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EFFECTS OF LANDING GEAR NONLINEARITIES ON GROUND RESONANCE OF HELICOPTER

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Abstract

One of key targets in helicopter design is to be free from ground-resonance, which may occur when lead-lag frequency of rotor is coupled with natural frequencies of rotor-fuselage-landing gear system. Typical analytical approach to this problem is based on simplified linear modeling for the landing gear. In practice, however, the landing gear has nonlinear characteristics not only when touching down but also when operating on ground, depending on stroke, velocity and friction of oleo-pneumatic shock absorber in the landing gear. In this study, a method is proposed to predict the vibration severity and unstable range of rotor speed, by including the nonlinear characteristics in the landing gear. Then, effects of landing gear nonlinearities will be discussed.

1. INTRODUCTION

To be simple, instability of a helicopter called ground resonance can be attributed to the coupling between natural frequencies of the fuselage with the blade lead-lag frequency of the rotor. To be more detail, instability arises from interaction of fuselage inertia, landing gear stiffness and dynamic properties of lead-lag motion which can be explained by the gyroscopic and centrifugal stiffening effects in the rotor. The usual solution for ground resonance is to add mechanical damper to lead-lag hinge and landing gear. The analytical approaches based on linear ground resonance model is often used while, in fact, most common landing gears have nonlinear characteristics. Nonlinear characteristics in the landing gear (especially oleo-pneumatic shock absorber) affects both roll and pitch motion of fuselage. In some publications [7,8] on the ground resonance, the nonlinear effects were taken into account but they used a very simple model which consists of viscous damper, fiction damper and spring element. Typically, the oleo-pneumatic shock absorber is composed of two chambers and an orifice. The friction due to tightness of the seal and restricted flow in the orifice works also as a damper.

The gas in the chamber acts as a spring element [1]. In this paper, stability analysis for ground resonance of helicopter is conducted by incorporating two types of nonlinearity in the landing gear: dry friction and hydraulic resistance proportional to the squared velocity.

2. DEVELOPMENT OF GROUND RESONANCE MODEL

For the ground resonance phenomenon, the basic mathematical description is provided by five differential equations that respectively model the responses in the fuselage z translation, roll and pitch rotations, blade cyclic in-plane(lead-lag wise) modes in ε_x, ε_y directions. In order to clearly identify the effects of landing gear nonlinearities, an articulated rotor is used for ground resonance model. Referring to Fig. 1, the position of p_{cg} which is the centre of a gravity of k-th blade is given by

$$\overline{op_{cg}} = \begin{Bmatrix} eR\cos\Omega t + r_g\cos(\Omega t + \varsigma_k) + h_2\theta_y \\ eR\sin\Omega t + r_g\sin(\Omega t + \varsigma_k) - h_2\theta_x \\ 0 \end{Bmatrix} \tag{1}$$

By differentiating with respect to time, the velocity of point p_{cg} is written as following:

$$\vec{V}_p = \begin{Bmatrix} -\Omega eR\sin\Omega t - r_g(\Omega + \dot{\varsigma}_k)\sin(\Omega t + \varsigma_k) + h_2\dot{\theta}_y \\ \Omega eR\cos\Omega t + r_g(\Omega + \dot{\varsigma}_k)\cos(\Omega t + \varsigma_k) - h_2\dot{\theta}_x \\ 0 \end{Bmatrix} \tag{2}$$

The displacements of landing gears due to fuselage motions in direction z, θ_x, θ_y are

$$E_1 = z + l_1\theta_y \ , E_2 = z - d\theta_x - l_2\theta_y \ , \ E_3 = z + d\theta_x - l_2\theta_y \tag{3}$$

The kinetic energy, the potential energy and the dissipation function of the system are then

$$\begin{aligned} T &= \sum_{k=1}^{b}\frac{1}{2}m_b\left|\overline{V_{pk}^2}\right| + \frac{1}{2}(I_{xx} + m_b h_2^2 + bI_b)\dot{\theta}_x^2 + \frac{1}{2}(I_{yy} + mx_{cg}^2 + m_b h_2^2 + bI_b)\dot{\theta}_y^2 + \frac{1}{2}(m + bm_b)\dot{z}^2 \\ U &= \sum_{k=1}^{b}\frac{1}{2}k_s\varsigma_k^2 + \frac{1}{2}k_n(z + l_1\theta_y)^2 + \frac{1}{2}k_m(z - d\theta_x - l_2\theta_y)^2 + \frac{1}{2}k_m(z + d\theta_x - l_2\theta_y)^2 \\ \mathbb{F} &= \sum_{k=1}^{b}\frac{1}{2}2(\kappa\Omega + \kappa_s)\delta\dot{\varsigma}_k^2 + \frac{1}{2}c_n(\dot{z} + l_1\dot{\theta}_y)^2 + \frac{1}{2}c_m(\dot{z} - d\dot{\theta}_x - l_2\dot{\theta}_y)^2 + \frac{1}{2}c_m(\dot{z} + d\dot{\theta}_x - l_2\dot{\theta}_y)^2 \end{aligned} \tag{4}$$

The equations of motion of k-th blade can be derived by using Lagrange equation.

$$\begin{aligned} &\frac{d}{dt}\left(\frac{eT}{e\dot{\varsigma}_k}\right) - \frac{eT}{e\varsigma_k} + \frac{eU}{e\varsigma_k} = \mathbb{F} \\ &\ddot{\zeta}_k + 2(\kappa\Omega + \kappa_s)\delta\dot{\zeta}_k + (\kappa^2\Omega^2 + \kappa_s{}^2)\zeta_k = +\frac{m_b r_g(h_2\ddot{\theta}_y)\sin\Psi_k}{I_b} + \frac{m_b r_g(h_2\ddot{\theta}_x)\cos\Psi_k}{I_b} \end{aligned} \tag{5}$$

The periodic terms in Eq.(5) can be removed by using Coleman co-ordinates[5] which let

$$\varepsilon_x = -\frac{2}{b}\sum_{k=1}^{b}\varsigma_k\sin\Psi_k, \ \varepsilon_y = -\frac{2}{b}\sum_{k=1}^{b}\varsigma_k\cos\Psi_k \tag{6}$$

Then, the equations of motion are obtained as following:

$$\ddot{\varepsilon}_x + 2(\kappa\Omega + \kappa_s)\delta\dot{\varepsilon}_x + (\kappa^2\Omega^2 + \kappa_s^2 - \Omega^2)\,\varepsilon_x - 2\Omega\dot{\varepsilon}_y - 2(\kappa\Omega + \kappa_s)\Omega\delta\varepsilon_y + \frac{m_b r_g h_2}{I_b}\ddot{\theta}_y = 0$$

$$\ddot{\varepsilon}_y + 2(\kappa\Omega + \kappa_s)\delta\dot{\varepsilon}_y + (\kappa^2\Omega^2 + \kappa_s^2 - \Omega^2)\,\varepsilon_y + 2\Omega\dot{\varepsilon}_y + 2(\kappa\Omega + \kappa_s)\Omega\delta\varepsilon x - \frac{m_b r_g h_2}{I_b}\ddot{\theta}_x = 0$$

$$(I_x + bI_b + 2m_b h_2^2)\ddot{\theta}_x + 2c_m d^2\dot{\theta}_x + 2k_m d^2\theta_x + \frac{b}{2} m_b r_g h_2 \ddot{\varepsilon}_y = 0 \tag{7}$$

$$(I_y + mx_{cg}^2 + bI_b + 2m_b h_2^2)\ddot{\theta}_y + (c_n l_1^2 + 2c_m l_2^2)\dot{\theta}_y + (k_n l_1^2 + 2k_m l_2^2)\theta_y$$

$$- (2c_m l_2 - c_n l_1)\dot{z} - (2k_m l_2 - k_n l_1)z + \frac{b}{2} m_b r_g h_2 \ddot{\varepsilon}_x = 0$$

$$(m + bm_b)\ddot{z} + (c_n + 2c_m)\dot{z} + (k_n + 2k_m)z + (c_n l_1 - 2c_m l_2)\dot{\theta}_y + (k_n l_1 - 2k_m l_2)\theta_y = 0$$

The stability of the rotor/fuselage system is determined by calculating the eigenvalues of these equations. A detailed discussion of the stability analysis will be undertaken in chapter 4.

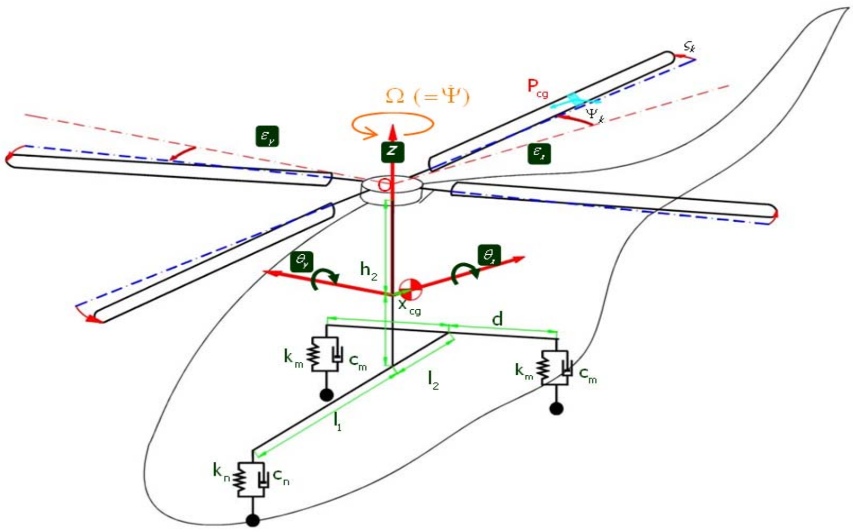


Figure 1. Schematic diagram of ground resonance model for helicopter

3. NONLINEAR CHARATERISTICS OF LANDING GEAR

Landing gear of helicopter consists of a oleo-pneumatic shock absorber and a wheel. In this paper, the oleo-pneumatic shock absorber which mainly has nonlinear characteristics is assumed as a landing gear.

3.1 Pneumatic stiffness in the oleo-pneumatic shock absorber

The pneumatic stiffness can be described from displacement and force relationship as given following Eq. (8):

$$k_p = \frac{dF}{d\delta} = A\frac{dP}{d\delta} \tag{8}$$

The pressure of the compressed gas in the upper cylinder can be described by the polytropic gas law for a closed system as:

$$PV^n = P_i V_i^n \tag{9}$$

where P_i and V_i denote initial pressure and volume at the equilibrium point. The V can be described as $V_i - A\delta$ and using Eq. (9), Eq. (8) can be rewrite as:

$$k_p = \frac{n P_i A^2}{V_i} \left(\frac{1}{1-(A/V_i)\delta} \right)^{n+1} \tag{10}$$

where the ( ) term can be neglected under assumption that the amplitude of vibration is infinitesimal. The pressure term in Eq. (10) is need to be related to the positional variable X. If the state of oleo-pneumatic shock absorber is changed from static state to fully extended state, as shown in Fig. 2, then the pneumatic stiffness is soften. It is shown in Fig. 3.

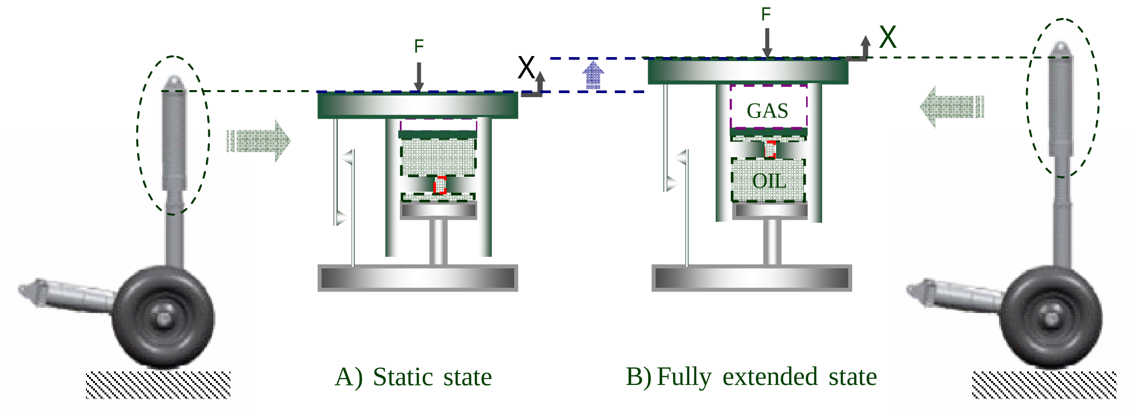


Figure 2. State variation of oleo-pneumatic shock absorber model

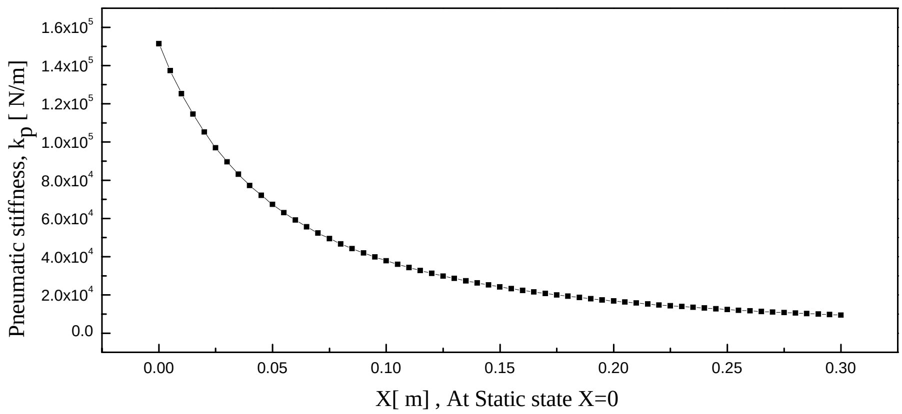


Figure 3. Pneumatic stiffness of gas in the upper chamber vs. absorber states

3.2 Complex stiffness of oleo-pneumatic shock absorber

In this chapter, the complex stiffness model is presented by taking two major considerations: amplitude and frequency dependencies which are related with frictional characteristics of the oil flow through the orifice and friction between cylinder and piston.

Fig. 4 shows the force acting on the cylinder. A_o is the orifice area, A is the piston (cylinder) area and L_o is the orifice length.

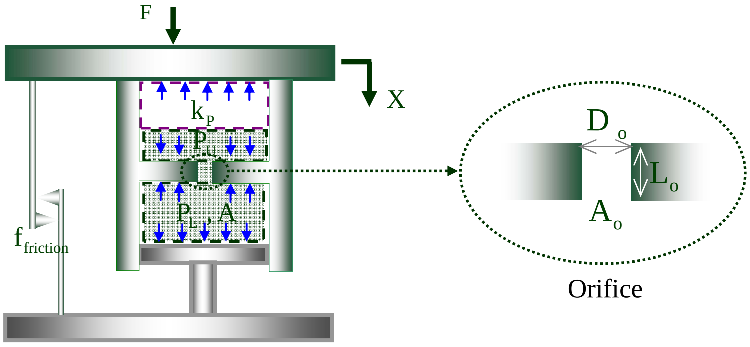


Figure 4. Schematic of oleo-pneumatic shock absorber model development.

Balancing the forces on the cylinder gives following equation.

$$F = k_p X + (A - A_o)(P_L - P_U) + f_{friction} \tag{11}$$

This equation is assumed that the fluid is incompressible and the fluid pressure in the upper cylinder is identical to the pneumatic pressure. From the momentum equation governing the oil flow through the orifice, the pressure loss between upper and low chamber can be derived as follow:

$$(A - A_o)(P_L - P_U) = (A - A_o)(L_o \rho \ddot{X}_o + \{\frac{L_o}{D_o} f + k_o\} \frac{\rho}{2} |\dot{X}_o| \dot{X}_o) \tag{12}$$

where, X_o is the flow displacement in the orifice, f is a friction coefficient in orifice (which is expressed as a function of X_o)and k_o is a minor loss coefficient at in and outlet of orifice[3]. Substitution of Eq. (12) into Eq. (11) and with the continuity equation $A_o \dot{X}_o = A\dot{X}$, Eq. (11) can be rewritten with the variable X. Linearization of the friction force term and the velocity square term, $|\dot{X}_o| \dot{X}_o$,are as following[4]:

$$\begin{aligned} f_{friction} &= C_f \text{ sign } \dot{X} \cong C_f \frac{4}{\pi X \omega} \dot{X} \\ |\dot{x}|\dot{x} &= (X_s \omega)^2 |\cos \omega t| \cos \omega t \approx \left(\frac{\omega}{\pi} \int_0^{\frac{2\pi}{\omega}} |\cos \omega t| \cos \omega t \cdot \cos \omega t \right) (X\omega)^2 = \frac{8}{3\pi} X\omega \dot{x} \end{aligned} \tag{13}$$

Then the complex stiffness of oleo-pneumatic shock absorber is

$$k^*(X,\omega) = \frac{F}{X} = k_p - m_e \omega^2 + j c_e(X,\omega)\omega \tag{14}$$

where,

$$c_e(X,\omega) = (A - A_o)\left(\frac{A}{A_o}\right)^2 \{\frac{L_o}{D_o} f + k_o\} \frac{4\rho X\omega}{3\pi} + C_f \frac{4}{\pi} \frac{1}{X\omega} \tag{15}$$

$$m_e = (A - A_o)\left(\frac{A}{A_o}\right) L_o \rho \tag{16}$$

From Eqs. (14)~(16), the complex stiffness of oleo-pneumatic shok absorber depends on frequency and dynamic amplitude. But the dependence of stiffness on frequency can be neglected, because the value of m_e is very small. It is shown in Fig.5,

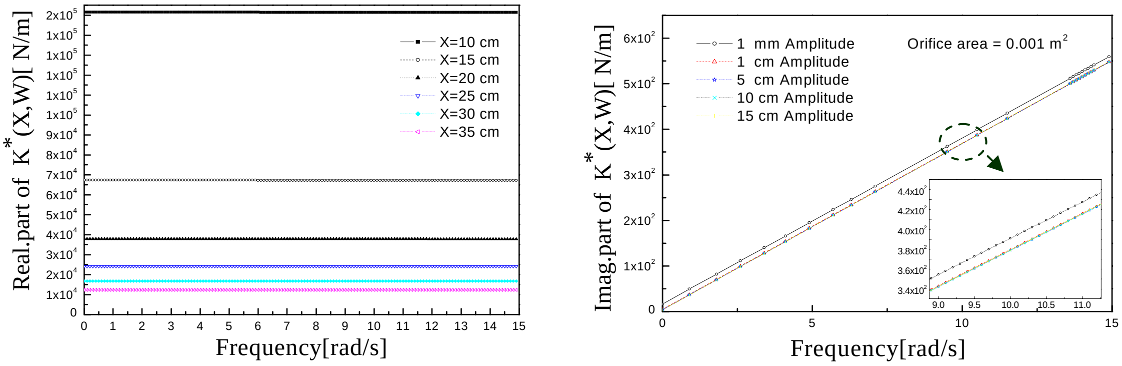


Figure 5. Complex stiffness of oleo-pneumatic shock absorber

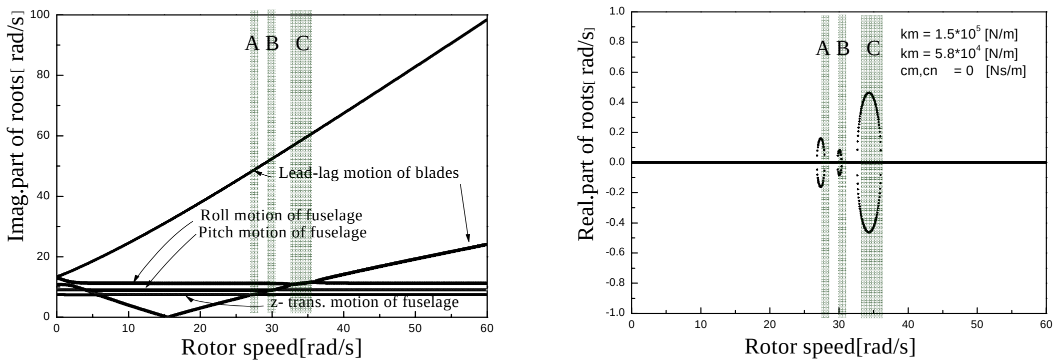
4. SIMULATION RESULTS FROM STABILITY ANALYSIS

Values of geometrical and physical parameters for the ground resonance model and oleo-pneumatic shock absorber are quoted from Refs. 1 and 6, which are not copied in this paper. The stability descriptors come up with solving the characteristic equation based on (Eqs. 7). Let the i-th root, λ_i , have the form:

$$\lambda_i = \sigma_i + j\omega_i \, . \tag{17}$$

Then the system stability is assured if $\sigma_i < 0$ for every value of the *i*. The stability diagram in Figure 6 shows variations with the rotor speed of resonance frequencies(imaginary part) and damping (real part) for linear landing gear model.

(a)



(b)

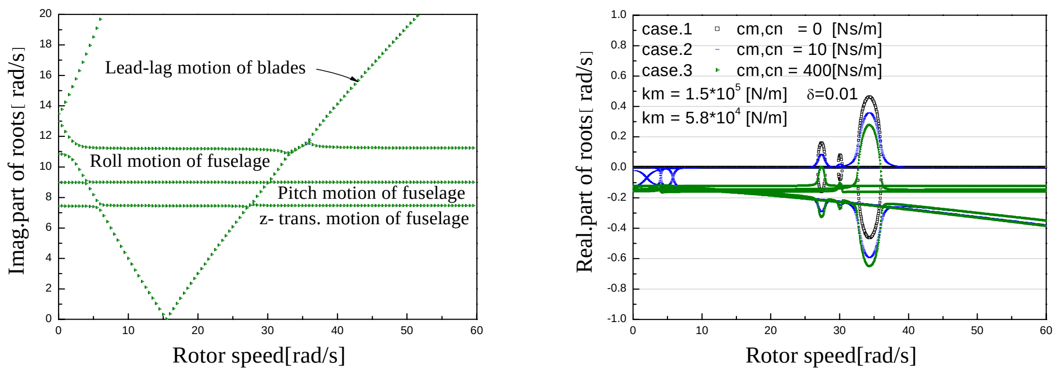


Figure 6 Stability diagram by linear modeling of spring & dashpot in landing gear
(a) no damping (b) damping

The severest ground resonance occurs in region C of rotor speed 30~40 rad/s where the fuselage roll mode and blade’s lead-lag mode are coupled to each other. Figure 6(b) indicates that the liability of ground resonance can be reduced expectedly by increasing damping in the landing gear.

(a)

(b)

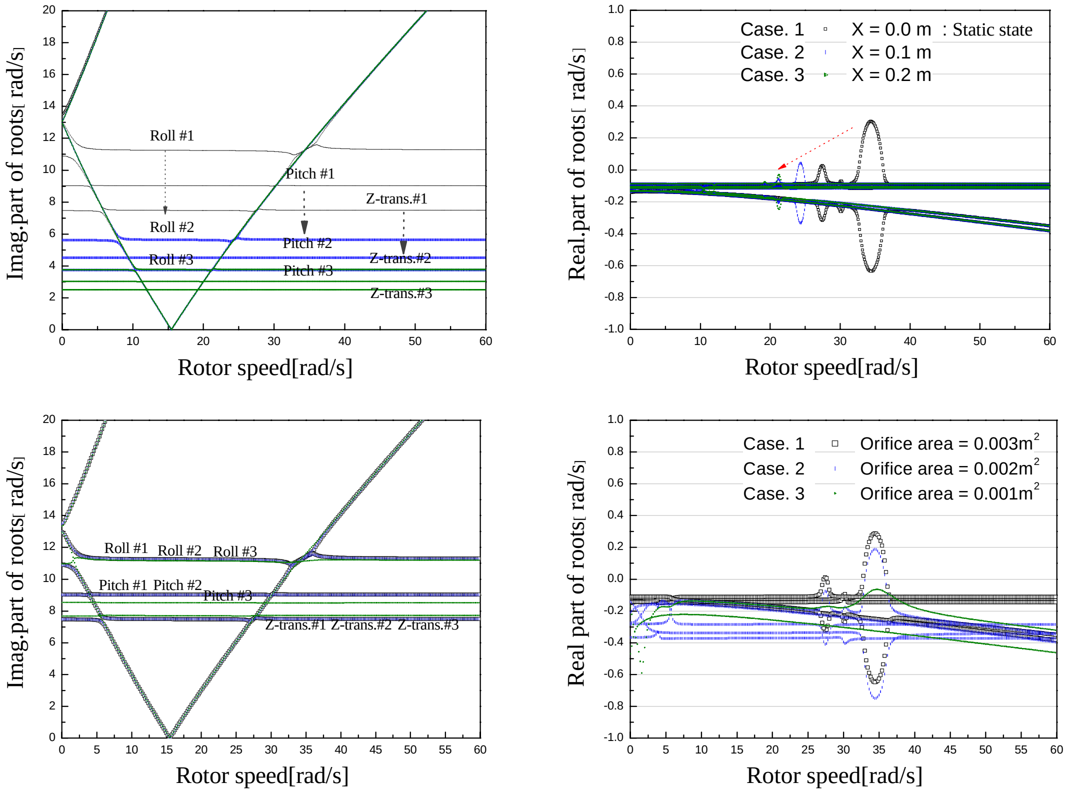


Figure 7 Stability diagram by nonlinear modeling of oleo-pneumatic shock absorber in landing gear (a) various static state (b) varyous orifice areas

Figure 7(a) shows effects of the oleo-pneumatic shock absorber's several static state on ground resonance instabilities. As the landing gear changes from static equilibrium state to extended ones, the speed ranges of instability get lower and the instability become less severe. It is to be noted, however, that, when the gear is fully extended up to the region where the load becomes less than 'break-out load for friction, the shock absorber can not dissipate any energy and, hence, the system may get into the most unstable state[8]. Damping of the oleo-pneumatic shock absorber can be increased by reduction of the orifice area. Figure 7(b) shows that the instabilities in ground resonance can avoided to some extent by such a design change.

6. CONCLUSIONS

The effects of nonlinear characteristics of landing gear on ground resonance in a helicopter were investigated in comparison with linear model. In linear analysis, it is shown that the instabilities occur over fixed regions of the rotor speed expectedly. In this paper, the ground resonance modeling of an oleo-pneumatic shock absorber which has nonlinear characteristics has been developed and the subsequent stability analysis has led to the following conclusions.

1. Damping of the nonlinear landing gear model has both amplitude and frequency dependency. Yet, dependence of stiffness on the frequency and damping on the amplitude is not so significant that they can be neglected.

2. Stiffness of the nonlinear landing gear model varies with the static position of the oleo-pneumatics shock absorber. As the shock absorber is more extended, the helicopter become more stable.

In further research, complex stiffness of an oleo-pneumatic shock absorber will be measured and feasibility of design improvements for the oleo-pneumatic shock absorber will be investigated.

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