ICSV14

Cairns • Australia

9-12 July, 2007

14th International Congress on Sound & Vibration

VIBRATION SUPPRESSION PERFORMANCES OF POWER HARVESTING SYSTEMS USING PIEZOELECTRIC TRANSDUCERS

Kimihiko Nakano[1], Tsuyoshi Arinaga [2]

[1] Institute of Industrial Science, the University of Tokyo, 4-6-1 Komaba, Meguro-ku, Tokyo 153-8505 Japan

[2]Graduate School of Science and Engineering, Yamaguchi University 2-16-1 Tokiwadai, Ube 755-8611, Japan

knakano@iis.u-tokyo.ac.jp

Abstract

Vibration suppression and power harvesting utilizing the piezoelectric shunt technique are analysed using equations that consider the effect of the dynamical properties of the transducer on the vibration of the structure. Numerical calculations show that the optimal conditions for suppression and harvesting do not agree; therefore, we need consider a trade-off between them before installing piezoelectric transducers.

1. INTRODUCTION

A piezoelectric transducer connected to a passive shunt circuit was utilized to produce damping in a vibrating structure and the optimal condition to suppress the vibration was discussed in the research of Hagood and Flotow [1]. In this shunt technique, the vibration energy is dissipated in the resistor; therefore, when t this energy is collected it is feasible to generate power from the structural vibration. Such systems are called power harvesting systems and have recently attracted the attention of many researchers [2]. To analytically discuss the performance of power harvesting systems, a two-port network model [3] was introduced to describe the electro-mechanical coupled dynamics and the power to be harvested, with consideration given to the effect of the dynamical properties of the transducer on the vibration of the structure [4]. However performance to suppress the vibration was not discussed in that report.
In this study, the transfer function of the transverse displacement of a simply supported beam from the point force is derived on the basis of a two-port network model, as well as from an equation to estimate the harvested power. The optimal conditions for suppressing the vibration are derived using a method presented in previous research [1]. We show the power to be harvested by the optimally tuned shunt system, as well as its suppression performance. The performances of power harvesting and vibration suppression under several conditions are represented by numerical analyses and the optimal conditions to satisfy these two performances are discussed.

2. MODELLING

2.1 Simply Supported Beam

We discuss the performance of the piezoelectric transducer on a simply supported beam as shown in figure 1. In this figure, f_p, f_t, l_b, l_t, p_f, p_t w and y represent point force, force produced by the transducer, length of the beam, length of the piezoelectric transducer, position of the point force, left edge of the transducer, transverse displacement, and the local coordinate of the beam, respectively.

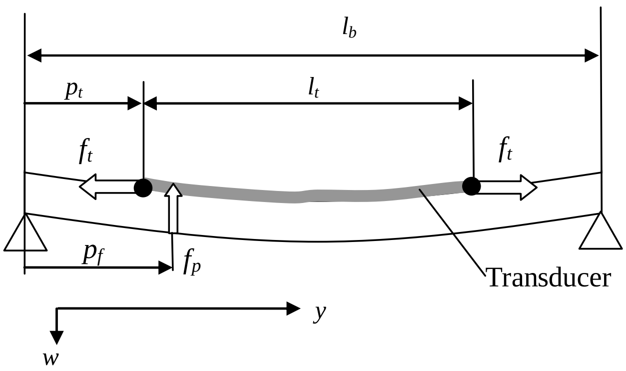

Figure 1. A schematic of the piezoelectric transducer on a simply supported beam.

When the transducer is thin enough, the moment distribution $M(y)$ induced by the transducer is

$$M(y) = -f_t \frac{t_b}{2}\left[H(y-p_t) - H(y-p_t-l_t)\right], \tag{1}$$

where t_b is the thickness of the beam and $H(y)$ is the unit Heaviside step function defined as

$$H(y) = \begin{cases} 1, & y > 0, \\ 0, & y < 0. \end{cases} \tag{2}$$

Then the governing equation is obtained by

$$\begin{aligned} EI\frac{d^4 w}{dy^4} + \rho t_b b_b \frac{d^2 w}{dt^2} &= \delta(y-p_f) f_p - \frac{d^2 M(y)}{d\,y^2} \\ &= \delta(y-p_f) f_p + \left[\frac{d}{dy}\delta(y-p_t) - \frac{d}{dy}\delta(y-p_t-l_t)\right] f_t \frac{t_b}{2} \end{aligned}, \tag{3}$$

where b_b, E, I, and ρ are the width of the beam, Young's Modulus of the beam, second moment of the beam cross section, and mass density, respectively, and δ(y) is the delta function defined as

$$\delta(y) = \begin{cases} 1, & y = 0, \\ 0, & y \neq 0. \end{cases} \tag{4}$$

Transverse displacement is written as

$$w(y,t) = \sum_{n=1}^{\infty} W_n \sin(k_n y) \cdot e^{st} \tag{5}$$

where t is time, $k_n = n\pi/l_b$, and s is Laplace parameter. By substituting equation 5 into 3, we obtain

$$\sum_{n=1}^{\infty} \left(k_n^4 EI + s^2 \rho t_b b_b\right) W_n \sin(k_n y) = \delta(y - l_f) F_p + \left[\frac{d}{dy}\delta(y - p_t) - \frac{d}{dy}\delta(y - p_t - l_t)\right] F_t \frac{t_b}{2} \tag{6}$$

where $f_p = F_p e^{st}$ and $f_t = F_t e^{st}$. When we multiply both sides of the equation by $\sin(k_n y)$ and integrate over the length of the beam, we obtain [5]

$$l_b \sum_{n=1}^{\infty} \left(k_n^4 EI + s^2 \rho t_b b_b\right) W_n = 2F_p \sin(k_n p_f) + F_t t_b \left\{\cos[k_n (p_t + l_t)] - \cos(k_n p_t)\right\}. \tag{7}$$

Considering the neighbourhood of the nth mode and the damping ratio of the nth mode is ζ_{msn}, W_n is given by

$$W_n = \frac{1}{L_{ms}} \frac{2F_p \sin(k_n p_f) + F_t t_b k_n \left\{\cos[k_n (p_t + l_t)] - \cos(k_n p_t)\right\}}{\omega_{msn}^2 + 2\omega_{msn}\varsigma_{msn} s + s^2} \tag{8}$$

where $L_{ms} = \rho t_b b_b l_b$ and $\omega_{msn} = k_n^2 \sqrt{\frac{EIl_b}{L_{ms}}}$.

2.2 Piezoelectric Transducer

A constitutive equation of the laminar design piezoelectric transducer is written as [6]

$$\begin{bmatrix} f_t \\ v \end{bmatrix} = \begin{bmatrix} \frac{1}{C_{mt}} & -D_{31} \\ -D_{31} & \frac{1}{C_{et}} \end{bmatrix} \begin{bmatrix} q_{mt} \\ q_{et} \end{bmatrix} \tag{9}$$

where q_{mt}, q_{et}, v, C_{mt}, C_{et} and D_{31} are mechanical deflection of the transducer, electrical charge, voltage, mechanical compliance, electric capacitance, and piezoelectric transducer constant, respectively. In the resonant shunt technique, the transducer is connected to a series LR circuit to produce the damping force. Then, the voltage v is written as

$$v = -\left(s^2 L_{eL} + sR_{eL}\right) q_{et}, \tag{10}$$

therefore

$$q_{et} = \frac{D_{31}}{s^2 L_{eL} + sR_{eL} + \frac{1}{C_{et}}} q_{mt} \,. \tag{11}$$

$$f_t = \left(\frac{1}{C_{mt}} - \frac{(D_{31})^2}{s^2 L_{eL} + sR_{eL} + \frac{1}{C_{et}}} \right) q_{mt} \tag{12}$$

When the transducer is thin enough, the deflection of the transducer is given by

$$\begin{aligned} q_{mt} &= \int_{p_t}^{p_t + l_t} -\frac{t_b}{2} \frac{d^2 q_{ms}(y)}{dy^2} dy \\ &= -W_n \frac{t_b}{2} k_n \{\cos[k_n(p_t + l_t)] - \cos(k_n p_t)\} e^{st} \end{aligned} \,, \tag{13}$$

From equations 8, 12, and 13, the amplitude of *n*th mode is derived as

$$W_n = \frac{2\sin(k_n p_f)}{L_{ms}\left(\omega_{msn}^2 + 2\omega_{msn}\varsigma_{msn} s + s^2\right) + \frac{1}{2} t_b^2 k_n^2 \left[\frac{1}{C_{mt}} - \frac{(D_{31})^2}{s^2 L_{eL} + sR_{eL} + \frac{1}{C_{et}}} \right] \{\cos[k_n(p_t + l_t)] - \cos(k_n p_t)\}^2} F_p \,. \tag{14}$$

2.3 Harvested power

Assuming the power consumed in the load impedance is the power to be harvested, the harvested power is written as

$$P_h = \frac{1}{2} R_{eL} \left| s\, q_{et} \right|^2 . \tag{15}$$

From equations 11, 13, 14 and 15, the harvested power is given by

$$P_h = \frac{R_{eL}}{2} \left| \frac{-sD_{31} t_b k_n \{\cos[k_n(p_t + l_t)] - \cos(k_n p_t)\} \sin(k_n p_f)}{L_{ms}\left(\omega_{msn}^2 + 2\omega_{msn}\varsigma_{msn} s + s^2\right)\left(s^2 L_{eL} + sR_{eL} + \frac{1}{C_{et}}\right) + \frac{1}{2} t_b^2 k_n^2 \left[\frac{1}{C_{mt}}\left(s^2 L_{eL} + sR_{eL} + \frac{1}{C_{et}}\right) - D_{31}^2 \right] \{\cos[k_n(p_t + l_t)] - \cos(k_n p_t)\}^2} \right|^2 F_p^2 \,. \tag{16}$$

3. PERFORMANCES

The optimal condition for vibration suppression is obtained referring to the resonant shunt technique. In this technique, the resistance R_{eL} and inductance L_{eL} of the load are tuned in a way similar to an optimally tuned vibration absorber. Then the optimal values of R_{eL} and L_{eL}, which are represented by R_{opt} and L_{opt}, respectively, are given by

$$R_{opt} = \frac{R_P + R_Q}{2} \tag{17}$$

$$L_{opt} = \frac{1}{\omega_m^2 C_{et} + \varphi_m^2} \tag{18}$$

where

$$\omega_m = \left(\omega_{msn}^2 + \frac{t_b^2 k_n^2}{2 L_{ms} C_{mt}} \left(1 - C_{et} C_{mt} D_{31}^2\right) \{\cos(k_n p_t) - \cos[k_n (p_t + l_t)]\}^2 \right)^2 \tag{19}$$

$$\varphi_m^2 = \frac{t_b^2 k_n^2 C_{et}^2 D_{31}^2}{2 L_{ms}} \{\cos(k_n p_t) - \cos[k_n (p_t + l_t)]\}^2 . \tag{20}$$

$$R_P, R_Q = \sqrt{\frac{3\varphi_m^2}{C_{et}\left(\omega_m^2 C_{et} + \varphi_m^2\right)\left[2\left(\omega_m^2 C_{et} + \varphi_m^2\right) \mp \sqrt{2\varphi_m^2\left(\omega_m^2 C_{et} + \varphi_m^2\right)}\right]}} . \tag{21}$$

Figure 2 shows the frequency response of the amplitude W_n from the point force in the 1st mode, where $L_{eL}=L_{opt}$ and $p_t = 0.5(l_b - l_f)$. Values of the parameters are written in Table 1. It is clear that the system has the best suppression performance when $R_{eL}=R_{opt}$. Figure 3 shows the power to be harvested per unit pin force input. The system with resistance less than the optimal value harvests power more than the optimal resistance. The optimal condition for suppression does not agree with the power harvesting. Frequency response and the harvested power of a system with a variable load inductance are represented in Figs. 4 and 5, respectively, where the load resistance is the optimal value. The system with an inductance less than the optimal value harvests more power. Figures 6 and 7 show the suppression and power harvesting performances, respectively, of shunt systems with variable instalment position p_t, where the resistance and inductance are optimal values. We can determine that installing at the mid point of the simply supported beam is the best solution for both suppression and harvesting.

Table 1. Values of the parameters.

Descriptions	Symbols	Values
length of the beam	l_b	1.0 m
length of the transducer	l_t	15.0cm
position of the point force	p_f	0.5 m
thickness of the beam	t_b	1.0 cm
capacitance of the transducer	C_{et}	0.156 μF
mechanical compliance of the transducer	C_{mt}	0.159 mm/kN
piezoelectric transducer coefficient	D_{31}	1.69 kV/mm
Young's modulus	E	206 GPa
moment of inertia of the beam cross section	I	33.3 cm^4
mass of the beam	L_{ms}	3.144 kg
damping ratio of *n*th mode	ζ_{msn}	0.01

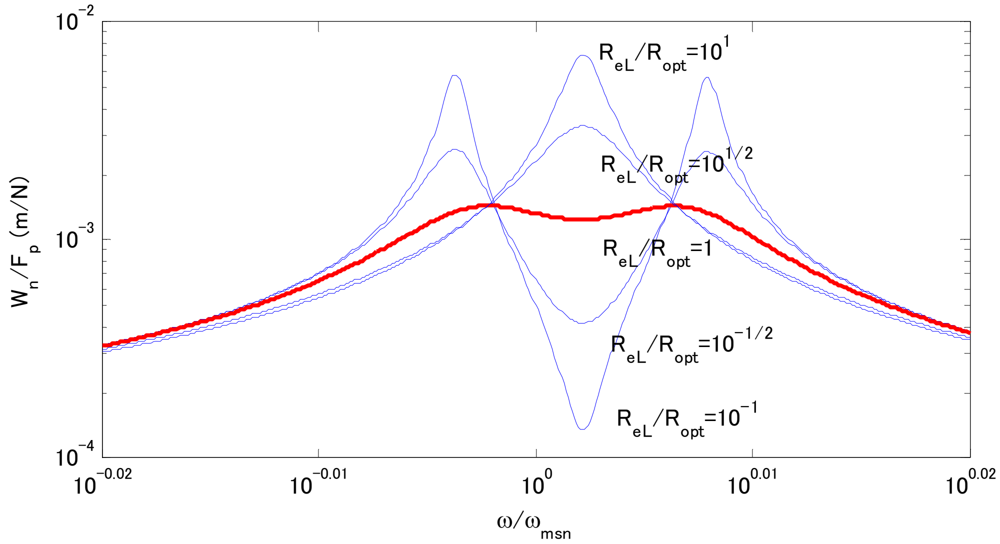

Figure 2 Suppression performances for variable R_{eL}.

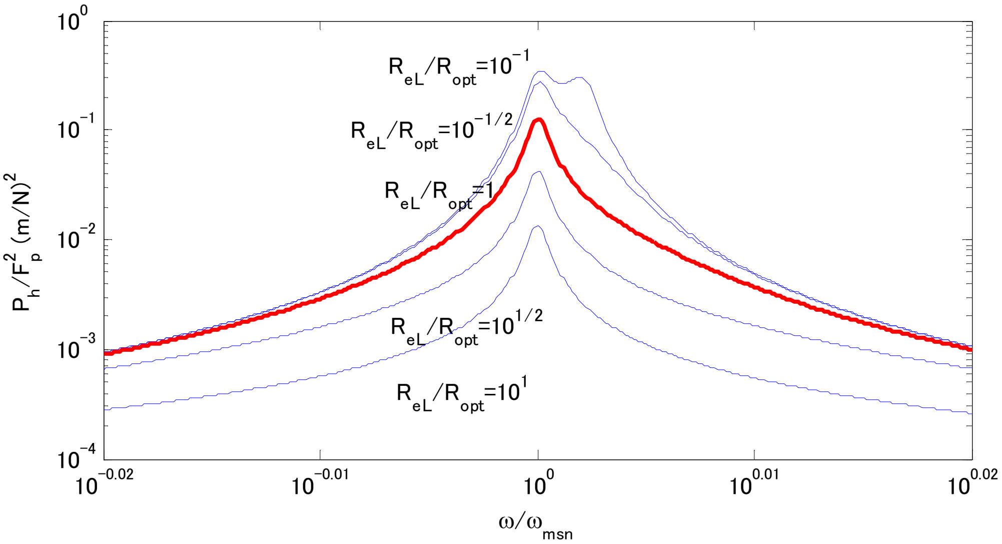

Figure 3 Power harvesting performances for variable R_{eL}.

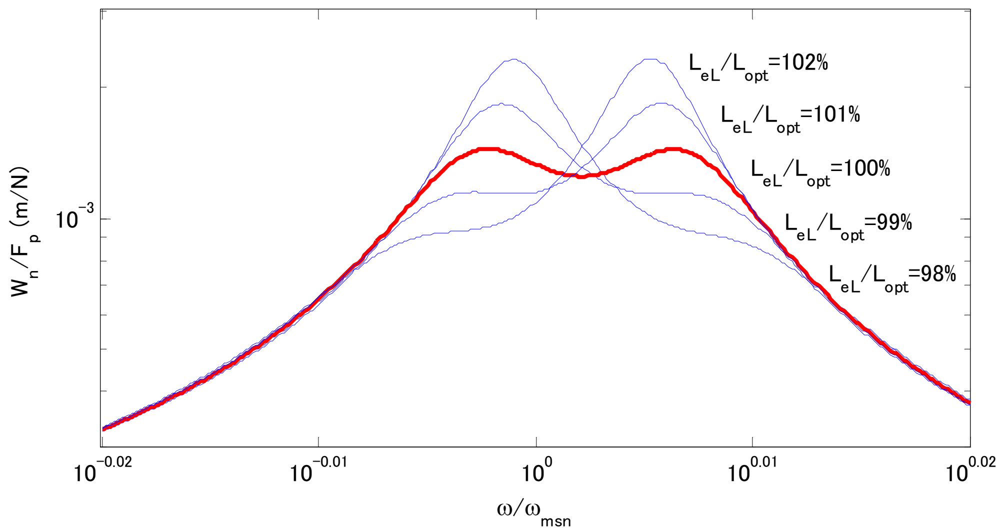

Figure 4 Suppression performances for variable L_{eL}.

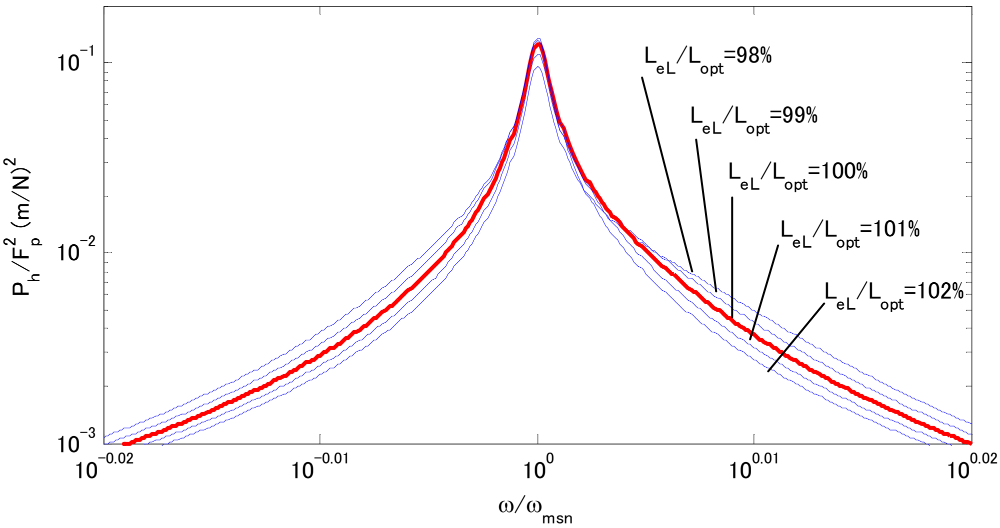

Figure 5 Power harvesting performancesfor variable L_{eL}.

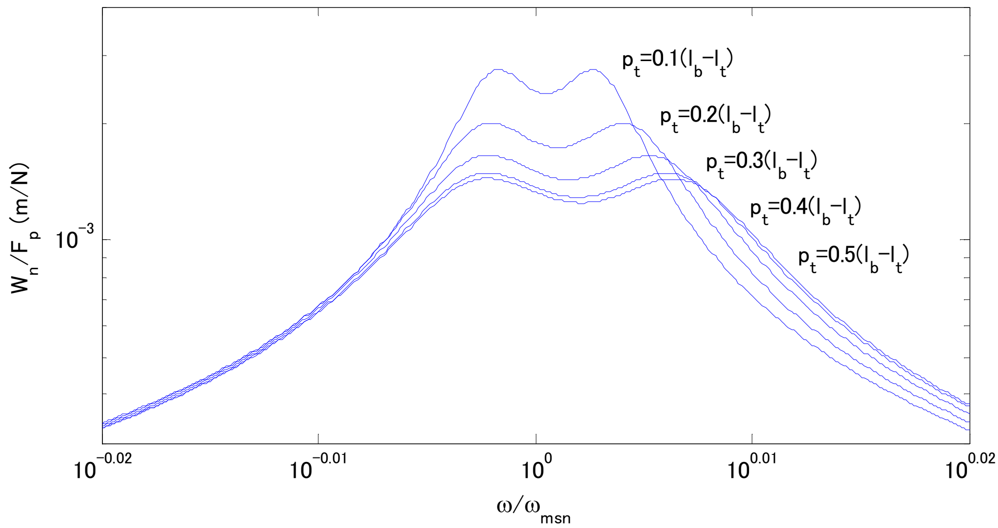

Figure 6 Suppression performances for variable instalment position p_t.

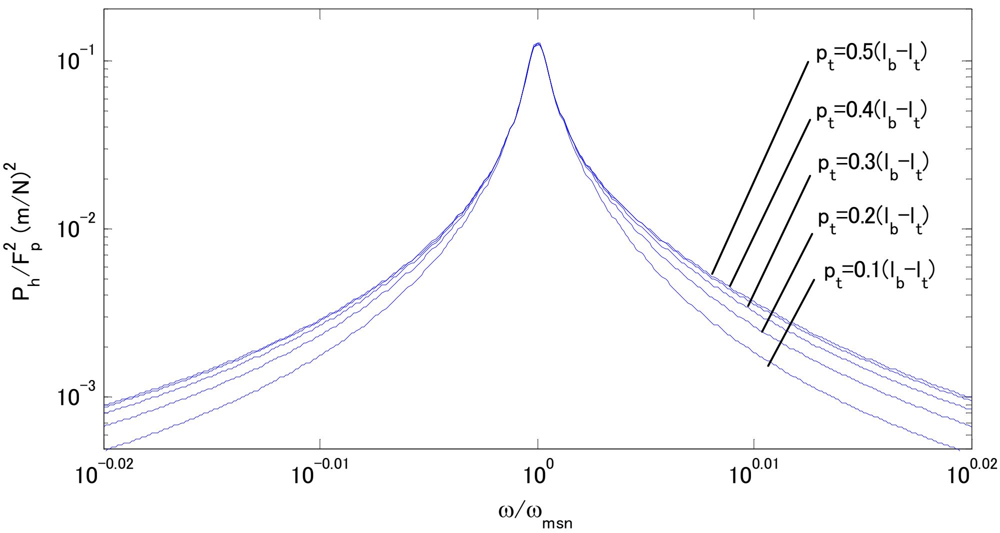

Figure 7 Power harvesting performances for variable instalment position p_t.

4. CONCLUSIONS

Using equations derived based on the two-port network model, suppression and power harvesting performances are discussed. The optimal conditions for suppression do not agree with those for power harvesting. A load resistance and inductance smaller than their optimal values provided more harvested power, although the optimal instalment position is the mid point between nodes for both suppression and power harvesting. We need to discuss a trade-off between suppression and harvesting performances before installing piezoelectric transducers.

ACKNOWLEDGEMENT

This work was supported by a grant from The Ministry of Education, Culture, Sports, Science and Technology of Japan, Grand-in-Aid for Young Scientists (B) (No. 18760169).

REFERENCES

[1] N.W. Hagood and A.V. Flotow, “Damping of structural vibrations with piezoelectric materials and passive electrical networks”, *Journal of Sound and Vibration* **146**(2), 243-268 (1991).

[2] H.A. Sodano, D.J. Inman and G. Park, “A review of power harvesting from vibration using piezoelectric materials”, *The Shock and Vibration Digest* **36**, 197-205 (2004).

[3] F.V. hunt, *Electroacoustics Second Printing*, Acoustical Society of America, New York, 1982.

[4] K. Nakano, S.J. Elliott and E. Rustighi, “A Unified Approach to Energy Harvesting Using Electro-magnetic and Piezoelectric Transducers”, *Proceedings. of IX International Conference on Recent Advances in Structural Dynamics*, 17-19 July, Southampton, UK.

[5] C.R. Fuller, S.J. Elliott and P.A. Nelson, *Active Control of Vibration*, Academic Press, London, 1996.

[6] A. Preumont, *Vibration Control of Active Structures Second Edition*, Kluwer Academic Publishers, Dordrecht, 2002.