ICSV14
Cairns • Australia
9-12 July, 2007

14th International Congress on Sound & Vibration

MICROPHONE ARRAY BEAMFORMING AND INVERSE METHODS FOR THE RECONSTRUCTION OF DUCTED ACOUSTIC SOURCES

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Abstract

Methods for noise source identification are a key point to develop research in nacelle noise reduction technologies. In this work, two different approaches, beamforming and inverse methods, have been assessed for the characterisation of in-duct sources. An analytical model has been implemented for the sound field due to elementary sources in a hard-walled duct with non-reflecting terminations. The data field is provided by a uniformly-distributed array of microphones located in a cross section of the circular duct. A comparison between both approaches has been performed as a function of the two main parameters that influence the reconstruction results: the axial distance between the sensor array and the sources, and the frequency range considered. Different beamforming techniques and regularised inverse methods have been tested over a set of coherent and incoherent sources. Results have been obtained which show that the beamformers are not able to accurately locate the sources when these are highly correlated or in the low frequency range. However, above a certain frequency threshold, it does not seem to be limited by the axial separation distance. On the other hand, although the inverse method is limited at low frequencies and in the far field, it allows the reconstruction of source strengths amplitudes. Under certain conditions, both methods can be used in a complementary way for a complete source characterisation.

1. INTRODUCTION

The aeronautic, aerospace and automotive industries require improved noise reduction techniques to meet the current normative and increase the acoustic comfort at acceptable cost. For example, noise of axial engine fans constitutes one of the dominant sources for current high bypass ratio turbofans. The development of future solutions is based on a complete source characterisation as for location and strengths, and it might not be directly attainable from the engine static tests.

As the direct measurement of pressure fluctuations on the fan blades requires delicate experimental techniques such as the use of several flush-mounted miniature piezoresistive sensors, an alternative indirect approach is to investigate the relation between the measurements of the pressure inside the duct to an assumed source distribution, and the sensitivity of the reconstruction to different generation mechanisms. The question addressed in this paper is the practical capability of model-based methods to reconstruct a given acoustic source distribution from pressure-based measurements at field points inside the duct. A comparison between two methodologies, inverse methods[1] and beamforming estimation techniques,[2] is presented for different problem parameters.

The main objective for inverse techniques is to reconstruct the original source strengths from a set of field measurements obtained at discrete points and a suitable model of the propagation path between the sources and the sensors. Although the problem formulation is rather simple, the ideal solution in practical cases can be extremely difficult to retrieve due to the properties of the propagation operator which filters out the high wavenumber components related to the source. Under certain conditions, some near-field properties cannot be detected from measurements in the far field, and this causes the inverse problem to be ill-conditioned. Most of the efforts concerning the inverse propagation have then been conducted towards the study of new techniques for regularising the inversion of the forward operator, the inverse of which is usually unbounded.[3]

Considering the inverse method limitations for the source strength reconstruction, a new approach to the problem is proposed in this work based on a prior analysis of the pressure field by means of a beamforming microphone array technique. This methodology has received much attention in reactive environments where interferences are present since microphone arrays are able to achieve high directionality and improved signal to noise ratio (SNR). A microphone array consists of a number of individual microphones distributed in space with a determined geometry. The outputs signals are processed according to a beamforming algorithm which determines the spatial filtering characteristics of the array. In this way it is possible to simulate any desired directivity for achieving a selective response pattern. Beamformers have been used previously for inferring the presence or the absence of a signal that impinges on the array and the main directions of arrivals (DOAs) for speech pickup in noisy environments.[4,5] It has been shown that beamforming microphone arrays can be used for the identification of the DOAs of the earliest reflections in closed rooms. When these directions can be traced back to locations in the room containing the array, they can indicate the sources of the detected sound.

A comparison between the inverse method and the focused beamforming has been studied previously[6] for sound source characterization in a semi-anechoic chamber. In this work, an analytical study is presented for comparing both methodologies when the field measurements are taken under guided wave propagation conditions, i.e. inside a hard-walled duct with non-reflecting terminations. In the next section, a model of the system is outlined in terms of a set of in-duct normal modes propagating upstream and downstream. The acoustic prediction results are then used as input data to the inverse method and to three different beamformer estimators for a complete source characterisation. The sensitivity of the reconstruction to different generation conditions and source configurations has been illustrated in terms of numerical simulations.

2. ANALYTICAL MODEL

The first step towards the analysis of the reconstruction problem is the derivation of an accurate and realistic acoustic model for predicting the sound pressure field due to an arrangement of ducted sources. A schematic representation of the physical set-up considered in this work is presented in Figure 1: a hard-wall cylindrical duct of infinite length containing a set of point monopole sources which are assumed to be stationary. The duct has a circular cross-section of radius R, and an axis aligned along the z-direction. The source section comprises up to nine acoustic drivers which can be flush-mounted on the duct wall. In-duct measurements can be conducted by means of an electro-mechanical system which permits the displacement of two pressure-velocity probes mounted on radial and circumferential rigs in order to scan the duct cross section, in the far-field and in the near-field of the sources. This set-up corresponds to an anechoic duct facility of the laboratory.

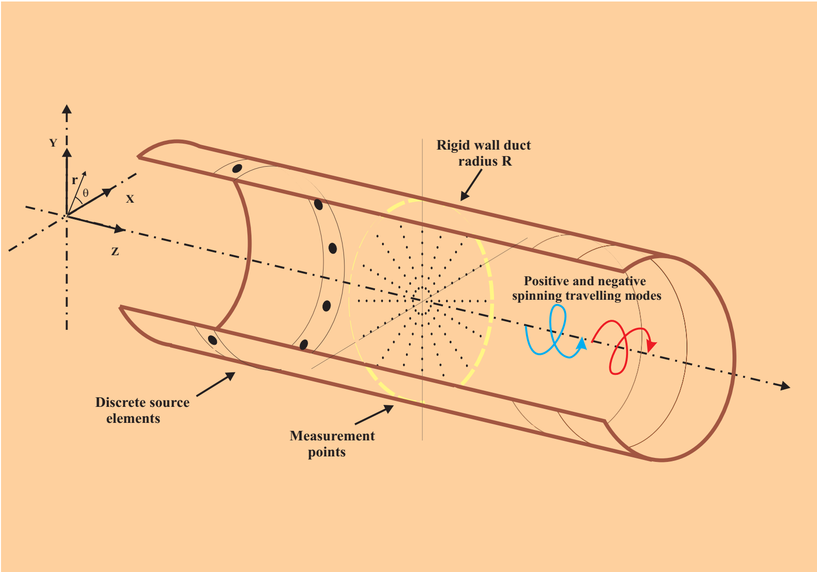

Figure 1. Schematic diagram of the set-up considered with the source and microphone positions into a hard-walled infinite duct.

The direct model is obtained by solving the Helmholtz equation which satisfies the rigid boundary conditions imposed at the duct walls. The pressure field in the duct can be expressed as a sum of modal contribution of the form[7]

$$p(r,\theta,z) = \sum_m \sum_n A_{mn}(z)\psi_{mn}(r,\theta)\,. \tag{1}$$

In this equation, $A_{mn}(z)$ are the modal amplitudes and $\psi_{mn}(r,\theta)$ are the duct normal modes that have the expression

$$\psi_{mn}(r,\theta) = e^{im\theta} J_m(\kappa_{mn} r), \tag{2}$$

where $J_m(\kappa_{mn} r)$ is the Bessel function of the first kind of order m and κ_{mn} corresponds to the values of $\kappa_{mn}R$ for which the gradient of the Bessel function is zero. The detailed expression for the modal coefficients and an explanation of the physical meaning of the different factors are provided in the reference.[7]

3. INVERSE METHOD

The discrete inverse problem of reconstructing sources inside the duct can be analysed from the inversion of the analytical model described in the previous section. We assume that a real source strength distribution can be modelled as a finite number of discrete assumed sources whose complex strength is unknown. The radiated sound field is measured by an array of sensors. Using the numerical model developed above, the transfer function between each assumed source and each measurement position can be predicted. The difference error between the model output, $\mathbf{p}$, and the measured pressure field due to the real source distribution, $\hat{\mathbf{p}}$, at the sensor positions can be expressed as[8]

$$\mathbf{e} = \hat{\mathbf{p}} - \mathbf{p} = \hat{\mathbf{p}} - \mathbf{Zq}\,, \tag{3}$$

where $\mathbf{Z}$ is the complex transfer matrix between the sources and the sensors, and $\mathbf{q}$ is a vector of the unknown complex source strengths associated to each of the elemental sources. Assuming broadband sources, the traditional approach to this type of problem is to find the least-squares solution for the source strength cross- spectral matrix, that is given by[8]

$$\mathbf{S}_{qq} = \lim_{T\to\infty} E\left[\frac{1}{T}\mathbf{q}\mathbf{q}^{\mathrm{H}}\right] = \mathbf{Z}^{+}\mathbf{S}_{\hat{p}\hat{p}}\mathbf{Z}^{+\mathrm{H}}\,, \tag{4}$$

where $\mathbf{S}_{\hat{p}\hat{p}}$ is the matrix of measured acoustic pressure auto- and cross-spectra. The matrix of optimal source strengths is calculated simultaneously for all the assumed model sources by the inversion of the transfer matrix. The error in the resulting $\mathbf{S}_{qq}$ will then be dependent on the square of the condition number of the matrix $\mathbf{Z}$.

4. BEAMFORMING

The main concept behind DOA estimation using a set of microphones with a known geometry is to use the phase information present in the array sensor signals. With the proper selection of the microphones data weights, the beamformer can enhance signals from a selected direction and reject noise and interference. This property can be used to perform DOA estimation. The output power of the array can be computed for each direction of interest, and plotted against the look directions. When a signal is present, the output power will exhibit a peak that can be taken as an estimation of the DOA of the desired signal.

Considering an array of M sensors immersed in a sound field. For a fixed frequency ω, the output $x_m(\omega)$ of sensor m, consists of a signal component $s_m(\omega)$ and a noise component $n_m(\omega)$

$$x_m(\omega) = s_m(\omega) + n_m(\omega)\,. \tag{5}$$

The Fourier transform array output response can be expressed as the sum of the weighted sensor outputs

$$\mathbf{y}(\omega) = \mathbf{w}^{\mathrm{H}}\mathbf{x}(\omega)\,, \tag{6}$$

where $\mathbf{w}$ is the complex weight vector for the array. The array power spectral can then be written as

$$P(\omega) = \mathbf{w}^{\mathrm{H}}\mathbf{R}\mathbf{w}\,, \tag{7}$$

where $\mathbf{R}$ is the spatial correlation matrix between the field array signals, i.e. $\mathbf{S}_{\hat{p}\hat{p}}$.

Different beamforming approaches correspond to different selections of the weight vectors $\mathbf{w}$. In this work, we have employed three different estimators that are applicable to arbitrary array geometries, namely the Bartlett beamformer, the Capon estimator and the MUSIC (MUltiple SIgnal Classification) method. The two first can be classified into the category of steered beamformer based methods, in which the array is "steered" to the different directions by forming a combination of the output response. The Bartlett and Capon spectrum are obtained respectively by

$$P_{\text{Bartlett}}(r,\theta,z) = \frac{\mathbf{a}^{\text{H}}(r,\theta,z)\,\mathbf{R}\,\mathbf{a}(r,\theta,z)}{\mathbf{a}^{\text{H}}(r,\theta,z)\,\mathbf{a}(r,\theta,z)}, \tag{8}$$

$$P_{\text{Capon}}(r,\theta,z) = \frac{1}{\mathbf{a}^{\text{H}}(r,\theta,z)\,\mathbf{R}^{-1}\mathbf{a}(r,\theta,z)}, \tag{9}$$

where $\mathbf{a}(r,\theta,z)$ is the beamformer steering vector focussed at a source position of coordinate (r,θ,z). For the problem considered here, this has been taken as the pressure evaluated at the microphone sensor positions due to a source situated at the particular evaluation position, and it has been deduced from Equation (1).

The MUSIC algorithm belongs to another group of estimators known as high-resolution subspaces methods. The cross correlation matrix of the array signals is divided into two orthogonal subspaces, the signal subspace and the noise subspace by means of an eigen-decomposition of $\mathbf{S}_{\hat{p}\hat{p}}$. The corresponding spectrum is given by

$$P_{\text{MUSIC}}(r,\theta,z) = \frac{\mathbf{a}^{\text{H}}(r,\theta,z)\,\mathbf{a}(r,\theta,z)}{\mathbf{a}^{\text{H}}(r,\theta,z)\,\Pi^{\perp}\mathbf{a}(r,\theta,z)}, \tag{10}$$

where $\Pi^{\perp}$ is the orthogonal projector onto the noise subspace.

5. NUMERICAL SIMULATIONS

The set of equations derived in the previous sections has been implemented in a numerical program to illustrate the performances of both approaches. The system analysed in this work has been motivated by an experimental investigation in a real duct laboratory facility situated in the anechoic chamber of the Université de Technologie de Compiègne Acoustic Group. The physical parameters have been chosen to match those of the experimental set-up. The test rig is made of a steel 0.015 m thick duct of radius R = 0.0748 m.

In order to compare the two methods, a planar array of field measurements in a duct cross-section has been created combining 16 equally spaced angular positions and 8 equally spaced radial positions, as indicated in Figure 1. It has been shown[8] that the axial distance between the assumed source array and the sensor array is a key factor that influences the capacity of the inverse method for an accurate reconstruction of the source strengths. In the source near field, the information provided by both the evanescent and propagating waves is captured by the sensor array, therefore leading to a good estimate of the real source from the measurements. When increasing the separation distance between the sources and the measurement positions, the measured amplitudes of the evanescent waves will decrease leading to an ill-conditioned problem. For this reason, and to test the complementarity between the different estimators, the sensor array has been situated in the far field location, at an axial separation distance of 2.5 m from the sources.

The primary field is supposed to be due to the generation of four monopole sources with source strength equal to unity and situated over a duct cross section, at a radial distance of $R/2$ with an angular separation of 15°. This set of sources has been assumed to be either fully coherent or incoherent. For the position and source strength estimation, Bartlett, Capon and MUSIC estimators have been compared with the inverse method when 5% of noise is introduced in the input data. For the beamformers, a grid of exploration positions has been defined in the source cross section, and the steering vector has been calculated using the analytical expression (1) for each source position. Equations (8-10) have been used consequently for calculating the output power spectra. For the inverse method, a model of reconstructed sources has been defined, and the matrix of transfer functions between all the sources-microphones pairs has been inverted for each frequency. The mean square error for the estimation of the source positions and strengths has been plotted as a function of frequency for the different estimators considering that the four sources are either coherent or incoherent. The results obtained are presented in the Figures 2 and 3 respectively.

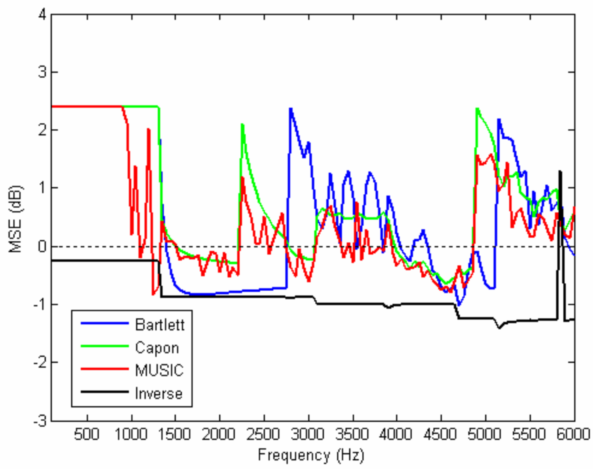

Figure 2. Mean square error for the strength estimation of four coherent sources using the inverse method (black) and three different beamformers: Bartlett (blue), Capon (green) and MUSIC (red).

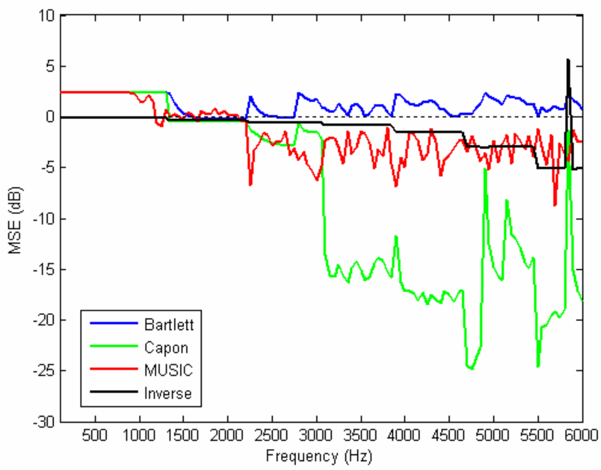

Figure 3. Mean square error for the strength estimation of four incoherent sources using the inverse method (black) and three different beamformers: Bartlett (blue), Capon (green) and MUSIC (red).

As expected, the performance of the beamforming methodology is rather poor when the sources are fully coherent. The reconstruction performance of these spectral-based methods is much reduced for scenarios involving correlated signals as the final result is a rank deficiency in the spatial correlation matrix. However, when working with incoherent signals, their performance improves considerably and outperforms the results obtained with the inverse method (excepting for the Bartlett estimator). The inverse method is less influenced by the degree of coherence between the primary sources, although this result has been found to be very dependent on the reconstructed source model proposed.

In order to better visualise the practical reconstruction capability of the different methods, the auto-power spectra of the source strengths have been compared at a fixed frequency (2750 Hz) for all the scanned positions in the original source duct cross section. The results obtained are presented in Figure 3.

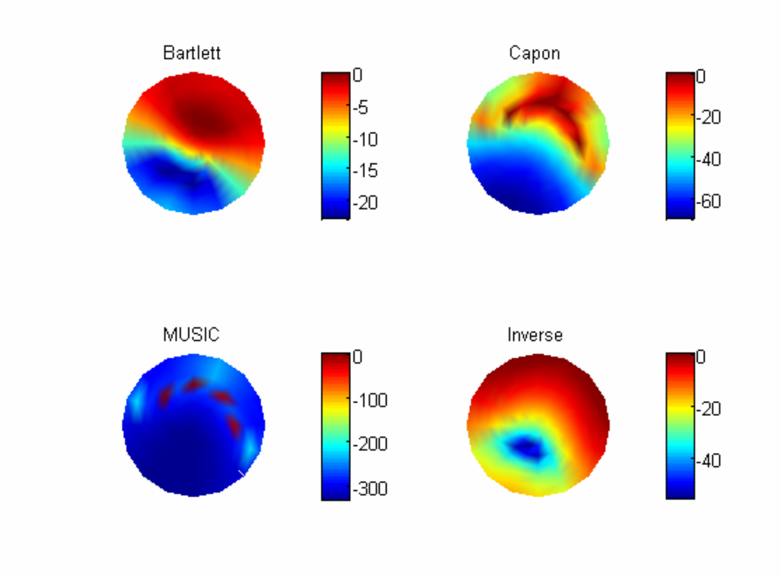

Figure 4. Estimated auto-power spectra of the source strengths distributions over the duct cross section using three different beamformers (Bartlett, Capon and MUSIC) and the inverse method.

As for spatial resolution, the MUSIC algorithm is able to outperform the inverse method and the other beamformer estimators. This might appear to be in opposition with Figure 3, that indicates that the minimum error corresponds to the Capon estimator. This difference can be understood by noting that the spectrum calculated by the MUSIC algorithm (Equation 10) is not a true spectrum (it is the distance between two subspaces), and it does not provide a good estimation of the real source strengths, but it exhibits well-resolved peaks in the proximity of the real source positions defined by the steering vector.[2] The Capon estimator is able to identify the right source strength unit amplitudes, but it is not so accurate for the positioning of the sources. Two types of mean square errors could then be defined, considering the capacity of the techniques to appropriately locate the original sources, and the capacity to provide the correct source strength amplitudes. With this idea in mind, the beamformers could be used in a first step to provide an estimation of the position of the reconstructed sources required for the inverse method.

6. CONCLUSIONS

The main objective of source identification techniques in aero-acoustic problems is the reconstruction of noise source distributions from available pressure field measurements. The estimation of the acoustic source distribution can help to improve noise control techniques, therefore constituting a development tool essential for research in nacelle noise reduction. In this study, a set of pressure measurements has been predicted over a planar array located inside an infinite rigid cylinder, and two different estimation methods have been compared, the inverse method and beamforming estimators.

For the inverse reconstruction techniques, a reconstructed source model has been assumed and a propagation operator has been inverted for deducing the position and the strength of the original sources distribution. However, the ducted source reconstruction problem becomes ill-conditioned in the low frequency range and when increasing the axial separation distance between the original source array and the microphone sensors. The performance results are also highly dependent on the choice of the reconstructed assumed model.

The use of directive microphone arrays to design beamformers that can be oriented in all possible directions and focussed at different source candidate positions could already provide a prior model of the assumed source locations for the inverse method. The position that gives the maximum output power to the MUSIC-based beamformer can be considered as a reliable estimate of the real source positions although this estimate is not well-resolved when the sources are highly coherent or under plane wave propagation conditions (below the first cut-on frequency of the duct). Further work in the future will be conducted for the implementation and testing of other beamformers that can overcome this problem, like the so-called parametric array processing methods.

ACKNOWLEGMENTS

This research has been supported by the European Commission under the Sixth Framework programme, contract INDUCT-MEIF-CT-2006-022579.

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