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MODELING OF THE SOUND TRANSMISSION LOSS OF LEAKS AND OPENINGS

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Abstract

The airborne sound transmission performance of structures may be strongly affected by the presence of apertures. These apertures might be designed on purpose to allow for the circulation of matter (machinery enclosures) or for the passage of wires and pipes through the structure (vehicles, buildings). They are then referred to as openings. They could also be unwanted or not part of the initial design due to bad assembly and/or mounting conditions. They are then called leaks. These two kinds of apertures differ by their size relative to the acoustic wavelength. To predict the insulating performance of a structure involving apertures, it is often required to know the oblique incidence or diffuse field sound transmission loss (TL) of the aperture. Numerous models exist for the normal incidence TL but rigorous models for oblique incidence and diffuse field excitations have rarely been considered. The diffuse field TL is usually obtained using a correction factor applied to the normal incidence TL which ranges from 0db to 5dB depending on the authors. The purpose of this paper is to propose a general efficient model based on the description of the field inside the aperture in terms of propagating and evanescent acoustic modes together with an efficient computation of modal radiation impedance matrices to predict the oblique incidence and diffuse field TL of rectangular and circular apertures of finite depth. The model is validated by comparison with existing experimental data and models. Numerical results illustrating the normal incidence, oblique incidence and diffuse field TL of apertures are then provided and the relationships between these indicators are discussed.

1. INTRODUCTION

The airborne acoustic transmission performance of enclosures strongly depends on the presence of leaks and openings. The leaks are the result of a bad assembly and denote apertures whose size is small compared to the acoustic wavelength. The openings have generally larger dimensions and are rather designed on purpose to allow for the circulation of matter between the external environment and the machinery through the enclosure. During the acoustic design of enclosures using simple analytical or energy based models, predicting the effect of an aperture on the overall acoustic transmission can become crucial and requires the knowledge of the aperture diffuse field sound transmission loss. Despite its importance, relatively few authors have dealt with this problem and there is little consensus on which formulations work best. In previous studies, apertures were considered as rectangular, slit or circular shapes with negligible or finite thickness since these geometries are representative of most practical cases. In a recent paper, the authors have presented a comprehensive literature review and comparisons of the principal approaches available in the literature. They have pointed out several issues (i) the lack of a model that would predict accurately and rapidly the diffuse field sound transmission loss in the acoustical frequency range whatever the size of the aperture; (ii) the relevance of using the normal incidence transmission loss (with or without corrections) to predict the diffuse field TL (iii) the lack of an efficient tool for predicting the broadband diffuse field sound transmission loss of both circular and rectangular apertures in the frequency range covered in most noise control applications (100Hz-5000Hz); (iv) the lack of numerical results regarding the diffuse field sound transmission loss of openings (large apertures). To address these questions, they have introduced a general and rigorous formulation to predict the oblique incidence and diffuse field sound transmission loss (TL) of rectangular and circular apertures of finite depth. The developed tool is simple, user-friendly, efficient and can easily be utilized to provide input data to SEA codes for instance. In the present paper, additional validation and numerical results are provided. The rest of the paper is organized as follows. Firstly, the theoretical approach developed in [1] is briefly recalled. The model is based on the description of the field inside the aperture in terms of propagating and evanescent acoustic modes together with an efficient computation of modal radiation impedance matrices. Secondly, comparisons between existing experimental results and calculations are given. Finally, numerical results illustrating the normal incidence, oblique incidence and diffuse field TL of apertures are presented and the relationships between these indicators are discussed.

2. THEORY

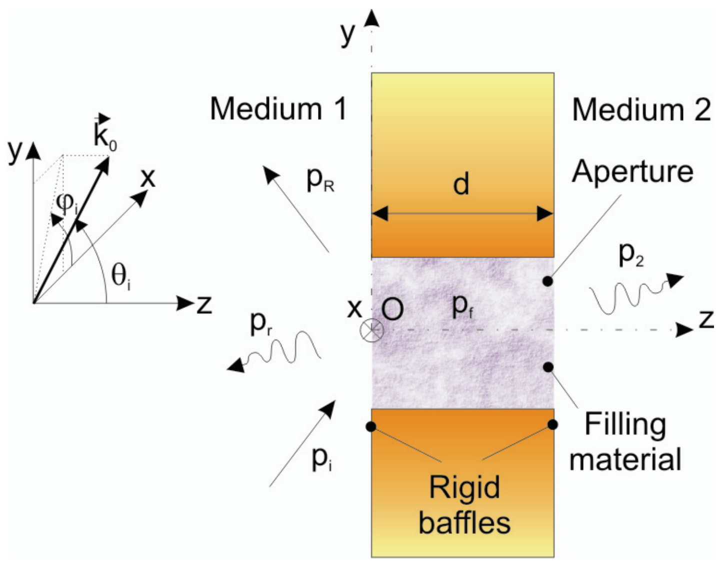

Figure 1. Scheme of an aperture filled with an absorbing material inserted into a rigid planar baffle

Consider a rectangular or circular aperture of area S and depth d inserted into a rigid planar baffle and excited acoustically by an oblique plane wave with incidence angles (θ_i,φ_i) as illustrated in Fig.1.

The total acoustic field $\hat{p}_1$ in medium 1 is given by :

$$\hat{p}_1 = \hat{p}_i + \hat{p}_R + \hat{p}_r = \hat{p}_b + \hat{p}_r \tag{1}$$

with the convention $p(t) = \hat{p}\, e^{j\omega t}$. p_i is the incident acoustic pressure (of amplitude A_i), p_b is the blocked acoustic pressure ($\hat{p}_b = 2\hat{A}_i e^{-jk_0(\sin\theta_i \cos\varphi_i x + \sin\theta_i \sin\varphi_i y)} \cos\left(k_0 \cos\theta_i z\right)$), p_r is the acoustic pressure radiated by the front face of the aperture (excitation side) and p_2 the acoustic pressure radiated in medium 2 by the back face of the aperture. p_r and p_2 are classically given by Rayleigh's integrals.

The acoustic field inside the aperture is noted p_f=p_{f1}+p_{f2}. The aperture can be filled with an acoustic material of characteristic impedance $\hat{Z}_f$ and complex propagation constant $\hat{k}_f$. For simple shapes such as rectangular and circular cross sections, the acoustic field inside the aperture can be expanded analytically in terms of propagating and evanescent modes:

$$\hat{p}_f\left(x,y,z\right) = \sum_P \left(\hat{A}_P e^{-j\hat{k}_P(z-d)} + \hat{B}_P e^{j\hat{k}_P(z-d)}\right)\phi_P\left(x,y\right) \tag{2}$$

where for a rectangular cross section of size 2a×2b $\hat{k}_P = \hat{k}_{pq} = \sqrt{\hat{k}_f^2 - \left(\frac{p\pi}{2a}\right)^2 - \left(\frac{q\pi}{2b}\right)^2}$, $\phi_P\left(x,y\right) = \phi_{pq}\left(x,y\right) = \cos\left(\frac{p\pi}{2a}\left(x+a\right)\right)\cos\left(\frac{q\pi}{2b}\left(y+b\right)\right)$ and for a circular cross section of radius a, $\hat{k}_P = \hat{k}_{pq} = \sqrt{\hat{k}_f^2 - \frac{\lambda_{pq}^2}{a^2}}$, $\phi_P\left(x,y\right) = \phi_{pqs}\left(r,\gamma\right) = J_p\left(\frac{\lambda_{pq} r}{a}\right)\sin\left(p\gamma + s\frac{\pi}{2}\right)$, λ_{pq} being the zeroes of the derivative of Bessel function of order p ($J_p'\left(\lambda_{pq}\right) = 0$) and s being a symmetry index equal to 0 or 1.

The next step is to write down the boundary conditions on both sides of the aperture (continuity of pressure and normal particle velocity). Substituting Eq(2) and Rayleigh's integrals for p_r and p_2 into these equations, multiplying them all by $\phi_M\left(x,y\right)$ and integrating over the surface of the front and back faces of the aperture leads to the following linear system:

$$\begin{bmatrix} [\Psi_1] & [\Psi_2] \\ [\Psi_3] & [\Psi_4] \end{bmatrix} \begin{Bmatrix} \{\hat{A}_M\} \\ \{\hat{B}_M\} \end{Bmatrix} = \begin{Bmatrix} \{\hat{F}_M\} \\ \{0\} \end{Bmatrix} \tag{3}$$

where it has been assumed that medium 1 and 2 are identical ($\rho_1 = \rho_2 = \rho_0$ and $c_1 = c_2 = c_0$) and with

$$\Psi_{1,MP} = N_M^2 e^{j\hat{k}_M d}\delta_{MP} + \frac{S}{\hat{Z}_f}\frac{\hat{k}_P}{\hat{k}_f} e^{j\hat{k}_P d}\hat{Z}_{MP} \tag{4}$$

$$\Psi_{2,MP} = N_M^2 e^{-j\hat{k}_M d}\delta_{MP} - \frac{S}{\hat{Z}_f}\frac{\hat{k}_P}{\hat{k}_f} e^{-j\hat{k}_P d}\hat{Z}_{MP} \tag{5}$$

$$\Psi_{3,MP} = N_M^2 \delta_{MP} - \frac{S}{\hat{Z}_f}\frac{\hat{k}_P}{\hat{k}_f}\hat{Z}_{MP} \tag{6}$$

$$\Psi_{4,MP} = N_M^2 \delta_{MP} + \frac{S}{\hat{Z}_f} \frac{\hat{k}_P}{\hat{k}_f} \hat{Z}_{MP} \tag{7}$$

with

$$\begin{cases} \hat{F}_M = \int_S \hat{p}_b(x,y)\phi_M(x,y)dS \\ \hat{Z}_{MP} = \frac{j\hat{k}_f\hat{Z}_f}{S}\int_S\int_S \phi_M(x,y)G(x,y,x_0,y_0)\phi_P(x_0,y_0)dS(M_0)dS(M) \\ N_M^2 = \int_S \phi_M^2(x,y)dS(M) \end{cases} \tag{8}$$

$\hat{F}_M$ is the generalized force acting on mode M, $\hat{Z}_{MP}$ is the aperture cross modal radiation impedance between modes M and P and N_M^2 is the squared norm of mode M. For a rectangular section, $\hat{Z}_{MP}$ can be efficiently calculated using a change of variables [2] which transforms the quadruple integral into a double integral that can be resolved with a Gauss integration scheme. For a circular cross section, this quadruple integral can be reduced to a simple one if the problem is solved in the wavenumber domain. All the details are provided in [1].

Once the system has been solved, the transmitted acoustic power for oblique incidence excitation can be calculated as:

$$\Pi^t(\theta_i,\varphi_i) = \frac{1}{2}\Re\left[-\frac{1}{\tilde{\rho}_f^*\omega}\sum_M N_M \hat{k}_M^* \hat{C}_M \hat{D}_M^*\right] \tag{9}$$

with $\hat{C}_M = \left(\hat{A}_M + \hat{B}_M\right)$ and $\hat{D}_M = \left(-\hat{A}_M + \hat{B}_M\right)$.

The oblique incidence transmission coefficient $\tau(\theta_i,\varphi_i)$ is defined as:

$$\tau(\theta_i,\varphi_i) = \frac{\Pi^t(\theta_i,\varphi_i)}{\Pi^{inc}(\theta_i,\varphi_i)} \tag{10}$$

where $\Pi^{inc}(\theta_i,\varphi_i)$, the incident power, is given by $\Pi^{inc}(\theta_i,\varphi_i) = \frac{\left|\hat{A}_i\right|^2 \cos\theta_i S}{2\rho_0 c_0}$.

Using the previous equations, the oblique incidence transmission coefficient then reads :

$$\tau(\theta_i,\varphi_i) = -\frac{\rho_0}{k_0\cos\theta_i\rho_f^*\left|\hat{A}_i\right|^2 S}\Re\left(\sum_M N_M \hat{k}_M^* \hat{C}_M \hat{D}_M^*\right) \tag{11}$$

The diffuse field sound transmission coefficient can then be calculated numerically using a Gauss integration scheme :

$$\tau_d = \frac{\int_0^{2\pi}\int_0^{\theta_{\lim}} \tau(\theta_i,\varphi_i)\sin\theta_i\cos\theta_i d\theta_i d\varphi_i}{\pi\sin^2\theta_{\lim}} \tag{12}$$

where the limit angle $\theta_{\lim}$ is taken equal to 78° (field incidence) in all the following numerical examples. The sound transmission loss (TL) is finally calculated using $TL = -10\log_{10}(\tau)$.

3. RESULTS

4.1 Validations

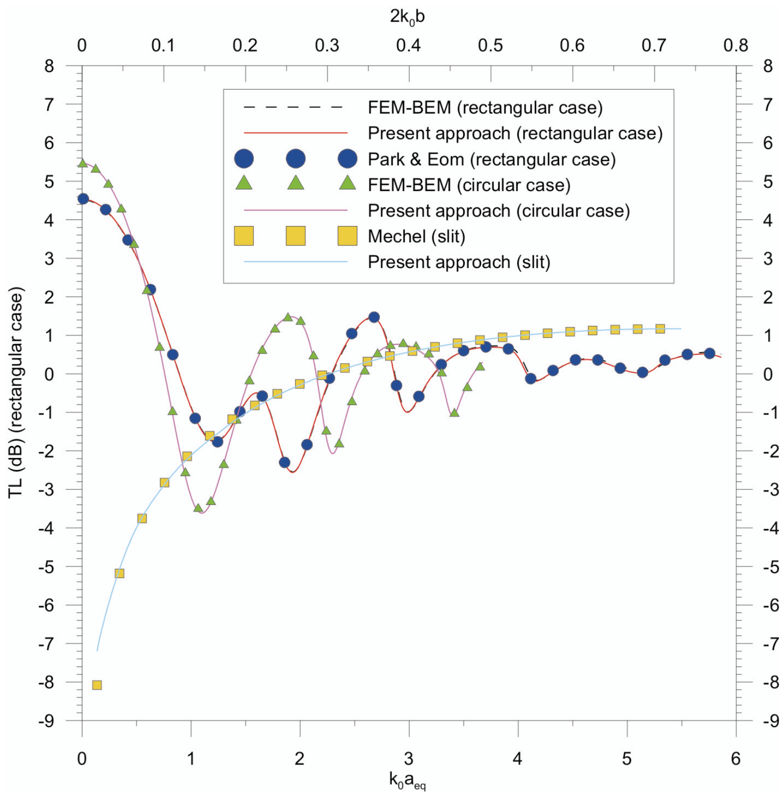

Figure 2. Oblique incidence (θ_i=45°,φ_i =0°) transmission losses of a rectangular aperture (b/a=1/2, d/a=1), a circular aperture (d/a=2/3) and a slit (d/b=2) as a function of normalized frequency $k_0 a_{eq}$ with $a_{eq} = \sqrt{4ab/\pi}$ the equivalent radius for rectangular shape and $a_{eq} = a$ for circular shape (bottom axis) and $2k_0 b$ for slits (top axis)

In [1], the proposed approach has been validated for various configurations (rectangular and circular cross section apertures and slit-shaped) using existing analytical formulas when available [3,4,5] or numerical models such as Park and Eom's [6] or FEM/BEM. As an illustration, Figure 2 displays the transmission losses calculated using the present approach together with numerical models (FEM/BEM and Park & Eom's method) for a rectangular and a circular aperture excited by an oblique incidence plane wave. The oblique incidence transmission loss of a slit shaped aperture calculated using Mechel's analytical model [5] and the present approach is also plotted. Note that for the analytical model the slit is assumed of infinite breadth whereas in the present approach it is considered of finite length with a=400b. A perfect agreement is observed for all the tested cases for example FEM/BEM results coincide with the present approach. The convergence study carried out in [1] has indicated that it was sufficient to keep modes up to the maximum frequency of interest to insure the convergence of the solution convergence within a 0.1dB error. In this paper, additional validation results are presented.

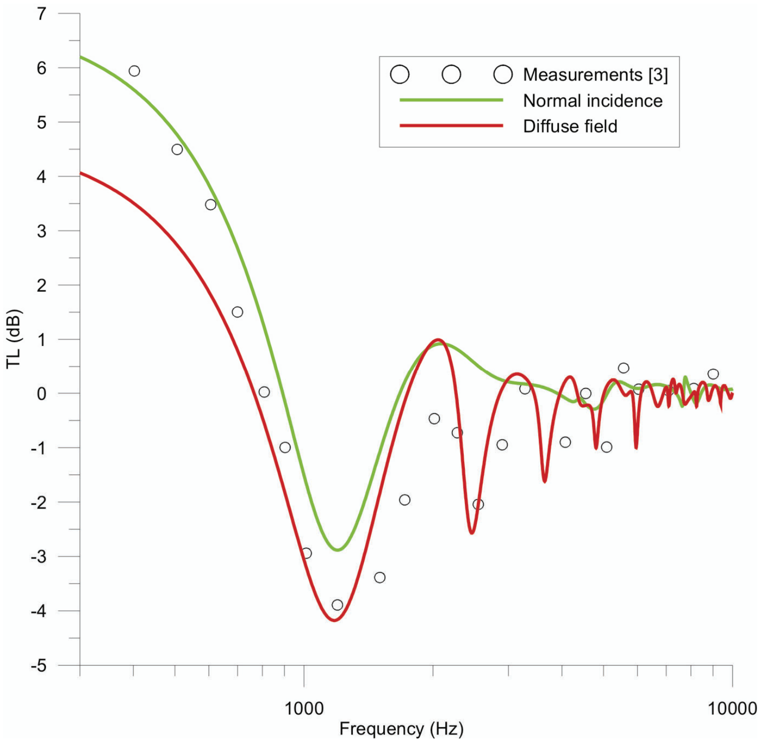

Figure 3. Comparisons between measured and calculated normal incidence and diffuse field transmission losses in the case of a circular aperture (a=5.08cm, d=7.62cm) as a function of frequency.

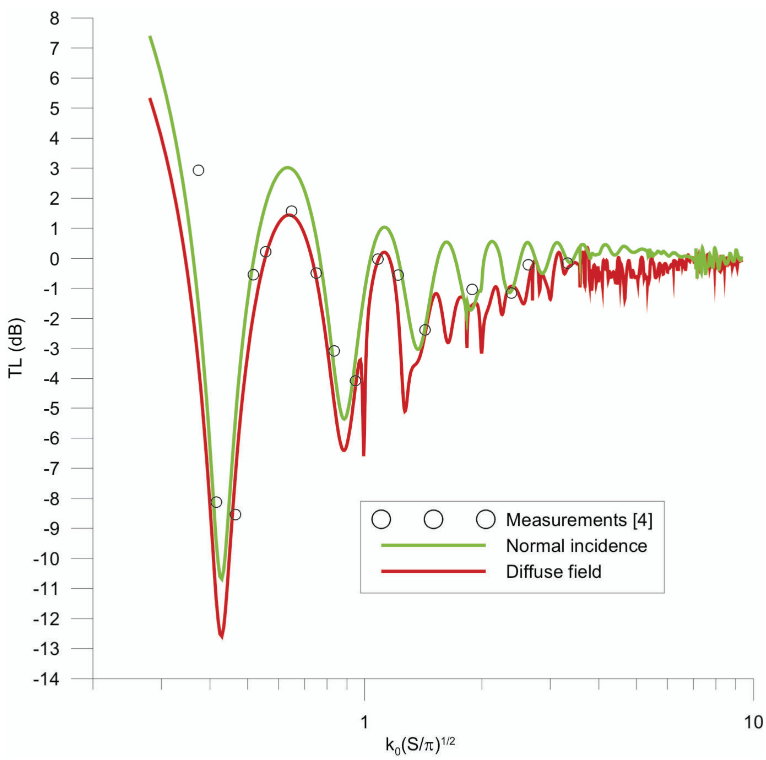

Figure 4. Comparisons between measured and calculated normal incidence and diffuse field transmission losses in the case of a rectangular aperture (2a=18cm, 2b=4.5cm, d=30.48cm) as a function of normalized frequency $k_0 a_{eq}$ with $a_{eq} = \sqrt{4ab/\pi}$ the equivalent radius.

Figures 3 and 4 display a comparison of calculations using the present approach and sound transmission measurements between two reverberant rooms carried out by Wilson and Soroka [3] and Sauter and Soroka [4] for circular and rectangular shaped apertures. Both the calculated normal incidence and diffuse field sound transmission loss are plotted. It is seen that there is a good agreement between the model and the experimental data. In particular, Figures 3 and 4 show that the diffuse field model reproduces very well the peaks and troughs observed in the measurements except maybe at low frequencies. In this frequency range, the acoustic field is indeed probably not diffuse enough for the diffuse field excitation model to be valid. In addition, these figures indicate as demonstrated in [1] that (i) the normal incidence model is able to capture most of the physics (ii) the difference between normal incidence and diffuse field excitations is small (of the order of 1 to 2 dB on average and up to 3dB locally for the cases investigated here).

4.1 Normal incidence versus diffuse field sound transmission loss

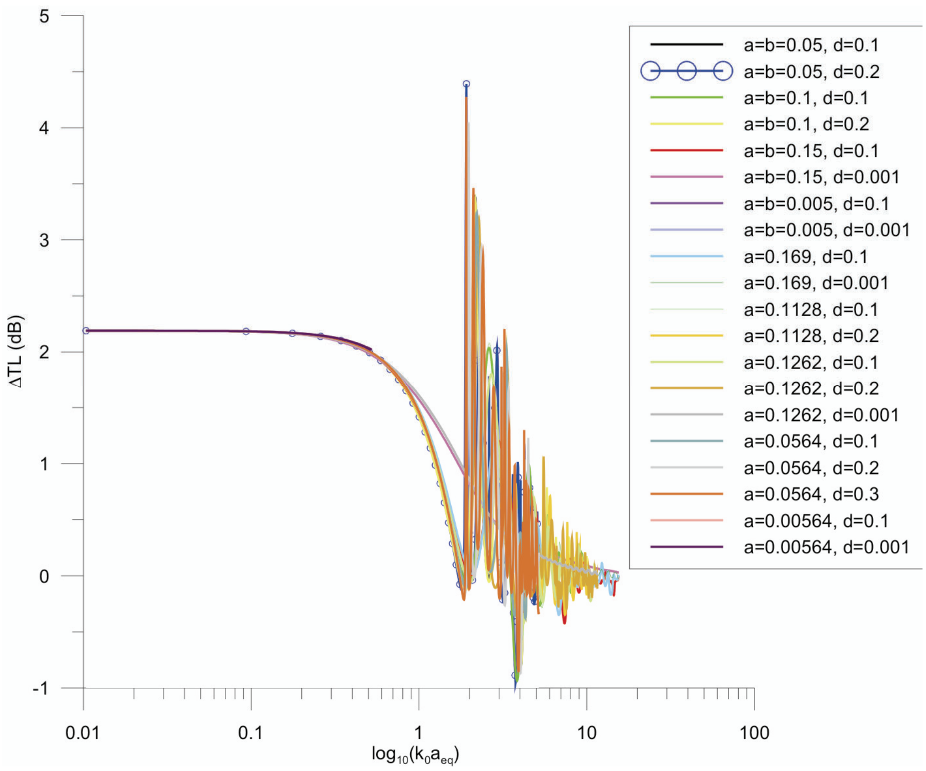

Figure 5. Narrow band differences between normal incidence and diffuse field sound transmission losses for investigated apertures calculated using the present approach as a function of normalized frequency

Fig.5 plots the narrow band calculated differences between TL_d and $TL(0)$ as a function of normalized frequency k_0a_{eq} for multiples size of rectangular and circular apertures whose characteristics are presented in the legend. At low frequencies $k_0a_{eq} < 0.3$, the difference between TL_d and $TL(0)$ is equal to 2.2dB when $\theta_{\lim} = 78°$ is used. At high frequencies $k_0a_{eq} > 7$, the difference between TL_d and $TL(0)$ is less than 1dB. In between, the differences oscillate and can reach up to approximately 5dB for very thick apertures. When the third-octave band averaged results are considered, the same observations remain but in the mid frequency zone, the difference between TL_d and $TL(0)$ is less than 2dB [1]. These conclusions are in agreement with the observations made by previous authors [3,4] on the differences between experimental results (diffuse field excitation) and their normal incidence model.

4. CONCLUSIONS

In this paper, a general and efficient numerical method to predict the diffuse field sound transmission loss of baffled apertures of rectangular and circular cross-sections has been introduced. The wave field inside the aperture is described in terms of propagating and evanescent acoustic modes and the acoustic radiation of the aperture is taken into account with a modal radiation impedance matrix whose calculation is carried out using efficient numerical algorithms. The coupled problem is solved in terms of modal contribution factors from which the transmission loss can be evaluated. The developed tool is simple, portable, does not require meshing tools such as those employed in BEM/FEM techniques and can easily be utilized to provide input data to SEA codes for instance.

The prediction tool has been validated by comparisons with existing analytical or numerical models in various configurations [1]. This paper has provided additional validation information by comparing numerical results and experimental existing data in the case of rectangular and circular apertures. The diffuse field model allows for a better agreement with experimental results. Results regarding the narrow band diffuse field sound transmission loss have been confronted to the classical normal incidence sound transmission loss models. It has been observed that a simple normal incidence transmission loss model for apertures can be used with a correction factor of about 2dB at low frequencies. The maximum difference between the narrow band diffuse field and normal incidence transmission losses is expected to be less than 5dB at medium frequencies, and 1dB at high frequencies. For averaged band indicators, the 5dB are reduced to 2dB.

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