applied forces, respectively, whereas $\mathbf{K}$, $\mathbf{V}$ and $\mathbf{M}$ are the $n \times n$ structural stiffness, damping and mass matrices. The structural-acoustic coupling required for an adequate modeling of the receiving structure is described in Nefske et al. [6]. In that case the discretized governing equations become

$$\begin{bmatrix} \mathbf{M} & 0 \\ (\rho c)^2 \mathbf{S}^T & \mathbf{M}_\mathrm{f} \end{bmatrix} \begin{Bmatrix} \ddot{\mathbf{q}} \\ \ddot{\mathbf{p}} \end{Bmatrix} + \begin{bmatrix} \mathbf{V} & 0 \\ 0 & \mathbf{V}_\mathrm{f} \end{bmatrix} \begin{Bmatrix} \dot{\mathbf{q}} \\ \dot{\mathbf{p}} \end{Bmatrix} + \begin{bmatrix} \mathbf{K} & -\mathbf{S} \\ 0 & \mathbf{K}_\mathrm{f} \end{bmatrix} \begin{Bmatrix} \mathbf{q} \\ \mathbf{p} \end{Bmatrix} = \begin{Bmatrix} \mathbf{f} \\ 0 \end{Bmatrix} \tag{3}$$

where $\mathbf{p}$ is the pressure at the grid points of the cavity mesh, $\mathbf{M}_\mathrm{f}$, $\mathbf{V}_\mathrm{f}$ and $\mathbf{K}_\mathrm{f}$ represent acoustic mass, damping and stiffness matrices, $\mathbf{S}$ is called the structural-acoustic coupling matrix, c is the speed of sound and ρ the air density. A model of the complete system may then be established through the regular assembly process of the FE method.

However, in practice some components may be difficult to model from first principles. The values of a large number of physical parameters have to be accurately assessed. An attractive solution is then to use models identified from experiments. From test data on each component, subsystem models may be obtained using methods for system identification, see Ljung [12]. The identified model may be in the form of a state-space model

$$\left\{ \begin{aligned} \dot{\mathbf{x}} &= \mathbf{A}\mathbf{x} + \mathbf{B}\mathbf{u} \\ \mathbf{y} &= \mathbf{C}\mathbf{x} + \mathbf{D}\mathbf{u} \end{aligned} \right. \tag{4}$$

where $\mathbf{u}$ and $\mathbf{y}$ are the time domain excitation and response vectors, respectively. An alternative description of the system is the non-parametric model

$$\mathbf{Y}(\omega) = \mathbf{H}(\omega)\mathbf{U}(\omega) \tag{5}$$

where $\mathbf{H}$ is the measured and possibly smoothed FRF and $\mathbf{U}$ and $\mathbf{Y}$ are the frequency domain excitation and response vectors, respectively. Methods to synthesize identified state-space models are treated in Su and Juang [13] and Sjövall and Abrahamsson [14]. Coupling of non-parametric models are formulated in e.g. Jetmundsen et al. [15], Otte et al. [16], Lim and Li [17] and Liu and Ewins [18].

4. NUMERICAL EXAMPLE

In order to evaluate the performance of the proposed TPA approach based on the parameterized VEF, a numerical example is studied. The investigated system is a plane frame, built-up by a

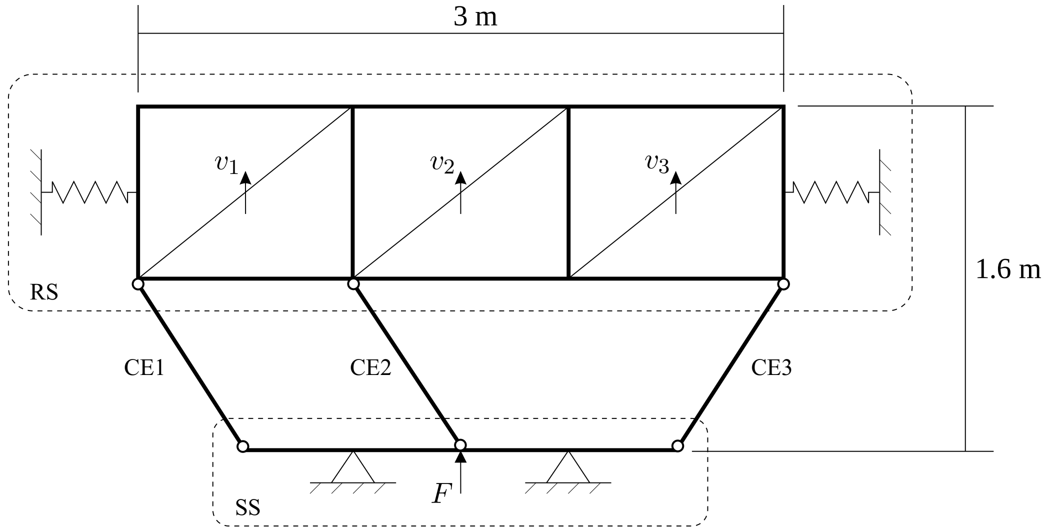

Figure 2. Plane frame built up by source structure (SS), receiving structure (RS) and coupling elements (CE1,CE2,CE3).

Table 1. Beam member properties: receiving structure except diagonal members (RSM), diagonal members of receiving structure (RSD), coupling elements (CE), source structure (SS).

Property	RSM	RSD	CE	SS
Young's modulus [GPa]	220.0	22.0	220.0	220.0
Cross section area [cm^2]	2.72	2.72	2.72	9.0
Cross section moment of inertia [cm^4]	3.4	3.4	3.4	6.75
Mass per unit length [kg/m]	2.14	2.14	2.14	7.08
Rayleigh damping mass coefficient [mNs/kgm]	1.0	0.01	10.0	1.0
Rayleigh damping stiffness coefficient [ms]	0.01	0.01	1.0	0.01
Spring stiffness [kN/m]	100.0	-	-	-

source structure which is connected to a receiving structure by three coupling elements, CE1, CE2 and CE3, see Fig. 2. Excitation is applied as a force in the center of the source structure with amplitude 1 N for all frequencies. The frame is modeled using two-dimensional FE beam elements and in Table 1 the member properties are presented.

The VEF is computed considering the velocity response in the vertical direction in the center of the three diagonals of the receiving structure. These three responses are weighted equally and for the frequency domain weighting the filter corresponding to the C-weighted sound pressure level (ISO 226 [11]) and defined as follows:

$$w_{\mathrm{f}}(\omega) = \left| \frac{5.91797 \times 10^9 (\mathrm{i}\omega)^2}{(\mathrm{i}\omega + 129.4)^2 (\mathrm{i}\omega + 76655)^2} \right| \tag{6}$$

To perform the transfer path analysis, the three coupling elements are parameterized. The

Table 2. Gradient of the VEF and the scalar power function.

Path no. i	1	1	2	2	3	3
Property, j	s	d	s	d	s	d
$10^3 \times \partial f_{\text{VEF}}/\partial\theta_i^j$	0.3283	-0.1487	1.7102	-0.8780	0.0928	-0.0429
$10^3 \times \partial f_{\text{PWR}}/\partial\theta_i^j$	0.3652	0.0215	0.8575	-0.5118	-0.0866	-0.0059

element stiffness and damping matrices of coupling element i are parameterized as

$$\mathbf{K}_{\text{CE}i} = (1+\theta_i^{\text{s}})\mathbf{K}_{\text{CE}i}^{\text{nom}} \tag{7}$$

$$\mathbf{C}_{\text{CE}i} = (1+\theta_i^{\text{d}})\mathbf{C}_{\text{CE}i}^{\text{nom}} \tag{8}$$

where $\mathbf{K}_{\text{CE}i}^{\text{nom}}$ and $\mathbf{C}_{\text{CE}i}^{\text{nom}}$ are the nominal element stiffness and damping matrices corresponding to the physical values in Table 1. Hence, there are one stiffness parameter θ_i^{s} and one damping parameter θ_i^{d} for each path (coupling element) i, in total six parameters.

In Table 2, the VEF gradient with respect to these six parameters is presented. According to this analysis the second coupling element, i.e. the one in the middle, is the dominant transfer path. The VEF is most sensitive to the parameters associated with this element. This should be compared to Fig. 3 where the power exchange in the interface between the receiving structure and the three coupling elements is plotted as function of frequency. The power exchange in each path is shown and it can be observed that the path responsible for the dominant energy exchange is the third path, i.e. the right-most coupling element. Hence, the two TPA approaches give deviating results.

This is further stressed when we consider a scalar power function defined as

$$f_{\text{PWR}}(\theta) = \sum_k w_{\text{f}}(\omega_k)^2 P(\omega_k, \theta) \tag{9}$$

where $P(\omega_k, \theta)$ is the total supplied power to the receiving structure at frequency ω_k. The gradient of $f_{\text{PWR}}(\theta)$ with respect to the coupling element parameters is also presented in Table 2. It is clear that the two gradients are not co-linear, although they both indicate that the parameters of the second coupling element as most important. Hence, it is possible to do a structural modification that reduce the power function but actually increase the VEF. This case is illustrated

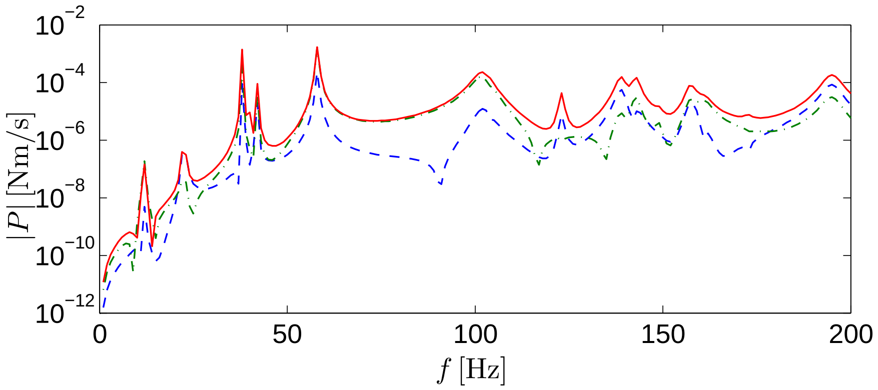

Figure 3. Transfer path power exchange: CE1 (dashed), CE2 (dash-dotted), CE3 (solid).

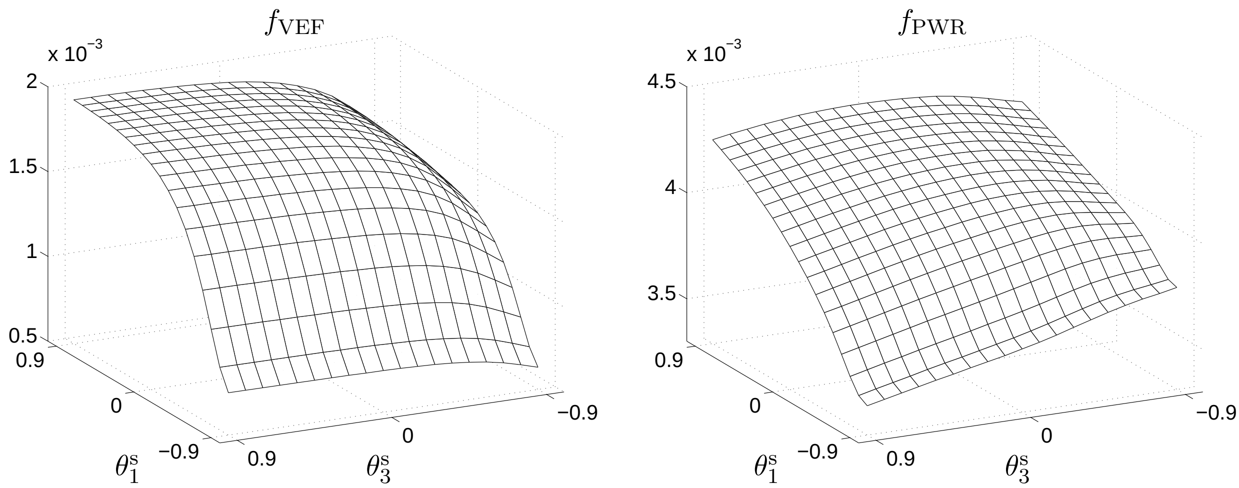

Figure 4. The scalar $f_{\rm VEF}$ (left) and $f_{\rm PWR}$ (right) as function of the damping in the coupling elements CE1 and CE2.

in Fig. 4 which shows the scalar $f_{\rm VEF}$ and $f_{\rm PWR}$ as functions of the parameters $\theta_1^{\rm s}$ and $\theta_3^{\rm s}$. It should be remarked that the behavior of these functions are affected by the frequency spectrum of the applied load. Different combinations of broadband and narrowband excitations have also been used (not shown here). In none of these cases the computed $f_{\rm VEF}$ and $f_{\rm PWR}$ gradients were co-linear. Hence, the power function can not replace the VEF in terms of predictive ability.

5. CONCLUSIONS

An alternative TPA approach has been proposed, which is based on the parameterized scalar function called VEF, defined as a combined spatial and frequency domain average of the vibration and/or noise levels of the structural-acoustic system. Vibration transfer paths are ranked based on the gradient of the VEF with respect to parameters associated to each path.

The VEF approach is based on quantities associated with the experienced response in specific locations whereas energy based TPA methods are based on response quantities of the transfer paths, i.e. at the interface of the receiving structure. Hence, the former direct approach can be considered as more correct than the latter indirect approach. A numerical example illustrated that the presented approach and the energy based TPA method give deviating results in terms of transfer path ranking. Also, regarding the ability to assess the result of structural modifications, it was shown that the energy based approach may give incorrect predictions.

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