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THEORETICAL AND NUMERICAL STUDY ON HYDROELASTIC VIBRATION OF PLATE FOR VARIOUS BOUNDED FLUID FIELD

Kuksu Kim[1] , Suhyun Choi[1] and Hyundong Cho[1]

[1]Dawoo shipbuilding & marine engineering co., ltd.
1, Aju-dong, Geoje-si, Gyeongsangnam-do, 656-714, Korea
kuksu@dsme.co.kr

Abstract

In this paper, a theoretical and numerical case study is carried out on the hydroelastic vibration of a rectangular plate in contact with various fluid fields. It is assumed that the tank wall is clamped along the plate edges. The fluid velocity potential is used for the simulation of the fluid domain and to obtain the added mass due to the wall vibration. The Assumed Mode method is utilized for the plate model and the hydrodynamic force is thus obtained. The proposed analytical approach was found to be in good agreement with the results of a well-known commercial three-dimensional finite element program. In addition, the natural frequencies are calculated and compared for rectangular plates in contact with various fluid fields. These include an infinite fluid, a finite fluid, a change in length of the finite fluid, and various fluid contacting conditions.

1. INTRODUCTION

This paper deals with the free vibration analysis of a rectangular plate in contact with various bounded fluids. This work arises as a part of a project developing a local vibration analysis and sloshing analysis program of a tank in a ship. In the many part of a ship, there exist so many tank structures contacting with water or oil. If there structures exhibit excessive vibration, it takes a lot of cost to improve the vibration situation because of required welding and special painting jobs. It is therefore very important to predict precise vibration characteristic for the structures at the design stage, however it is not easy to evaluate vibration characteristic of the structures in contact with a fluid by ship designer.

The plate vibration in contact with fluid has recently been studied. Kim[1] calculated on the added mass of a thin rectangular plate vibrating elastically in an infinite ideal fluid. Lee[2] investigated experimentally on transverse vibration characteristics of rectangular plates in water having an inner free cutout. Jeong[3][4] derived a formulation on a plate using normalized admissible functions satisfying all the plate boundary conditions. However, the identical two plates coupled with a bounded fluid without free surface was dealt with. Lee[5] derived an expression of the fluid domain using the velocity potential imposing the simple free surface boundary condition. On the other hand, as for a stiffened plate in air, Kim[6] investigated the natural frequency of a thin elastic rectangular plate with various boundary conditions. Han[7] calculated the natural frequency of a stiffened plate having concentrated masses using the Receptance method. The theoretical study is performed on the natural frequency of a stiffened plate of a tank by Kim[8], and the result is compared with the three-dimensional finite element method.

This paper describes an application of the hydroelasticity theory to the fluid-structure interaction problems in a rectangular tank for various bounded fluids. Tank wall is modeled using the Assumed Mode methods and the velocity potential method to assess the added mass. Then effect of various bounded fluids for a plate is discussed and compared each other.

2. THEORETICAL BACKGROUND

2.1 Plate modelling

Figure 1 shows a plate with stiffeners that can be attached to the plate in x –direction and/or y-direction.
For the modeling of a rectangular plate, the Assumed Mode method is adopted and mode shapes of beam are used.

$$w(x,y,t)=\sum_{m=1}^{p}\sum_{n=1}^{q}A_{mn}(t)X_m(x)Y_n(y) \tag{1}$$

where,

$$X_m(x)=A_m(\cosh\frac{a_m x}{a}-\cos\frac{a_m x}{a})+B_m\sinh\frac{a_m x}{a}+\sin\frac{a_m x}{a}$$

$$Y_n(y)=A_n(\cosh\frac{a_n y}{b}-\cos\frac{a_n y}{b})+B_n\sinh\frac{a_n y}{b}+\sin\frac{a_n y}{b}$$

$X_m(x), Y_n(y)$ are mth and nth mode shapes of a beam for x and y direction respectively, which is a spatial coordinate of a plate, and $A_{mn}(t)$ is the time dependent generalized coordinate.
a,b and a_m, a_n are the breadth or the length of a plate and characteristic value of mth ,nth vibration mode, respectively.

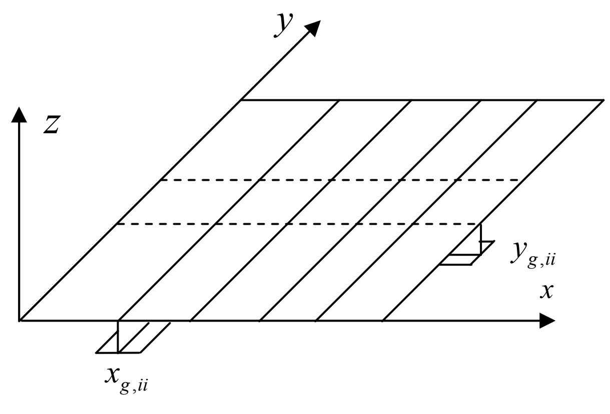

Fig.1 A plate with stiffeners and coordinate system

Using the Assumed Mode method, kinetic and potential energy can be calculated. This approach can be extended to the stiffened plate. In that case, kinetic and potential energy of the plate itself can be expressed as follows:

2.2.1 Kinetic energy

$$T_p(t) = \frac{1}{2}\int_0^a\int_0^b \rho h\, \dot{w}(x,y,t)^2 dxdy$$
$$T_g(t) = \frac{1}{2}\rho\sum_{ii=1}^{n_g}\left[A_{x,ii}\int_0^a \dot{w}(x,y_{g,ii},t)^2 dx + I_{px,ii}\int_0^a\left(\frac{\partial \dot{w}(x,y_{g,ii},t)}{\partial y}\right)^2 dx\right]$$
$$T(t) = T_p + T_g \tag{2}$$

V_p, V_g are kinetic energy of a plate and a stiffener.

a, ρ, A_{xii}, I_{pxii} and $y_{g,ii}$ are the stiffener length, density, sectional area, polar moment of inertia, y-axis location of the ii^{th} stiffener.

2.2.2 Potential energy

$$V_p(t) = \frac{1}{2}\int_0^b\int_0^a D_E\left(\left(\frac{\partial^2 w}{\partial x^2}\right)^2 + \left(\frac{\partial^2 w}{\partial y^2}\right)^2 + 2\upsilon\frac{\partial^2 w}{\partial x^2}\frac{\partial^2 w}{\partial y^2} + 2(1-\upsilon)\left(\frac{\partial^2 w}{\partial xy}\right)^2\right)dxdy,$$
$$V_g(t) = \frac{1}{2}\sum_{ii=1}^{n_g}\left[EI_{x,ii}\int_0^a\left(\frac{\partial^2 w(x,y_{g,ii},t)}{\partial x^2}\right)^2 dx + GI_{px,ii}\int_0^a\left(\frac{\partial^2 w(x,y_{g,ii},t)}{\partial x\partial y}\right)^2 dx\right]dy$$
$$V(t) = V_p + V_g \tag{3}$$

V_p, V_g are potential energy of a plate and a stiffener.

D_E, υ are the bending rigidity of plate, poission's ratio and EI_x, GJ_x, n_g are the equivalent bending rigidity, torsional rigidity of x-axis stiffener and the number of stiffeners respectively.

$$D_E = \frac{Eh^3}{12(1-\upsilon^2)},\quad EI = EI_0 + \frac{EShe^2}{1-\upsilon^2},\quad GJ = \frac{G}{3}\sum_i d_i t_i^3 \tag{4}$$

The natural frequency of the stiffened panel can be obtained from Eq. (5) applying Lagrange equation to kinetic and potential energies.

$$[K]q + [M]\ddot{q} = 0, \tag{5}$$

It should be noted that non-zero off-diagonal terms exists in Eq. (5) because the admissible functions are not the eigen-function of the considered problem.

2.2 Hydrodynamic modeling

Figure 2 shows a rectangular tank filled with liquid and the coordinate system. a, b, H and L are the breadth, height of the plate, liquid filling level and length of the tank, respectively. The length of the tank is the length of two opposite sides between the face A and the face B in the y direction. The bottom plate is taken into account for the formulation. However, application to other plates will be very similar and straightforward. As for a side plate, the detail formulation is described in Reference [8].

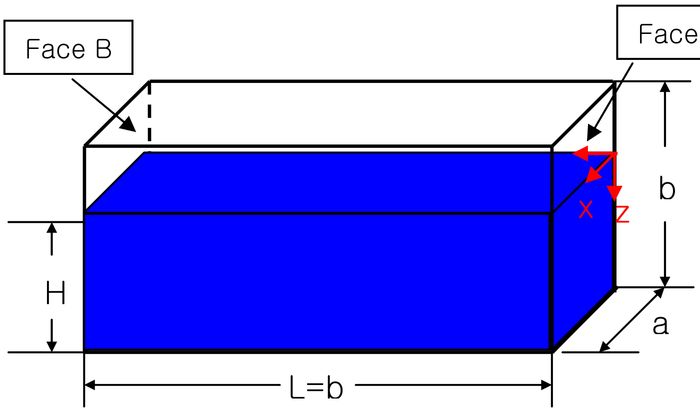

Fig.2 A rectangular tank filled with liquid

The velocity potential is used for the estimation of the hydrodynamic force assuming the incompressible and irrotational flow. Consequently, the Laplace equation is the governing equation as shown in Eq. (6).

$$\nabla^2\Phi = \frac{\partial^2\Phi}{\partial x^2} + \frac{\partial^2\Phi}{\partial y^2} + \frac{\partial^2\Phi}{\partial z^2} = 0 \tag{6}$$

Boundary conditions are given in Eqs. (7)~(12).

$$\frac{\partial\Phi}{\partial x} = 0 \quad \text{at } x = 0 \tag{7}$$

$$\frac{\partial\Phi}{\partial x} = 0 \quad \text{at } x = a \tag{8}$$

$$\frac{\partial\Phi}{\partial y} = 0 \quad \text{at } y = 0 \tag{9}$$

$$\frac{\partial\Phi}{\partial y} = 0 \quad \text{at } y = L \tag{10}$$

$$\Phi = 0 \quad \text{at } z = 0 \tag{11}$$

$$\frac{\partial\Phi}{\partial z} = \frac{\partial w}{\partial t} \quad \text{at } z = H \tag{12}$$

In the above equations, (7)~(10) imply the rigid wall conditions at the other walls except the elastic plate wall of interest and free surface. Equation (11) means no surface wave on the free surface. Equation (12) shows a fluid attached a vibrating wall should move with the elastic plate. In general, the free surface boundary condition is expressed as in Eq. (13). However, it should be noted that because the elastic wall vibration is relatively high compared with a water wave frequency, the high frequency approximation can be applied to the general free surface boundary condition (13).

$$\frac{\partial^2\Phi}{\partial t^2} + g\frac{\partial\Phi}{\partial y} = 0 \quad \text{at } z = 0 \tag{13}$$

Through the Eqs. (6)~(12), the velocity potential can be obtained as Eq. (14).

$$\Phi(x,y,z,t)=\sum_{i=0}^{\infty}\sum_{j=0}^{\infty}H_{ij}(t)\cdot\cos p_i x\cdot\cos q_j y\cdot\sinh\lambda_{ij}z \tag{14}$$

where,

$$p_i=\frac{i\pi}{a},\quad q_j=\frac{j\pi}{L},\quad \lambda_{ij}=\sqrt{p_i^2+q_j^2}$$

$H_{ij}(t)$ is a time function of the velocity potential, which is obtained from Eq. (12).
The force applied to the wall can also be calculated with the Bernoulli's equation (15) and velocity potential (14).

$$p=-\rho\left.\frac{\partial\Phi}{\partial t}\right|_{z=H} \tag{15}$$

Consequently, the force can be obtained as Eq. (16).

$$\begin{aligned}F_s=&\int_0^L\int_0^a-\rho_w\sum_{i=0}^{\infty}\sum_{j=0}^{\infty}\cos p_i x\cdot\cos q_j y\cdot E\cdot R_{ij}\\&\cdot\sum_{r=1}^{\infty}\ddot{a}(t)_r\int_0^L\int_0^a W_r(x,y)\cdot\cos p_i x\cdot\cos q_j y\,dxdy\cdot W_s(x,y)\ dxdy\end{aligned} \tag{16}$$

ρ_w is the density of water.
Because it is known that the added mass is proportional to the acceleration of the wall, the added mass can be extracted from Eq. (16)

$$\begin{aligned}A_{rs}=&\sum_{r=1}^{\infty}\sum_{s=1}^{\infty}\cdot\sum_{i=0}^{\infty}\sum_{j=1}^{\infty}-\rho_w\cdot E\cdot R_{ij}\int_0^H\int_0^a W_r(x,y)\cdot\cos p_i x\cdot\cos q_j y\,dxdy\\&\int_0^H\int_0^a W_s(x,y)\cdot\cos p_i x\cdot\cos q_j y\,dxdy\\&M_{add,rs}=A_{rs}\end{aligned} \tag{17}$$

where,

$$W_r(x,y)=\sum_{m=2}^{\infty}\sum_{n=2}^{\infty}a_{mn}^r X_m(x)Y_n(y)$$

$$R_{ij}=\frac{1}{\lambda_{ij}\coth\lambda_{ij}H}$$

$$E=\begin{cases}\dfrac{1}{aL} & when\ i=0,j=0\\ \dfrac{2}{aH} & when\ i=0,j\neq0\ \ or\ \ i\neq0,j=0\\ \dfrac{4}{aH} & when\ i\neq0,j\neq0\end{cases}$$

2.3 Natural frequency of a stiffened plate in contact with liquid

The natural frequency of a stiffened plate can be calculated with the structure modal mass, modal stiffness and added mass.

$$[K_{\text{modal}}][a_{add}]-[\Lambda_{add}][M_{\text{modal}}+M_{add}][a_{add}]=0, \quad [\Lambda_{add}]=[\varpi^2_{add}] \tag{18}$$

where

$$M_{add,rs} = A_{rs}$$

3. COMPARISON AND VALIDATION

Based upon the proposed methodology, a couple of numerical calculations are carried out and compared with NASTRAN, which is a three dimensional finite element program. Figure 3 shows a rectangular steel tank filled with water. The thickness of the wall is 1.4 mm. Calculation was carried out for 2 cases, at the 100% water filling level of tank depth.

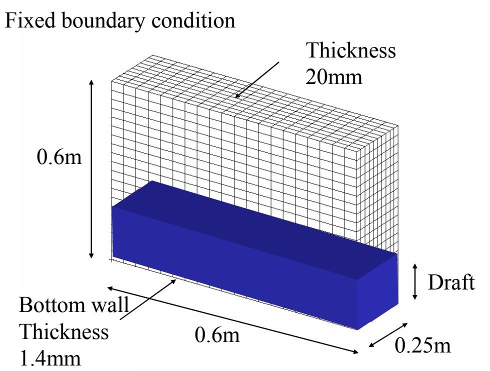

Fig. 3 A rectangular tank filled with water for validation

The fixed boundary condition was imposed on all edges and walls except wall of interest. Table 1 shows the natural frequencies obtained by the proposed theory and NASTRAN.

Table 1 Natural frequencies of side and bottom plates for the tank

Fluid contact / Mode	Bottom plate			Side plate		
	Theory	NASTRAN	Discrepancy (%)	Theory	NASTRAN	Discrepancy (%)
1	22.8	23.0	0.9	26.8	27.2	1.5
2	47.0	47.4	0.8	50.7	51.7	1.9
3	74.1	76.2	2.8	77.9	80.5	3.2

It is found that the proposed theory agrees very well with the NASTRAN result within a 3.2% discrepancy range as shown in Table 1. Figure 4 shows the mode shapes of the bottom and side plate of the tank respectively.

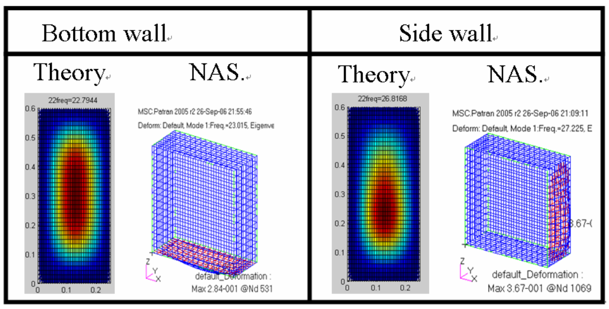

Fig. 4 The mode shapes of bottom and side wall

4. EFFECT OF VARIOUS BOUNDED FLUID ON ADDED MASS

- **Effect of the tank-length on a side plate**

As for a study on a hydroelactic vibration of a tank, little attention has been shown to the effect of a tank-length on the natural frequencies of a side plate. As shown in Fig. 2, the tank-length is the length between the plate of interest (face A) and it's the opposite wall (face B) in the y direction. The natural frequencies of the side elastic plate (face A) are affected by variation of the tank-length(L). According to Eq.(17), effect of the tank-length on the added mass is solely dependent upon R_{ij} .

The added mass effect on a side plate is exponentially decreased as a tank length is increased and it is converged when the tank-length (L) is reached to about the same length of the draft (H) as shown in Fig.5. Figure 5 shows effect of the tank-length on the natural frequency for each mode at the draft 0.6m. It can be seen that an increase in the length of the tank results in the higher natural frequencies of the side plate, which tend to approach constant values, when the tank-length(L) reaches the similar dimension of the draft (H).

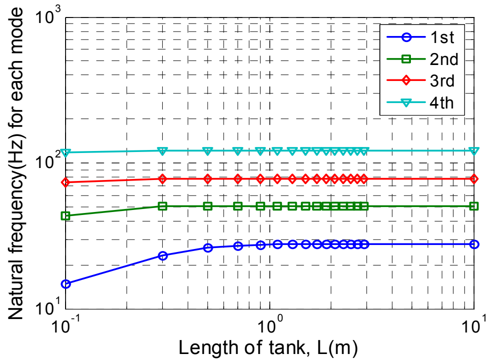

Fig. 5 Effect of the tank length on a side plate

- **Effect of semi-infinite and finite fluid on a side plate**

The natural frequencies are calculated by using NASTRAN and compared for a rectangular side plate in contact with an infinite fluid and a finite fluid, varying the water filling level. As shown in Fig. 6, it is found that about a 50% of discrepancy of the fundamental natural frequency between plates coupled with an infinite and a finite fluid at 90% filling level.

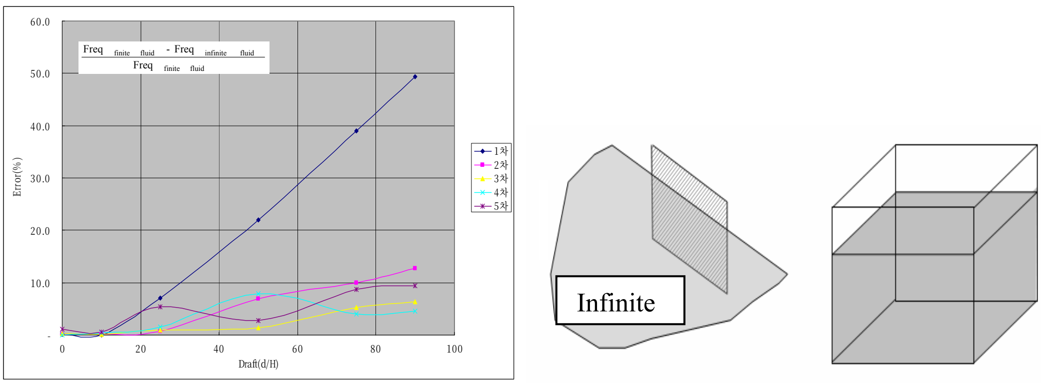

Fig. 6 Natural frequency difference of a plate in contact with an infinite and a finite fluid

- **Effect of breadth on a side plate**

Figure 7 shows effect of the breadth on the natural frequency for each mode at the draft 0.6m. The natural frequency of a side plate is exponentially increased as the fluid-breadth is increased and it is converged when the fluid-breadth(c) is reached to about the same length of the tank-breadth (a) as shown in Fig.7.

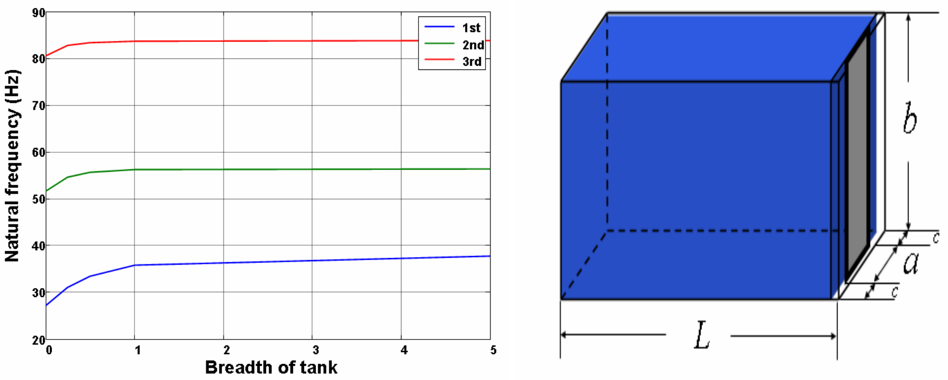

Fig. 7 Effect of tank breadth on a side plate

5. CONCLUSIONS

An analytical method to estimate the natural frequency of a bottom and side wetted plate is developed. Through out the tests, it is found that this approach can properly deal with the fluid-structure interaction of a tank. The calculation results show very good agreement with the three-dimensional finite element method. In addition, the change of the natural frequency is investigated for the various bounded fluids. And it is found that the natural frequency is increased when the fluid field becomes wider, that is when the fluid-breadth and length is bounded narrower, the natural frequencies of a wetted plate become lower.

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