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THEORETICAL AND EXPERIMENTAL INVESTIGATION ON POWER FLOW OF A RAFT VIBRATION ISOLATION SYSTEM

Guangming Wu[1], Qiang Wang[2], Rongying Shen[1]

[1] State Key Laboratory of Mechanical System and Vibration , Shanghai Jiao Tong University, Shanghai 200030, P.R.China
[2] Shanghai Marine Equipment Research Institute, Shanghai 200031, P.R.China
shenry@sjtu.edu.cn

Abstract

The raft vibration isolation system, one of the most effective isolation systems, is widely adopted to isolate vibration and reduce radiation sound of engines and auxiliary machines on ships. In this paper, the modeling of the raft vibration isolation system has been paid much more attention on considering the flexibility of the raft and the foundation. The general equations of motion for the raft vibration isolation system have been successfully derived by means of a new substructure method. The power flow from the machine to the raft and further to the foundation has been studied theoretically and experimentally. An experimental rig of the raft isolation system, including elastic raft and non-rigid foundation, has been set up. The power flow through the experimental model has been successfully measured. The comparison between the theoretical predicted power flow and the experimental ones has been completed and analyzed. The agreement of the power flow results demonstrates the correctness of the new modeling theory and power flow theory of the raft vibration isolation system.

1. INTRODUCTION

The raft vibration isolation system is widely adopted to isolate vibrations of engines and auxiliary machines in ships. The traditional modeling method of the isolation system mostly neglects the flexibility of the raft and/or foundation. However, a lot of works about the flexibility of the isolation system has been done[1-2]. Sciulli and Inman derived the dynamic equation of single mass-spring system that mounted on non-rigid foundation and discussed the influence of the mount frequency on the isolation system[3]. Li and Lavrich derived the 3D dynamic equations of machine mounted on elastic flat plate and discussed its power flow property[4]. Li et al. further derived the equations of machine mounted on elastic cylinder and also studied the power flow characteristics of this kind of the isolation system[5]. Li and Yam established the general dynamic equations of single body mounted on flexible foundation by the mode synthesis method[6]. Xiong derived generalized the mobility/impedance matrix formulations for three-dimensional rigid and elastic structures of general configuration, which is an important theory in the complicated elastic coupling system[7].

In this paper, the general equations of motion for the raft vibration isolation system have been successfully derived by means of a new substructure method. The dynamic characteristics of the raft isolation system have been explored. Based on the above model, the power flow from the machine to the raft and further to the foundation has been studied theoretically and experimentally. An experimental rig of the raft isolation system, including elastic raft and non-rigid foundation, has been built up and the power flow through the experimental model has been successfully measured. The comparison between the theoretically predicted values and the experimental ones has been completed and analyzed, and some useful conclusions have been obtained.

2. DYNAMIC EQUATION OF A RAFT VIRATION ISOLATION SYSTEM

Figure1 shows a general 3D raft vibration isolation system. The isolation system can be divided into three parts: vibratory machines(A, D et al), elastic raft(B) and non-rigid foundation(C).

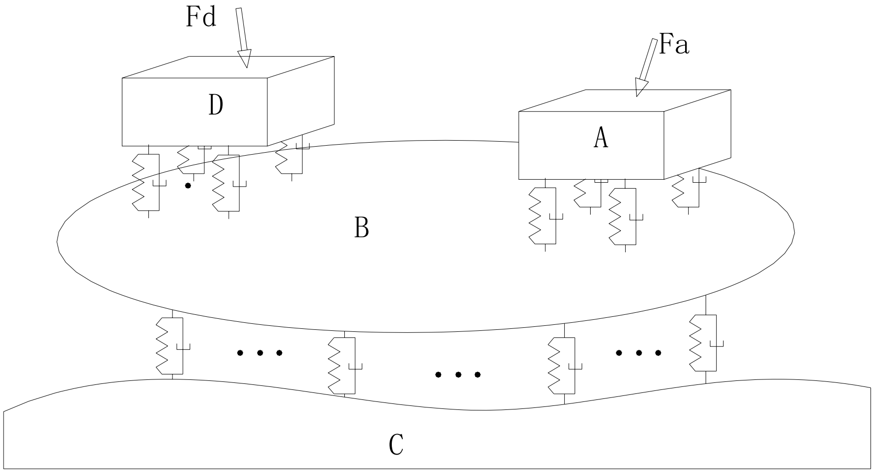

Figure 1. Three dimensional raft vibration isolation system

To establish the dynamic equation of the raft vibration isolation system consists of three steps.

1. Dividing substructure. Each machine or isolated equipment is generally regarded as a rigid substructure. The middle raft and non-rigid foundation can be considered as an elastic substructure respectively.
2. Establishing dynamic equations of each substructure.
3. Synthesizing substructure dynamic equations and then obtaining the whole system dynamic equations.

The raft vibration isolation system in figure 1 can be divided into four substructures: rigid substructures A and D, elastic substructure B and C. Rigid substructure A connects elastic substructure B with $N1$ isolators. Rigid substructure D connects elastic substructure B with $N2$ isolators. Elastic substructure B connects with elastic substructure C by $N3$ isolators. M_a , K_a , C_a and M_d , K_d , C_d are the physical parameters of substructures A and D. M_b , K_b , C_b , W_{bi} and M_c , K_c , C_c , W_{ci} are the modal parameters of substructures B and C respectively. Through establishing dynamic equations of each substructure and synthesizing them, the dynamic equation (1) of the raft vibration isolation system can be derived[8].

In equation (1), $X = \{X_a \quad X_d \quad q_b \quad q_c\}^T$ is the general coordinate vector, q_b and q_c are the modal vectors of substructure B and C respectively. K_i , C_i , T_i and R_i are the stiffness matrix, damping matrix, translation matrix and rotation matrix of the i th isolator. W_{bi} and W_{ci} are the elastic deformation matrixes of two substructures B and C respectively，in which taking the n_b truncated modes of substructure B and n_c truncated modes of substructure C.

$$
\begin{bmatrix} M_a & & & \\ & M_d & & \\ & & M_b & \\ & & & M_c \end{bmatrix}
\begin{Bmatrix} \ddot{X}_a \\ \ddot{X}_d \\ \ddot{q}_b \\ \ddot{q}_c \end{Bmatrix} +
$$

$$
\begin{bmatrix}
\sum_{i=1}^{N1} T_i R_i C_i R_i^T T_i^T & & -\sum_{i=1}^{N1} T_i R_i C_i R_i^T T_i^T W_{bi} & \\
& \sum_{j=1}^{N2} T_j R_j C_j R_j^T T_j^T & -\sum_{j=1}^{N2} T_j R_j C_j R_j^T T_j^T W_{bj} & \\
-\sum_{i=1}^{N1} W_{bi}^T T_i R_i C_i R_i^T T_i^T & -\sum_{j=1}^{N2} W_{bj}^T T_j R_j C_j R_j^T T_j^T & \begin{array}{l} C_b + \sum_{i=1}^{N1} W_{bi}^T T_i R_i C_i R_i^T T_i^T W_{bi} \\ + \sum_{j=1}^{N2} W_{bj}^T T_j R_j C_j R_j^T T_j^T W_{bj} \\ + \sum_{l=1}^{N3} W_{bl}^T R_l C_l R_l^T W_{bl} \end{array} & -\sum_{l=1}^{N3} W_{bl}^T R_l C_l R_l^T W_{cl} \\
& & -\sum_{l=1}^{N3} W_{cl}^T R_l C_l R_l^T W_{bl} & C_c + \sum_{l=1}^{N3} W_{cl}^T R_l C_l R_l^T W_{cl}
\end{bmatrix}
\begin{Bmatrix} \dot{X}_a \\ \dot{X}_d \\ \dot{q}_b \\ \dot{q}_c \end{Bmatrix}
$$

$$
+\begin{bmatrix}
\sum_{i=1}^{N1} T_i R_i K_i R_i^T T_i^T & & -\sum_{i=1}^{N1} T_i R_i K_i R_i^T T_i^T W_{bi} & \\
& \sum_{j=1}^{N2} T_j R_j K_j R_j^T T_j^T & -\sum_{j=1}^{N2} T_j R_j K_j R_j^T T_j^T W_{bj} & \\
-\sum_{i=1}^{N1} W_{bi}^T T_i R_i K_i R_i^T T_i^T & -\sum_{j=1}^{N2} W_{bj}^T T_j R_j K_j R_j^T T_j^T & \begin{array}{l} K_b + \sum_{i=1}^{N1} W_{bi}^T T_i R_i K_i R_i^T T_i^T W_{bi} \\ + \sum_{j=1}^{N2} W_{bj}^T T_j R_j K_j R_j^T T_j^T W_{bj} \\ + \sum_{l=1}^{N3} W_{bl}^T R_l K_l R_l^T W_{bl} \end{array} & -\sum_{l=1}^{N3} W_{bl}^T R_l K_l R_l^T W_{cl} \\
& & -\sum_{l=1}^{N3} W_{cl}^T R_l K_l R_l^T W_{bl} & K_c + \sum_{l=1}^{N3} W_{cl}^T R_l K_l R_l^T W_{cl}
\end{bmatrix}
\begin{Bmatrix} X_a \\ X_d \\ q_b \\ q_c \end{Bmatrix} = \begin{Bmatrix} F_a \\ F_d \\ 0 \\ 0 \end{Bmatrix} \tag{1}
$$

3. POWER FLOW OF RAFT VIBRATION ISOLATION SYSTEM

Since all substructures in the raft isolation system are connected by the isolators, the power flows, including the power flow into the system, the power flow transmitted between substructures and the power flow into the foundation, can be classified into two kinds: power flow input to the system and power flow through the isolators.

3.1 Power flow input to the system

The power flow input to the system is

$$P = \frac{1}{2}\mathrm{Re}\left\{F \cdot V^*\right\} \tag{2}$$

Assume the displacement response of mass centre of machine A is X_c, the response at the exciting force position is

$$X_{a\_p} = T_{a\_p} X_c \tag{3}$$

in which T_{a_p} is the translation matrix of the exciting force point.

The velocity response at the same point is

$$V_{a\_p} = i\omega X_{a\_p} \tag{4}$$

Substituting equations (3) and (4) into equation (2) gives the power flow into the raft isolation system

$$P_{in\_p} = \frac{1}{2}\mathrm{Re}\left\{i\omega F_p \cdot (T_{a\_p} X_c)^*\right\} \tag{5}$$

3.2 Power flow through the isolator

Suppose isolator k is one of the isolators that connect substructure i with substructure j. There are two cases: 1. substructure i is rigid, and substructure j is elastic; 2. both of the two substructures are elastic.

In case 1, assume that X_c is the response of substructure i at mass centre, and q_j is the modal response of substructure j. T_{i_k} and W_{j_k} are the translation matrix of isolator k at its mounting position on substructure i and the modal deformation matrix of isolator k at its mounting position on substructure j. Then the responses of the two mounting points are

$$X_{i\_k} = T_{i\_k} X_c, \quad X_{j\_k} = W_{j\_k} q_j \tag{6}$$

The isolator reaction force can be written as

$$F_{tr\_k} = R_k (K_k + iG_k) R_k^T (X_{i\_k} - X_{j\_k}) \tag{7}$$

in which R_k is rotation matrix, $K_k + iG_k$ is complex stiffness.

Substituting equations (6) into equation (7), then substituting into equation (2) gives

$$P_{tr\_k} = \frac{1}{2}\mathrm{Re}\left\{i\omega R_k (K_k + iG_k) R_k^T (T_{i\_k} X_c - W_{j\_k} q_j) \cdot (W_{j\_k} q_j)^*\right\} \tag{8}$$

Equation (8) expresses the power flow through the isolator k.

In case 2, assume that q_i is the modal response of substructure i, and W_{i_k} is the modal deformation matrix of isolator k at its mounting position on substructure i. Equation (6) becomes

$$X_{i\_k} = W_{i\_k} q_i, \quad X_{j\_k} = W_{j\_k} q_j \tag{9}$$

Therefore, the power flow through the isolator k is

$$P_{tr\_k} = \frac{1}{2} \mathrm{Re}\left\{ i\omega R_k (K_k + iG_k) R_k^T (W_{i\_k} q_i - W_{j\_k} q_j) \cdot (W_{j\_k} q_j)^* \right\} \tag{10}$$

4. EXPERIMENTAL STUDY

4.1 Experimental Model

An experiment model of the raft vibration isolation system is built in figure 2. It is made up of four substructures. Substructures A and D are two steel blocks of dimensions 150×180×50mm to simulate the machine setup No.1 and No.2. Substructure B, simulating raft, is made up of two 460×460×2mm steel plates connected by four cylinders with 50 mm high and diameter 30mm at four corners. Substructure C, simulating the non-rigid foundation, is a 460×460×2mm steel plate installed on a flat rigid foundation by four cylinders with 50mm high and diameter 30mm at four corners. Each machine is connected the raft by four isolators, and the raft is connected the non-rigid foundation by eight isolators. The stiffness of the isolators along three directions are $k_x = 2e4N/m$, $k_y = 1e4N/m$, $k_z = 2.51e4N/m$.

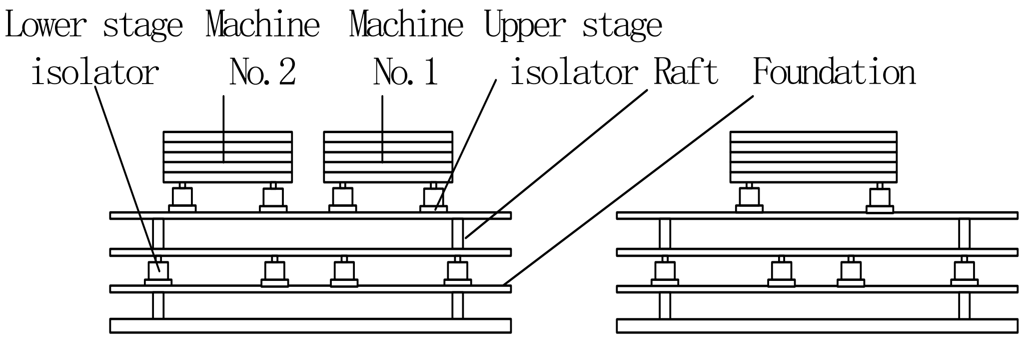

Figure 2. Model of the raft vibration isolation system

4.2 Theoretical Calculation of The Power Flow

The power flow characteristics of the model of the raft isolation system between 0~1000Hz are investigated. Assume the exciting force $F = 5e^{i\omega t}$ N acting on the mass centre of the machine No.1 vertically. The theoretical calculation of the power flow of the model shows in figure 3. It can be found that the power flow is distinctly decreased from the machine to the foundation, that denotes the vibration is effectively attenuated through two stages isolators. It also can be found that the low frequency energies dominate over the whole frequency ranges in all of the power flows. This phenomenon shows the rules in designing the raft vibration isolation system, i.e. trying to reduce the low frequency peaks as much as possible.

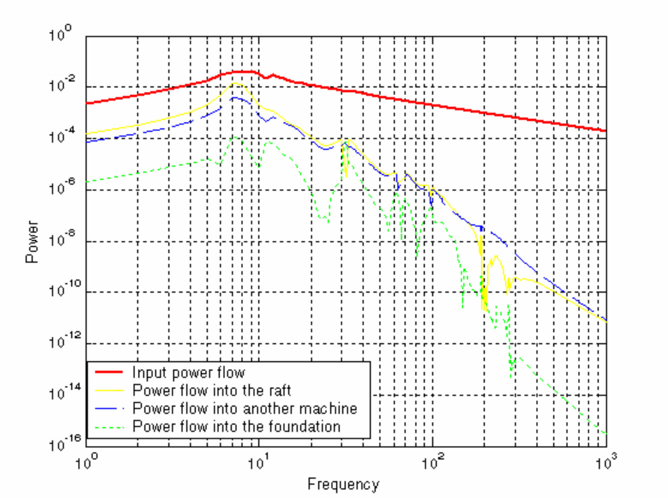

Figure3. Power flow of the elastically coupled vibration isolation system

4.3 Experimental Results

The input power flow at input position can be measured by[9]

$$P_{in}/Hz = 1/\omega \cdot \mathrm{Im}\{G_{Fa}\} \tag{11}$$

where G_{Fa} is the cross spectral density between acceleration and force at the input position.

The power through the isolator is

$$P_{tr}/Hz = 1/\omega \cdot \mathrm{Im}\{A_{12} \cdot G_{a1a2}\} \tag{12}$$

Where $A_{12} = \left(k/\omega^2\right) \cdot (1 + i\eta)$ is apparent mass of the isolator, k and η are the stiffness and damping ratio of the isolator, G_{a1a2} is the cross-spectral density of the acceleration between the input position and output position of the isolator.

Figure 4 shows the experimental rig of the raft isolation system. During the test the exciter always acts at the same position and the spectral density of the exciting force keeps the same. Every two isolators are regarded as a group, input force and acceleration signals at the input position and acceleration signals of the two isolator input and output positions are picked up. Since there are 16 isolators in the raft vibration isolation system, it needs 8 times to complete the power flow measurement.

After obtaining the power flows through all the isolators, adding the power flows through the isolators that connect the machine No.1 and the raft, the power flow to the raft is obtained. The power flows from the raft to the machine No.2 and to the foundation can be obtained by the same method.

The experimental results of the measured power flow are shown in figures 5~9. For the convenience of the comparison the theoretical predicted results of the power flow are also plotted out. It can be seen clearly that the experimental results agree with the theoretical predicted results satisfactorily.

Figure 4. Power flow testing of the raft vibration isolation system

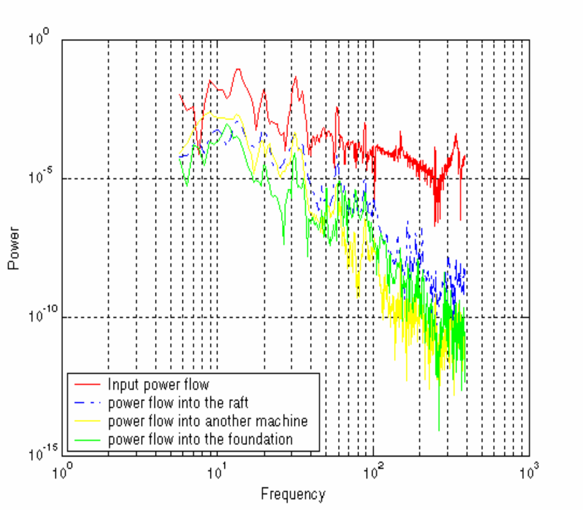

Figure 5 Power flow of the raft isolation system

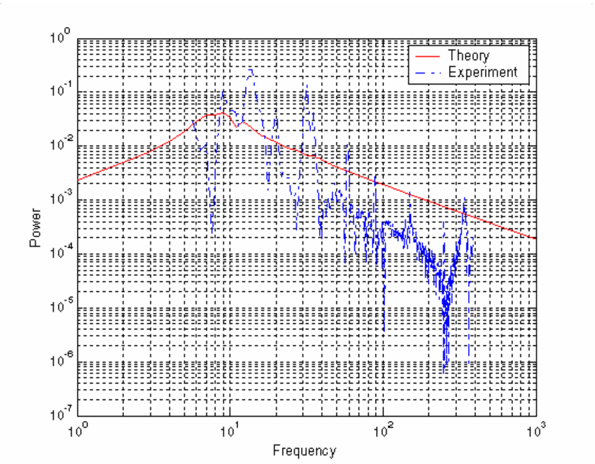

Figure 6. Power flow input to the vibration isolation system

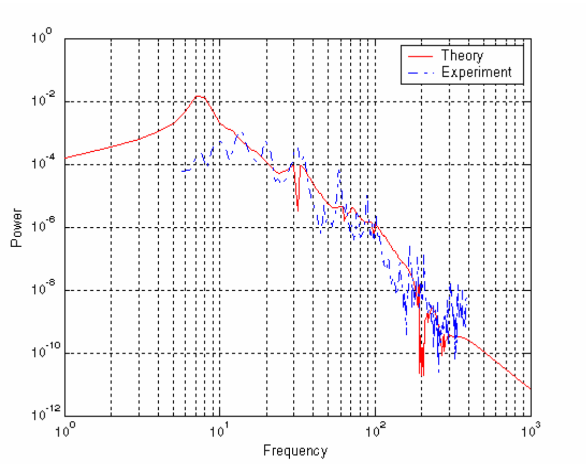

Figure 7. Power flow to the raft

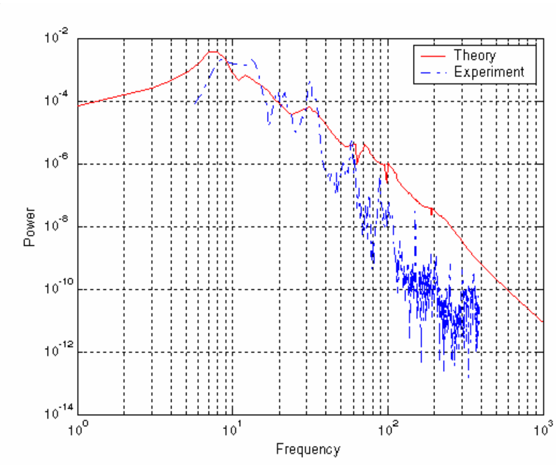

Figure 8. Power flow to the machine No.2

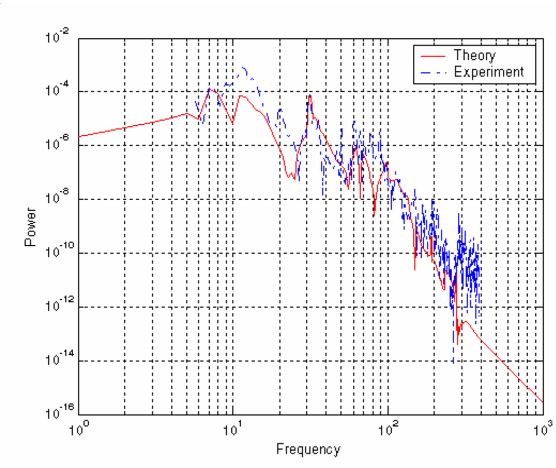

Figure 9. Power flow to the foundation

5. CONCLUSIONS

A new substructure method to complete the modeling of the raft isolation system has been derived. Based on the dynamic equations of the raft vibration isolation system the power flow theory has been studied. The power flow of a model of the raft isolation system has been calculated by the new modeling method and measured by the experiment. The agreement of the predicted and measured values of the power flow demonstrates the correctness of the new modeling theory and power flow theory of the raft vibration isolation system.

The power flow curves of the model also reveal the power flow characteristics of the raft isolation system, i.e., the low frequency energy dominates over the whole frequency range. To design a high quality raft isolation system, it needs to avoid low frequency peaks in power flow curve.

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