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SYMMETRY METHODS APPLIED TO THE CONDITION-MONITORING OF SLOW SPEED SLEW BEARINGS

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Abstract

All data has symmetry and anti-symmetry embedded in it. This paper discusses various aspects of symmetrical transformations that can be applied to data to give machine condition information. These transformations lead to novel views of the information. Combining these symmetrical transformations with the more traditional methods of Eigenvalue/vector and Fast Fourier Transform analyses provides views of the source and extent of the problem. To illustrate the various views of the vibration information, results from a large (4.2m diameter) Coal Reclaimer slow speed (4rpm) slew bearing are presented. Many of the transformations are also currently being evaluated on an experimental test-rig specifically designed for the monitoring of horizontal slow speed (1 to 4 rpm) slew bearings.

1. INTRODUCTION

The use of vibration data from slow-speed slew-bearings has been notoriously unsuccessful in predicting bearing failure. There are a number of reasons for this. Primarily, the very slow speeds involved (1 - 4 rpm) lead to very low rotational energy release. The operation of a slew bearing is often intermittent and non-cyclic. The data used in this paper is from a Coal Reclaimer. This machine has 2 large 4.2m diameter, vertically mounted, slew bearings supporting the reclaiming buckets and rotates at approximately 4.3 rpm in one direction in a continuous mode.

Current condition monitoring methods provide a sample of data that is short in duration containing a few thousand samples (4098 max). Demodulation/Fourier type data analyses of vibration from slew bearings have been unsuccessful in determining the useful life and /or time to replace. Therefore, we have to ask two simple questions.

What can we extract from data that is new and informative? We can find information that involves destroying/modifying the original data and/or information that does not involve destroying/modifying the original data. Any process that modifies/destroys information can be ‘classified’ as a filter.

How can we determine a recognisable pattern in a set of data such that the data remains essentially the same? Symmetry/anti-symmetry springs to mind when we talk about a recognisable pattern. We introduce a number of operators that extract the symmetry/anti-symmetry that is embedded in data and illustrate the use of these operators on raw acceleration data obtained from a slow speed slew bearing on the Coal Reclaimer previously described. Surprisingly, McWeeny [1] and Cantwell [2] offer very little really useful information on symmetry of data methods. Pickover [3] and Wu [4] demonstrate an interesting visualisation which forces the data into a symmetrical state space pattern. However this image is not reliant on the symmetry embedded in the data.

2. SYMMETRY OF A VECTOR

Stenger [5] when discussing point-of-view invariance says that 'Often invariance is expressed by the term *symmetry*' and goes on to say 'If you wish to build a model using space-time as a framework and you formulate that model so as to be space-time symmetric, then that model will automatically contain what are usually regarded as the three most important "laws" of physics, the three conservation principles'. These three; energy, linear momentum and angular momentum, must be conserved. So it seems that symmetry is a desirable, fundamental property to be explored.

We begin by looking at a dataset $\overset{>}{x}$ containing two members x_1 and x_2. Note that this is a vector hence the > indicating a forward direction. We can define the symmetric vector $\overset{>+}{x} = 0.5\langle x_1 + x_2, x_2 + x_1 \rangle$ and the anti- (skew) symmetric vector $\overset{>-}{x} = 0.5\langle x_1 - x_2, x_2 - x_1 \rangle$ such that $\overset{>+}{x} + \overset{>-}{x} = \overset{>}{x}$. Obviously for completeness we could have a vector $\overset{<}{x}$. In general, any length vector $\overset{>}{x}$ has $\overset{>+}{x} = 0.5\langle x_i + x_{n-i-1} \rangle$ and $\overset{>-}{x} = 0.5\langle x_i - x_{n-i-1} \rangle$ for $i = 0,1,2...m = n-1$. This statement also says that any 'truly' random sequence has by default symmetry and anti-symmetry as properties; be it global or local, and hence the notion of randomness is not quite correct. Note that these operations can be classified as first order non-linear filters and that we now have two orthogonal vectors from a single vector. They are orthogonal because the vector dot product [6] $\overset{>+}{x} \cdot \overset{>-}{x} = 0$. As a consequence we can say all symmetric and anti-symmetric vectors are orthogonal. That is, any symmetric vector of length n is orthogonal with any n length anti-symmetric vector. For completeness $\overset{>\times}{x} = \langle x_i x_{n-i-1} \rangle$ is symmetric and a 2nd order filter whereas $\overset{>\div}{x} = \langle x_i \div x_{n-i-1} \rangle$ is neither symmetric nor anti-symmetric.

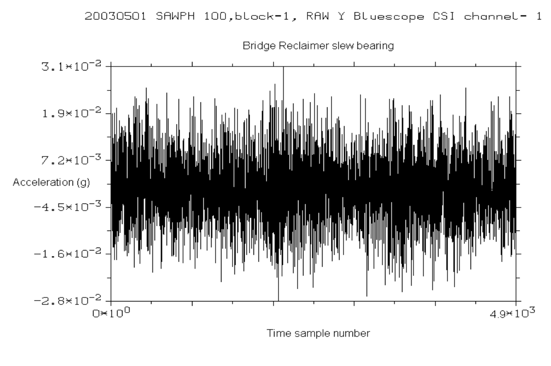


Figure1. $\overset{>}{x}$ Acceleration vs. time at 01/05/2003.

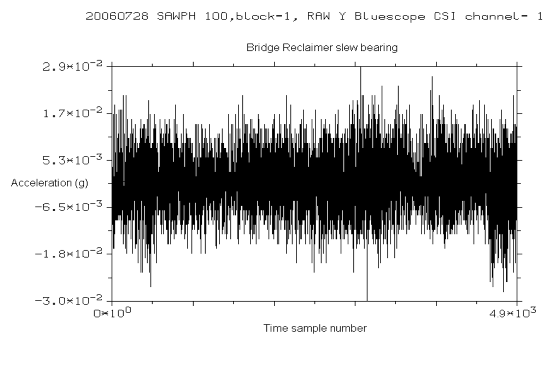


Figure2. $\overset{>}{x}$ Acceleration vs. time at 28/07/2006.

Figures 1 and 2 are the raw accelerometer vectors for the two dates 01/05/2003 and 28/07/2006

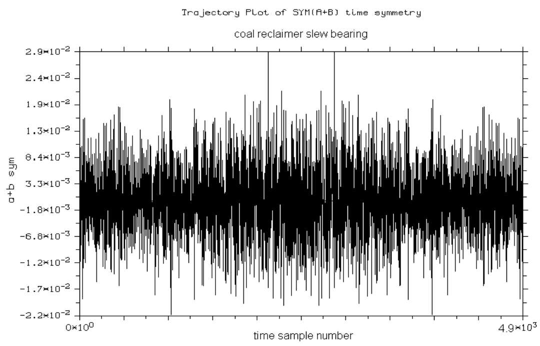


Figure3. $\overset{>+}{x}$ Acceleration vs. time at 01/05/2003.

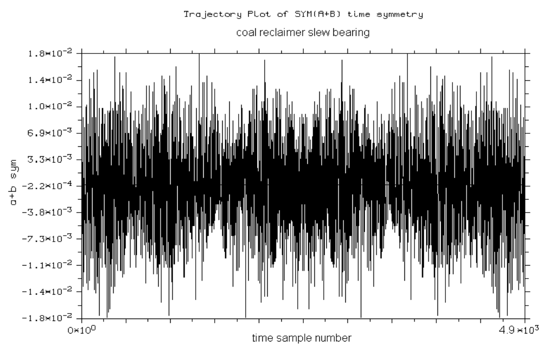


Figure4. $\overset{>+}{x}$ Acceleration vs. time at 28/07/2006.

Figures 3 and 4 are the global symmetric vectors and Figures 5 and 6 are the global anti-symmetric vectors for the two dates 01/05/2003 and 28/07/2006

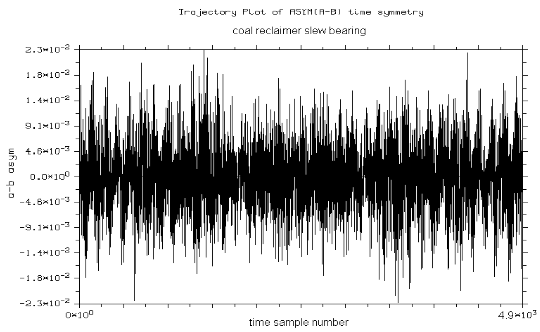


Figure5. $\overset{>-}{x}$ Acceleration vs. time at 01/05/2003.

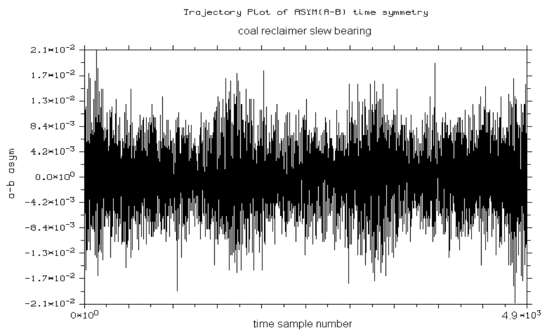


Figure6. $\overset{>-}{x}$ Acceleration vs. time at 28/07/2006.

Note in both instances, $\overset{>+}{x}$ and $\overset{>-}{x}$, we see more structure in the images. A visually more distinctive pattern is visible when compared to Figures 1 and 2. The $\overset{>+}{x}, \overset{>-}{x}$ pair allow us to produce a state space (probability) map that contains all this geometric symmetry. Figure 7 and Figure 8 are examples of this. They are formed by plotting $\overset{>+}{x}$ versus the $\overset{>-}{x}$.

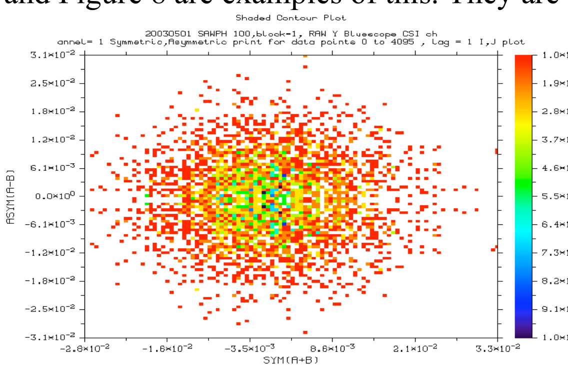


Figure7. $\overset{>+}{x}$ vs. $\overset{>-}{x}$ Accel'n space at 01/05/2003.

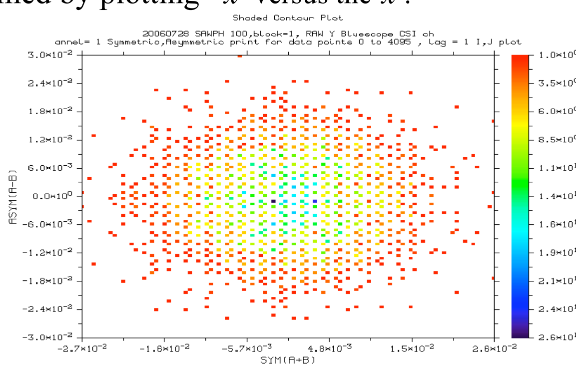


Figure 8. $\overset{>+}{x}$ vs. $\overset{>-}{x}$ Accel'n space at 28/07/2006.

You may note the spreading out of the image in Figure 8 in comparison to Figure 7. This is the result of progressive bearing wear.

We now introduce the following convention that $\overset{>+}{x}\,\overset{>-}{x}$ is the forward symmetric transform of a forward asymmetric transform of $\overset{>}{x}$. You may note that if we try to do this operation for $\overset{>+}{x}$ or $\overset{>-}{x}$ we get $\overset{>+}{x}\,\overset{>-}{x} = \overset{>-}{x}\,\overset{>+}{x} = 0$, $\overset{>-}{x}\,\overset{>-}{x} \neq 0$ and $\overset{>+}{x}\,\overset{>+}{x} \neq 0$. This result points us in two different directions. The first direction is to somehow extend $\overset{>+}{x}\,\overset{>-}{x}$ or $\overset{>-}{x}\,\overset{>+}{x}$ terms. The second direction is to extend $\overset{>+}{x}\,\overset{>+}{x}$ or $\overset{>-}{x}\,\overset{>-}{x}$ terms. This leads us to a discussion about symmetry breaking.

3. SYMMETRY BREAKING

We now return to the first direction on handling $\overset{>+}{x}\,\overset{>-}{x}$ and $\overset{>-}{x}\,\overset{>+}{x}$. Here we have considerably more choice. We can do this by considering the concept of breaking symmetry [7, 8]. It turns out that there are many different ways to 'break symmetry'. At this point we introduce the symbol ψ as our symmetry breaking operator and $\overset{>+\psi}{x}$ indicate that we have formed the forward symmetry of $\overset{>}{x}$ and broken it. We can break symmetry by considering the possible combinations that are permissible with removing only a minimum of data. Minimum here implies no data lost or perhaps the very last (or first) value in the dataset.

Whilst we consider the dataset as one fixed length vector we can only form one symmetry/anti-symmetry operation. We can treat the dataset as consisting of $p = \mathrm{int}(n \div m)$ sequential groups of sub-datasets where n, m are integers such that $p \times m = n$ and $m \leq n$ is the number of data elements in each subset. Our initial discussion on symmetric vectors, treated the case where $m = n$ and this is the condition we discuss predominantly in this paper. Other possibilities are a series of operations where we consider each factor m in $m! = n$ subject to $p \times m = n$. This method provides some continuity in the process. Alternatively the set of numbers p that satisfy $p \times m = n$ would not necessarily provide continuity between successive operations $\overset{>+\psi}{x}\;\overset{>-\psi}{x}$.

Yet another possibility of symmetry breaking is to consider subsets of data that are equally spaced, that is, $\overset{>+\psi}{x} = \left\langle x_{i-m}, x_i, x_{i+m} \right\rangle$ for $i = m, m+1, \ldots, n-m-1$. We use this form in the Symmetric Wave Decomposition (SWD) [9] algorithm which is a special form of symmetry breaking for $\overset{>+\psi}{x} = \overset{>+}{x}\,\overset{>+}{x}\,\overset{>+}{x}\,\overset{>+}{x}\ldots$. This is achieved by recursively visiting near neighbours in the set. This allows us to reconstruct in correct time sequence the individual waves contributing to the final measured waveform. No redundant information is generated.

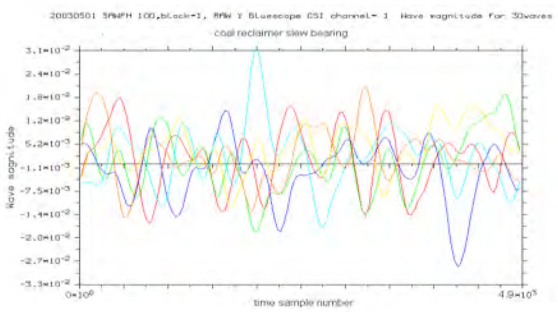


Figure9. SWD Accel'n vs. time at 01/05/2003.

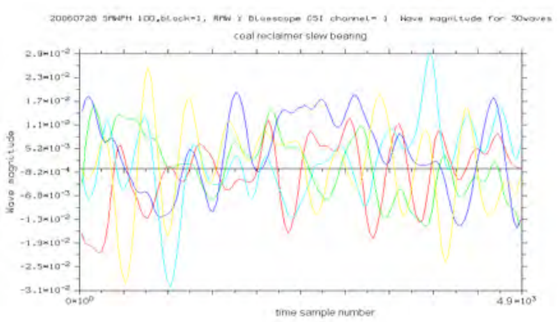


Figure10. SWD Accel'n vs. time at 28/07/2006.

The Figures 9 and 10 illustrate some of the output waves for the 2 datasets of the Coal Reclaimer. Each colour identifies an individual 'decomposed' wave. Note the number of waves whose peak magnitude exceeds 2×10^{-2} .The increase from 2 to 5 waves is significant but in a field of 30 possible waves implies that things are still quite acceptable.

3.1 Vector Compression

We now discuss a special version of the second direction which turns out to be a method for compressing data.

You may note that symmetry introduces a degree of redundancy, that is, one half of the data is the same as the other in $\overset{>+}{x}$ and that the second half of $\overset{>-}{x}$ is the negative of the other. We can exploit this redundancy by removing the last (or first) half of the data. This is another 'symmetry breaking'. If we repeatedly 'break symmetry' for $\overset{>+}{x}$ and $\overset{>-}{x}$ we compress the data to two values. The symmetric operator $\overset{>+\psi}{x}\ \overset{>+\psi}{x}\ \overset{>+\psi}{x}\ \overset{>+\psi}{x}\cdots\Rightarrow\sum_{i=0}^{n-1}\underset{i}{x}$ is the sum of numbers in the dataset. The anti-symmetric operator $\overset{>-\psi}{x}\ \overset{>-\psi}{x}\ \overset{>-\psi}{x}\ \overset{>-\psi}{x}\cdots\Rightarrow\sum_{i=0}^{n-1}-1^{i}\underset{i}{x}$ is the anti-sum of numbers in the dataset for datasets of length 2^{n}. The more general algorithm which includes all length datasets is more complex and not included here. This is a new statistic. The sum is a time invariant measure of the data as long as each element in the dataset is counted once. The anti-sum is not time invariant. The anti-sum is dependent on the order of the data. This number pair (sum, anti-sum) provides us with a state space instance for a dataset.

3.2 Vector Expansion

Note that so far we have decomposed one vector into two vectors. It is possible to break symmetry by 'joining' the two vectors $\overset{>+}{x},\overset{>-}{x}$ to form a new vector $\overset{>+\psi}{x}=\left\langle \underset{i}{\overset{>+}{x}},\underset{i}{\overset{>-}{x}}\right\rangle$ for $i=0,1,2...m=n-1$ that is now length $n'=2n$. This is a form of data expansion.

There are obviously many possibilities contained in all the above vector operations including the possibilities of vector normal products that lead to higher order filters.

4. SYMMETRY OF MATRICES

Up to this point we have only discussed issues to do with a vector $\overset{>}{x}$ .We will now proceed to outline the process of creating an invariant matrix from a vector $\overset{>}{x}$ and subsequent operations on that matrix that yield new information.

Initially we state that this invariant matrix form is a Ring Matrix $\overset{>0}{X}$ where o indicates the Ring form. For simplicity we will discuss the dataset $\overset{>}{x}=\left\langle x_1,x_2,x_3\right\rangle$. Note that all datasets have a beginning and an end. If we treat the dataset as a piece of string and join the ends we create a loop or ring. This operation on a dataset provides a mechanism for point of view invariance which allows us to form a matrix $\overset{>0}{X} = \begin{bmatrix} x_1 & x_2 & x_3 \\ x_2 & x_3 & x_1 \\ x_3 & x_1 & x_2 \end{bmatrix}$ from successive translations in a left to right sense, hence the forward arrow in $\overset{>0}{X}$. This operation is translational symmetry. The reverse ring matrix is $\overset{<0}{X} = \begin{bmatrix} x_3 & x_2 & x_1 \\ x_2 & x_1 & x_3 \\ x_1 & x_3 & x_2 \end{bmatrix}$. Note there is no filtering taking place. These matrices $\overset{>0}{X}$, $\overset{<0}{X}$ are symmetric (Hermitian) [10].That is, the matrix is symmetric about the forward diagonal. It follows from [11] that if a matrix $\overset{>0}{X}$ is Hermitian then the eigenvalues of $\overset{>0}{X}$ are real. The eigenvectors of $\overset{>0}{X}$ associated with the distinct eigenvalues are mutually orthogonal vectors.

Another feature is that the sum of the elements of each row is the same as the sum of elements of each column. However, the anti-sum of the elements of each row taken left to right or right to left are not equal. Likewise the anti-sum of the elements of each column; taken bottom to top or top to bottom, are not equal. Furthermore, the ring matrix also defines the possible state space combinations. So far we have defined a ring matrix for a sequence $\overset{>}{x} = \left\langle x_1, x_2, x_3 \right\rangle$ where each consecutive entry is lag =1. If we choose a lag = 2, $\overset{>}{x} = \left\langle x_1, x_3, x_2 \right\rangle$ and $\overset{>0}{X} = \begin{bmatrix} x_1 & x_3 & x_2 \\ x_3 & x_2 & x_1 \\ x_2 & x_1 & x_3 \end{bmatrix}$. We can define any $n \times n$ ring matrix from a dataset, length n, with a lag $l < n$.

In Figures 11 and 12 we show the principal modes that are formed by multiplying the sorted eigenvalues (principal components) by their respective eigenvectors and display the frequencies in each mode for the top 100 modes of the Coal Reclaimer slew bearing. Note that the low frequencies are not associated with a dominant mode and that a new frequency at 110 Hz encompasses more than one principal eigenmode.

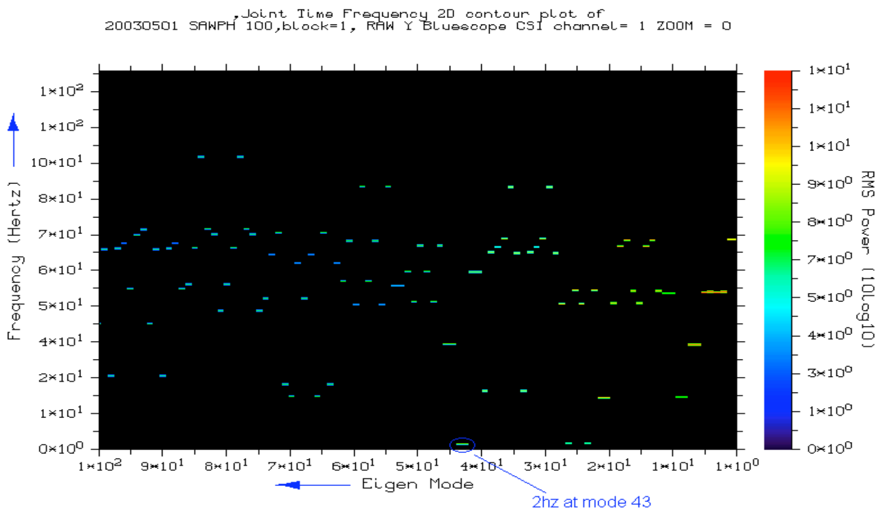


Figure11. $\overset{>}{x}$ Acceleration frequency, Eigen mode, RMS power at 01/05/2003.

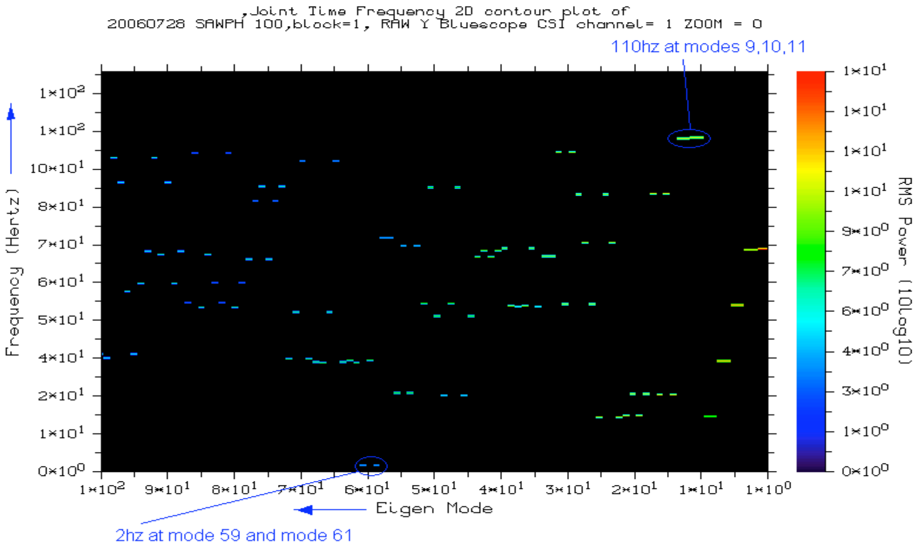
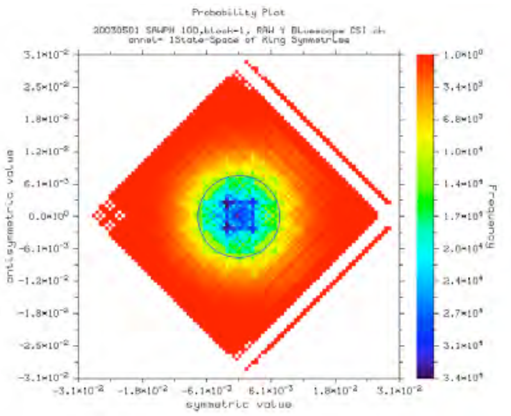


Figure12. $\overset{>}{x}$ Acceleration frequency, Eigen mode, RMS power at 28/07/2006.

Prior to extracting eigenvalues, the data in the matrix has not been ‘corrupted’ or filtered. Obviously the previous symmetry (filtering) operations on vectors; or for that matter any other linear/non-linear type filter, can be applied and subsequent conversion to the Ring Matrix form is quite simple. As an example we can form the horizontal (row wise) symmetry (H+) and anti-symmetry (H—) on the original data after it has been placed in Ring Matrix form. Figures 13 and 14 are examples of this operation for the same raw data in Figures 1 and 2. These are State-space images.Note the spreading out effect from the early time to the most recent. This is indicative of bearing wear even though the overall magnitudes have not varied significantly. As one would expect, this result is very similar to Figures 7 and 8.



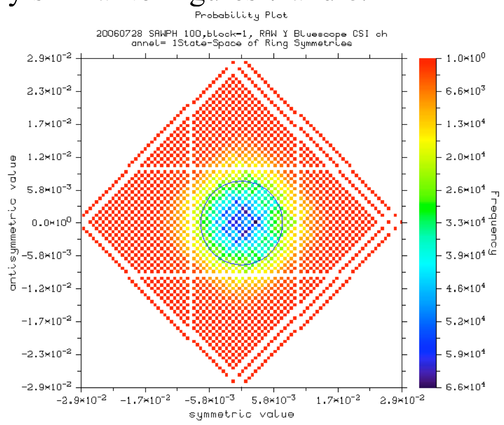


Figure13. $\overset{>oH+}{X}$ vs. $\overset{>oH-}{X}$ Accel’n space at 01/05/2003. Figure14. $\overset{>oH+}{X}$ vs. $\overset{>oH-}{X}$ Accel’n space at 28/07/2006.

This could also be done for the vertical (column wise) V+, V— Ring Matrix. The $\overset{>oH+V+}{X}$ Ring Matrix is a ‘super’ symmetric Ring Matrix. The Ring Matrix is not limited to two dimensions. Higher Tensor forms are possible via recursion.

6. CONCLUSIONS

The Coal Reclaimer slew bearing is still quite healthy although showing signs of wear. There appear to be no characteristic bearing defects dominating in the signal.

It is clear that symmetry operators are fundamental to all data and that there are many possible forms that involve both global and/or local operations. Symmetry breaking provides a mechanism to perform recursive operations on symmetric/anti-symmetric transforms. All symmetric/anti-symmetric transforms act like non linear filters and can be characterized into different orders 1, 2, 3...

The Symmetric Wave Decomposition (SWD) method appears to provide useful new information which may enable the determination of internal versus external faults.

New probability state-spaces give views of the structure of the data that provide a global view of behavioural changes.

The formation of a Ring Matrix, which is essentially an outcome of translation symmetry, provides us non-filtered 'point of view' invariance. The characteristic equation of the data can be determined via the eigenvalues of the Ring Matrix. Subsequent processing identifies the frequencies and power contained in each fundamental mode which can be displayed.

Symmetry operations allow us to extract pattern from any sequence. Symmetry suggests that there is a whole field of statistics based on the anti-sum waiting to be discovered.

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