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# DE-CONVOLUTION OF TIME-SERIES DATA USING TRANSLATIONAL SYMMETRY AND EIGENSTATE ANALYSIS

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**Abstract**

Translational Symmetry permits a time invariant matrix (a Ring Matrix) of a time series to be formed. Traditional Eigenvalue/vector and Fast Fourier Transform techniques applied to the Ring Matrix provide views of all the vibration modes, their magnitude and frequency content.

The Ring Matrix has many interesting features associated with it that enable it to be used as an additional tool in defining the 'condition state' of a bearing.

This paper introduces the Ring Matrix as a significant step in the 'Blind' de-convolution process. Application of the de-convolution method to vibration data obtained from both a large and small slow speed (1 to 4rpm) slew bearing is presented.

The Ring Matrix is being applied to the condition monitoring of a bearing which forms part of an experimental test rig specifically built to monitor slow speed (1 rpm) slew bearings with a view to predicting their remaining useful life.

Results, from a Coal Reclaimer which has slow speed (4rpm) slew bearings, are also presented to illustrate the various views of the vibration information.

## 1. INTRODUCTION

Machine condition assessment of slow-speed slew-bearings has generally been unsuccessful using vibration data. There are a number of reasons for this. Primarily, the very slow speeds involved (1 - 4 rpm) lead to very low rotational energy release. The motion of a slew bearing is often intermittent and non cyclic. Power spectra obtained via Fast Fourier Transform methods in combination with Demodulation methods and various filter techniques, are usually the only analysis techniques employed as they are non intrusive methods that will show up problems if they present as vibrations.

The data we will present comes from a large Coal Reclaimer slew bearing and a small slew bearing undergoing 'run to failure' testing on an experimental test-rig. These slew bearings contain two horizontal and one vertical row of cylindrical roller bearings. The Coal Reclaimer has two vertically mounted slew bearings (4.2m dia.) supporting the reclaiming buckets which rotate at approximately 4.5 rpm in one direction in a continuous mode. The slew bearing (0.3m dia.) in the test-rig is horizontally mounted and can operate in the speed range 1 to 10 rpm. The test-rig data presented in this paper is at 1 rpm.

Current condition monitoring methods applied to the Coal Reclaimer provide a sample of data that is short in duration containing a few thousand samples (4098 max) at 240 samples per second. The test rig provides 125,000 samples per channel per time block at 16 bit data resolution over the range of 2K-2M samples per second.

## 2. NOMENCLATURE

The following describes the notation adopted for this paper.

- All vectors are in lower case.
- All matrices are in upper case.
- > indicates a left to right direction.
- < indicates a right to left direction.
- o indicates the matrix is a ring matrix.
- $p$ indicates the horizontal direction and $q$ indicates the vertical direction.

## 3. DE-CONVOLUTION

Convolution [1] is defined as 'coiling, twisting; fold, twist'. De-convolution can be defined as the reverse, that is, to uncoil, untwist, and unfold. The mathematical representation for convolution is essentially an asterisk operator $X * Y \Rightarrow Z$ where $X, Y$ are the source items to convolve and $Z$ is our result or set of observations. The asterisk operator usually involves a folding, shifting, multiplication and finally a summation. This series of operations act like a sweeping filter. De-convolution is an inverse filtering process which identifies the $X * Y \Leftarrow Z \Rightarrow \lambda * V$ where $\lambda$ is a vector of characteristic values such that each value $\lambda_q$ is associated with a fundamental function described by the vector $\upsilon_p \in V$ where there are $p$ vectors and $q$ characteristic values. De-convolution is not the same as de-composition. De-convolution is a process that employs multiplication whereas decomposition is a process that employs subtraction. De-convolution is associated with a variety of methods called Blind Source Separation (BSS) [1, 2, 3], Independent Component Analysis (ICA) [4], Mean Field Independent Component Analysis (MFICA) [5] and Principal Component Analysis (PCA) [6].

In this paper we outline a method for finding $Z \Rightarrow \lambda * V$ where our known set of observations is $Z$.

The difficulty is in defining the matrix $Z$ which is a valid representation of the system given that all we know is the sensor output data. Our approach is to identify an invariant matrix that contains all the known information. For the moment we have to be content with the possibility of finding as many source functions as there are data points. We discuss this issue later when we identify the principal eigenvalues (components).

## 4. RING MATRIX

Initially for simplicity, we will discuss a dataset $\overset{>}{x} = \langle x_1, x_2, x_3 \rangle$. Note that all datasets have a beginning and an end. If we treat the dataset as a piece of string and join the ends we create a loop or ring as in Figure 1.

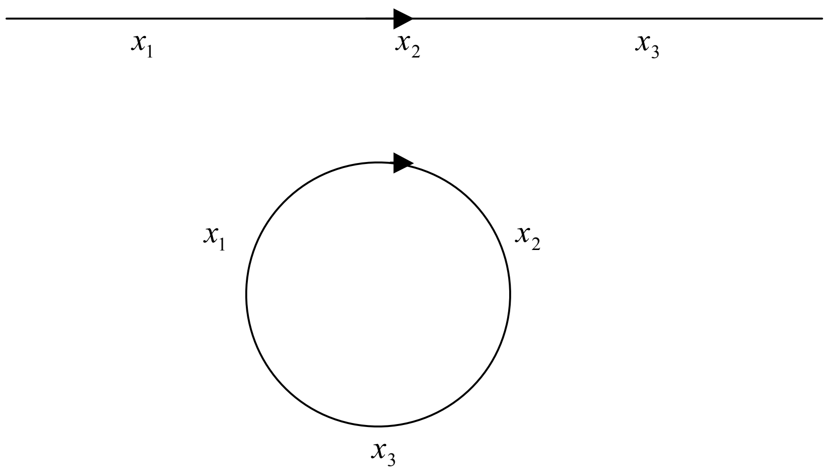


Figure1. Data string to invariant topology.

This operation on a dataset provides a mechanism for point of view invariance [7] which allows us to form a Ring Matrix $\overset{>0}{X} = \begin{bmatrix} x_1 & x_2 & x_3 \\ x_2 & x_3 & x_1 \\ x_3 & x_1 & x_2 \end{bmatrix}$ from successive translations in a left to right sense, hence the forward arrow in $\overset{>0}{X}$. This operation is translational symmetry [8]. The reverse Ring Matrix has the form $\overset{<0}{X} = \begin{bmatrix} x_3 & x_2 & x_1 \\ x_2 & x_1 & x_3 \\ x_1 & x_3 & x_2 \end{bmatrix}$. The Ring Matrix has the topology of a torus. These matrices $\overset{>0}{X}$, $\overset{<0}{X}$ are symmetric (Hermitian) [9], that is the matrix is symmetric about the forward diagonal. The Ring Matrix $\overset{>0}{X}$ is an invariant matrix form where o indicates the Ring form.

So far we have defined a Ring Matrix for a sequence $\overset{>}{x} = x_1, x_2, x_3$ where each consecutive entry is lag $l = 1$. If $l = 2$, $\overset{>}{x} = x_1\ , x_3,\ x_2$ and $\overset{>0}{X} = \begin{bmatrix} x_1 & x_3 & x_2 \\ x_3 & x_2 & x_1 \\ x_2 & x_1 & x_3 \end{bmatrix}$.

State (phase) space methods for time-series data [10, 11, 12, 13] involve embedded length groupings of $\overset{>}{x} = \langle x_i, x_{i+l}, x_{i+2l}, \cdots \rangle$ for $i = 0,1,2 \ldots m = n-1$ where the lag $l = 1$ or 2 or $3 \ldots m = n-1$. The Ring Matrix can also contain these groupings but it does so in a form that is time invariant. Therefore we can say that the ring matrix of a $n$ length time-series is time invariant for any lag $l < n \geq 1$. We can define any $n \times n$ ring matrix from an $n$ length dataset with a lag $l < n \geq 1$.

The data in the matrix may or may not come from a previously transformed or filtered dataset. All the results in this paper are associated with unfiltered data.

## 5. EIGENVALUES AND EIGENVECTORS

It follows from [14] that if a Ring Matrix $\overset{>0}{X}$ is Hermitian then the eigenvalues of $\overset{>0}{X}$ are real. The characteristic equation of the data $\overset{>}{x}$ can be determined via the eigenvalues of the Ring Matrix $\overset{>0}{X}$. The eigenvectors of $\overset{>0}{X}$ associated with the distinct eigenvalues are mutually orthogonal vectors.

We know the matrix is invariant since the sum of the elements of each row is the same as the sum of elements of each column. If we assume that $\overset{>0}{X}$ forms a homogeneous linear algebraic system of equations $\overset{>0}{X}\overset{>}{y}=0$ where we wish to solve for $\overset{>}{y}$. We can recast $\overset{>0}{X}\overset{>}{y}=0=\left[\overset{>0}{X}-\lambda I\right]\overset{>}{y}$ where $\lambda$ is the eigenvalue or characteristic value, $\overset{>}{y}$ is the eigenvector and $I$ is the unit matrix. This is the standard eigenvalue/vector formulation.

Now for every observation in the set $\overset{>}{x}$ there will be a corresponding eigenvalue and eigenvector.

Figure 2 is the eigenvalues; sorted into *principal components,* for a time instance of the Coal Reclaimer. Figure 3 is a small sample of the first two *principal* eigenvectors for the same time instance of the Coal Reclaimer. The *principal* eigenvector (eigen-mode) is the *eigenvector* multiplied by its associated *principal* eigenvalue.

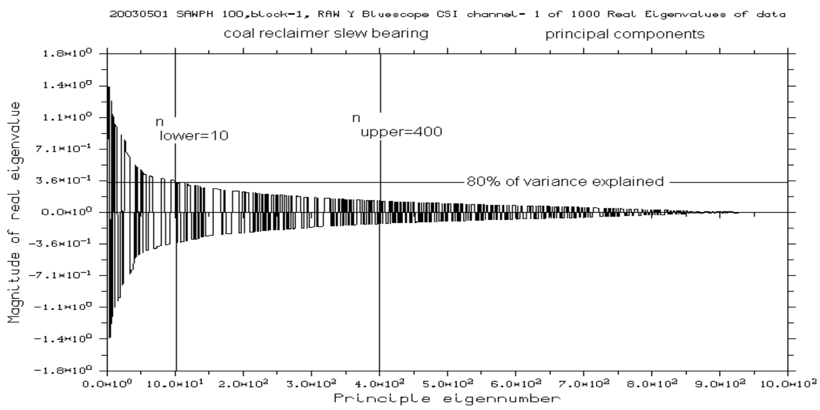


Figure2. $\overset{>0}{X}$ Acceleration eigenvalue versus eigennumber at 01/05/2003 for the Coal Reclaimer

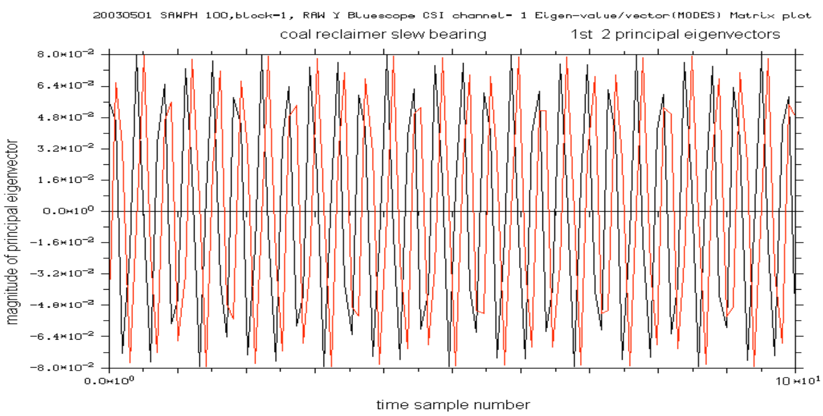


Figure 3. $\overset{>0}{X}$ First 2 Principal Acceleration Eigenvectors at 01/05/2003 for the Coal Reclaimer

It should be appreciated that finding the eigenvalues/eigenvectors is essentially a filtering operation. The data has been radically transformed. Discussions [15, 16] about how many of these eigenvalues are significant leads to a general rule where 80 % of the eigenvalue variance is explained by the *principal components*. We shall call this value $n_{lower}$. In structures containing many independent geometric elements we know that there are more 'active' components than there are *principal components*. For example in slew bearings there are often 300-400 rollers. Knowledge of this number, $n_{upper}$, gives us an upper bound on the number of eigenvalues. We know there are not likely to be any more because this represents the maximum number of possible forcing functions that could be active on and within the bearing.

In vibration analysis we usually restrict our immediate interest to the first 10 *principal components.* If we identify a particular bearing fault frequency in the eigenvectors of these first 10 principal components then we should have some cause for concern.

By multiplying the sorted eigenvalues by their respective eigenvectors; which have the same time axis as the measured signal, we can display the Fast Fourier frequencies (FFT) in each *principal* mode. Finding the Fourier frequencies is also a further filtering operation which may or may not be desirable. For example we have to be particularly cautious about filtering low amplitude, non-cyclic vibrations that typically emanate from slow speed slew bearings.

## 6. FREQUENCY BY EIGEN-MODE

Figures 4, 5, 6 and 7 are the frequency and magnitudes for each eigen-mode for the first 1000 data points in each dataset. The computed frequencies are placed into a hierarchy of importance. The dominant eigen-modes are indicated from right to left. The power in each frequency is indicated by the colour scale at the right hand side of the image. As a rule, red is on the right hand side and dark blue is on the left hand side.

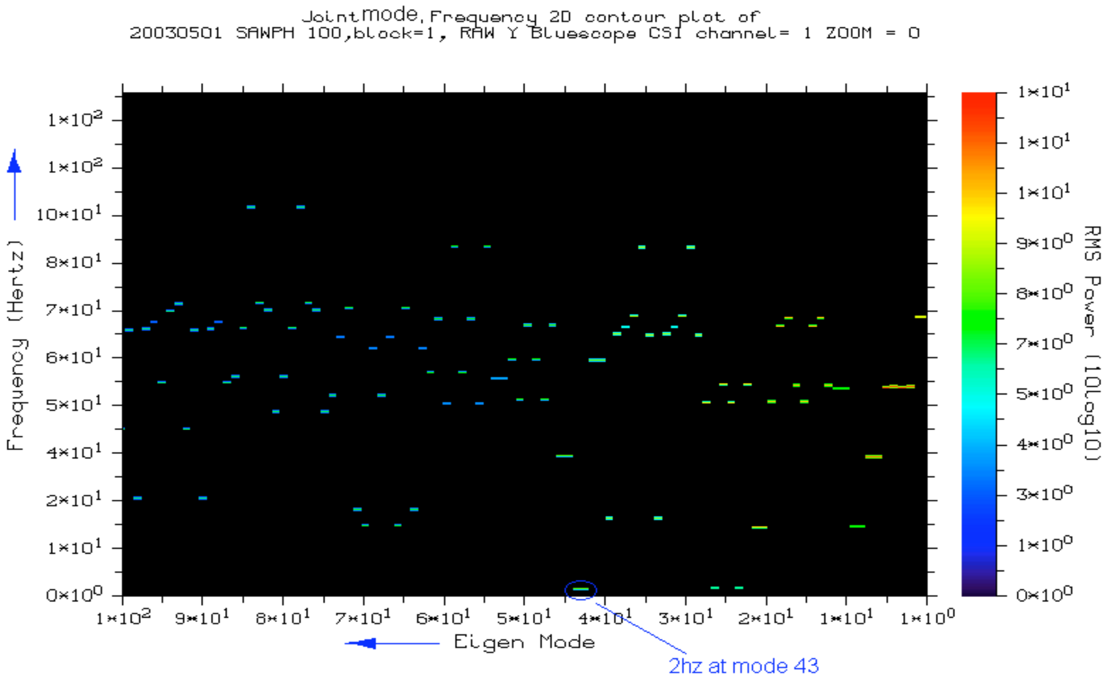

Figure4. Coal Reclaimer $\overset{>}{x}$ Acceleration frequency, eigen-mode, RMS Power at 01/05/2003

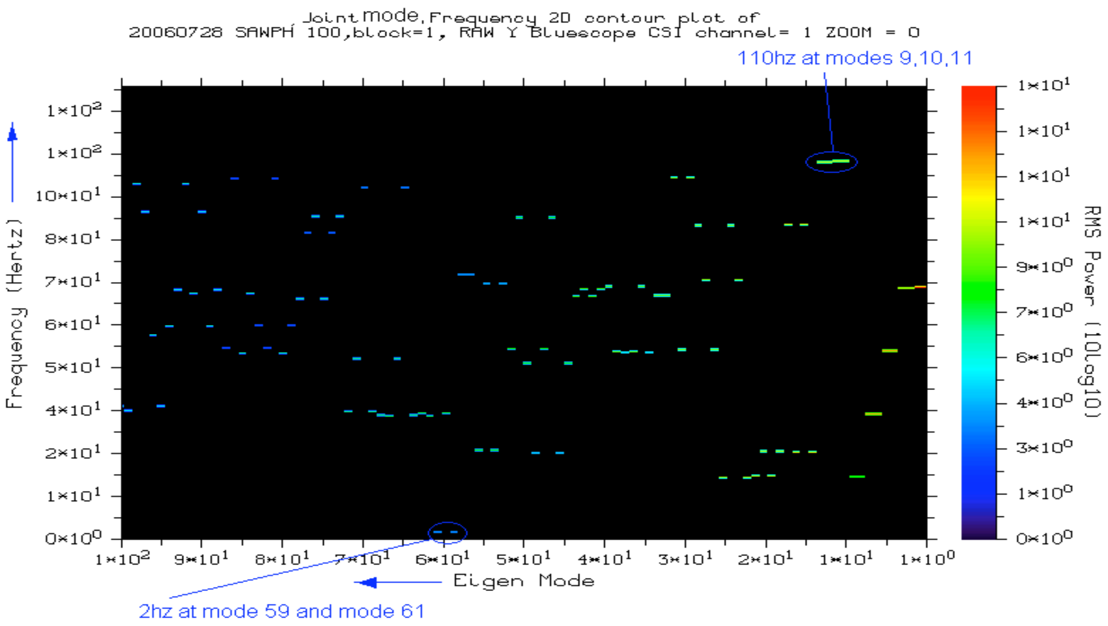


Figure5. Coal Reclaimer $\overset{>}{x}$ Acceleration frequency, eigen-mode, RMS Power at 28/07/2006

We limit the number of eigen-modes to the 100 dominant modes. This makes it easier to track a frequency of special interest across different time-series.

Figures 4 and 5; for the Coal Reclaimer, cover the frequency range 0-120Hz and represent the change over approximately 3 years. Figure 4 indicates that the fault frequency of interest (2 Hz) occurs at eigen-mode 43. Figure 5 shows that this frequency occurs in eigen-mode 59 and 61. However another dominant frequency has occurred at 110Hz. This indicates that the bearing is still good for service as a bearing fault frequency is not dominant.

For the test-rig, Figures 6 and 7 cover the frequency range 0-300kHz and represent the change over an equivalent simulated production period of 300 days.

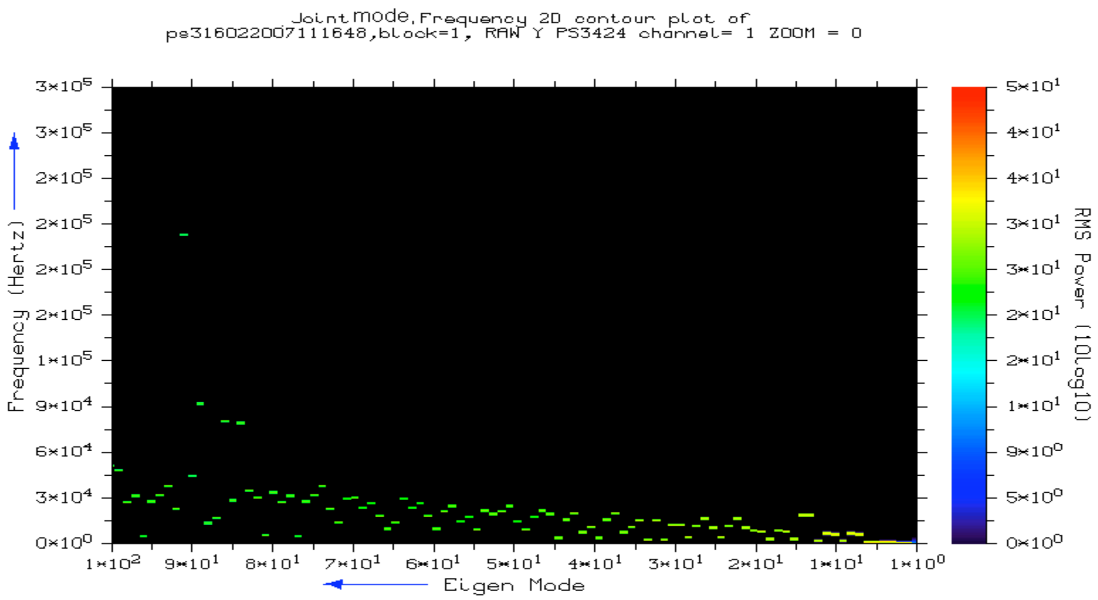

Figure6. Test-rig channel 1, $\vec{x}$ Acceleration frequency, eigen-mode, RMS Power at 016/02/2007

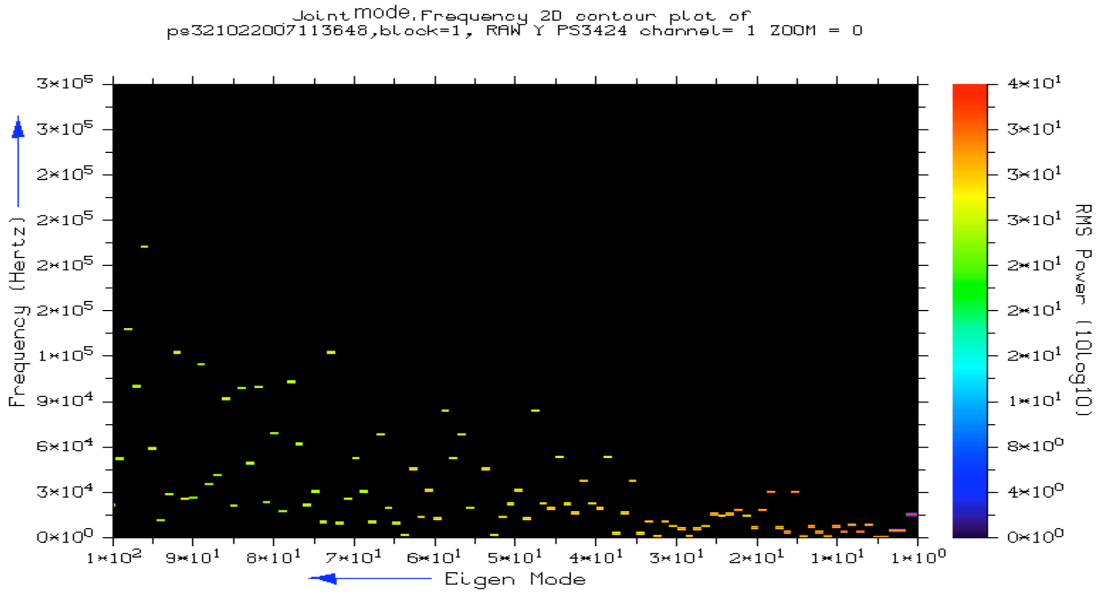


Figure7. Test-rig channel 1, $\vec{x}$ Acceleration frequency, eigen-mode, RMS Power at 21/02/2007

Figure 6 and 7 show that low frequencies (approx.10Hz) are dominant. These have been identified and are not fault frequencies. Figure 6 illustrates a bearing that has only recently been put into service.

An 'ideal' system would contain very few frequencies and the frequencies would have low power such that the state-space diagram would resemble a small well defined 'tight' ball shape. Over time; under load, the state-space diagram would become less well defined. The 'tight ball' would become a 'fuzzy ball'. This 'fuzziness' is associated with an increase in the frequencies of the vibration waveform. As we see in Figure 7 we have an increase in the occurrence of higher frequency waves which indicates that some wear is progressing. These ultrasonic frequencies are typically caused by scuffing of the rollers and or subsurface deformations of the roller and/or raceway metals. To obtain a better definition of the low frequencies requires sampling at a lower frequency.

In general if a fault frequency is contained in the first few eigen-modes then it is definitely time to take action. This action may be to replace the bearing or it may be the removal of a possible external forcing function creating the 'fault' frequency. At the present time, the de-convolution process does not identify whether the 'fault' is internal or external. To identify this effect requires a different analysis.

## 7. CONCLUSIONS

Any time-series can be de-convolved by placing the data into the Ring Matrix form. Any data can be placed into a Ring Matrix form. The Ring Matrix is not a filtering mechanism. The formation of a Ring Matrix, which is essentially an outcome of translation symmetry, provides us a 'point of view' invariant structure that enables the eigenvalues and eigenvectors representing the data to be determined.

The number of principal components represents a lower bound on the eigenvalue set. The number of known independent geometric components represents an upper bound on the eigenvalue set. Subsequent processing of the eigenvalues and eigenvectors allows us to identify the frequencies contained in each fundamental mode and their magnitude can be displayed.

The significance of theoretical fault frequencies can be indicated by monitoring their eigen-mode. The lower the eigen-mode the more severe/important the fault becomes.

Based on the analysis outlined in this paper both the Coal Reclaimer and test-rig bearings are still fundamentally sound.

## REFERENCES


1. R.Peled,S. Braun, M. Zacksenhouse. *A blind deconvolution separation of multiple sources, with application to bearing diagnostics.* Mechanical Systems and Signal Processing **19** (2005) 1181-1195.
2. Xianhua Liu, R. B. Randall. *Blind source separation of internal combustion engine piston slap from other measured vibration signals.* Mechanical Systems and Signal Processing **19** (2005) 1196-1208.
3. C. Servière, P. Fabry. *Blind source separation of noisy harmonic signals for rotating machine diagnosis.* Journal of Sound and Vibration **272** (2004) 317-339.
4. Sorin Drăghici. *Data Analysis Tools for DNA Microarrays.* Chapman & Hall/CRC. 2003. ISBN 1 58488 315 4. pp 259-261
5. Niels Pontoppidan, Sigurdur Sigurdsson, Jan Larsen. *Condition monitoring with mean field independent component analysis.* Mechanical Systems and Signal Processing **19** (2005) 1337-1347.
6. Sorin Drăghici. *Data Analysis Tools for DNA Microarrays.* Chapman & Hall/CRC. 2003. ISBN 1 58488 315 4. pp 246-259.
7. Victor J Stenger. *The Comprehensible Cosmos*. Prometheus Books.2006. ISBN-13 978 1 59102 424 8. pp 57-58.
8. http://en.wikipedia.org/wiki/Translational_symmetry
9. Anthony J. Pettofrezzo. *Matrices and Transformations.* Dover Publications.1978. ISBN 0 486 63634 8. pg 15.
10. G. L. Baker, J. P. Gollub. Chaotic Dynamics an introduction. Second edition. Cambridge University Press.1996. ISBN 0 521 47685 2.
11. Henry D.I. Abarbanel. Analysis of observed Chaotic Data. Springer.1996. ISBN 0 387 94523 7.
12. Holger Kantz, Thomas Schreiber. *Nonlinear Time series Analysis.* Cambridge University Press.1997. ISBN 0 521 65387 8.
13. W.J. Wang and R. M. Lin. *The application of pseudo-phase portrait in machine condition monitoring.* Journal of Sound and Vibration (2003) **259**(1),1-16
14. Anthony J. Pettofrezzo. *Matrices and Transformations.* Dover Publications.1978. ISBN 0 486 63634 8. pg 90-92.
15. J. Jolliffe. *Principal Component Analysis.* Springer Verlag.1986
16. R. B. Cattell. *The scree test for the number of factors.* Multivariate Behavioural Research 1. pp 245-276