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WAVE PROPAGATION IN THIN ORTHOTROPIC MEDIA

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Abstract

Wave motion in a fibre-reinforced thin orthotropic laminate is studied in the paper. A thin panel made of laminate is loaded in-plane by stress pulses. Theoretical solution is based on finite element approach. Experimental solution utilizes for noncontact measurements a laser vibrometer. It was found out that there are some discrepancies between theory and experiment. It turns out that elastic constants given by the producer of the laminate are somewhat incorrect. Measurements performed during these days indicate e.g. difference between Young's moduli in the direction of fibres about 10%. Further research is in progress.

1. INTRODUCTION

The problem concerned with wave propagation in a fibre-reinforced composite is discussed in the paper. The kind of composite material in mind is one in which a matrix material (e.g.epoxy resin) is reinforced by strong stiff fibres (e.g. carbon fibres) which are systematically arranged in the matrix. The fibres are considered to be long compared to their diameters and the fibre spacing, and to be quite densely distributed, so the fibres form a substantial proportion of the composite. Since the fibres are systematically orientated, a composite of this kind has strong directional properties, thus macroscopically for sufficiently long wavelength it can be regarded as a homogeneous anisotropic material.

The propagation of elastic waves in anisotropic media differs in many respects from that customarily attributed to elastic waves in isotropic media. For a given direction of wave propagation represented by a wave vector there will be generally three phase velocities, the three corresponding displacement (polarization) vectors will be mutually orthogonal but contrary to the isotropic case the displacements are neither truly longitudinal nor truly transverse in character. As the mechanical or material behaviour of the solid becomes more complicated, the description of non-stationary wave propagation starts to be analytically intractable and, consequently, such problems are often modelled by means of discretization techniques such as finite elements (FE) or finite differences. It is very useful to supplement the theoretical analysis by experimentally obtained results.

The aim of the paper is to study the wave propagation in thin orthotropic laminate strip loaded in-plane by stress pulses. Theoretical solution is based on FE approach. Experimental solution utilizes for noncontact measurements a laser vibrometer.

2. PROBLEM FORMULATION

We consider a thin strip of laminate. Each layer of the laminate is supposed to be a unidirectionally fibre-reinforced composite. It is also assumed that fiber diameters and laminate thickness are small compared to the shortest wavelength taken into account. Hence one can consider the material as orthotropic solid in the state of plane stress. The principal directions of orthotropy often do not coincide with coordinate directions that are geometrically natural to the solution of the problem. Therefore it is assumed that body axes x,y form a nonzero angle ϑ with principal material axes 1,2 as may be seen in Fig.1. Third axis z is identical with material axes 3 and constitutes axis of rotation of principal material axes 1,2 from body axes x,y.

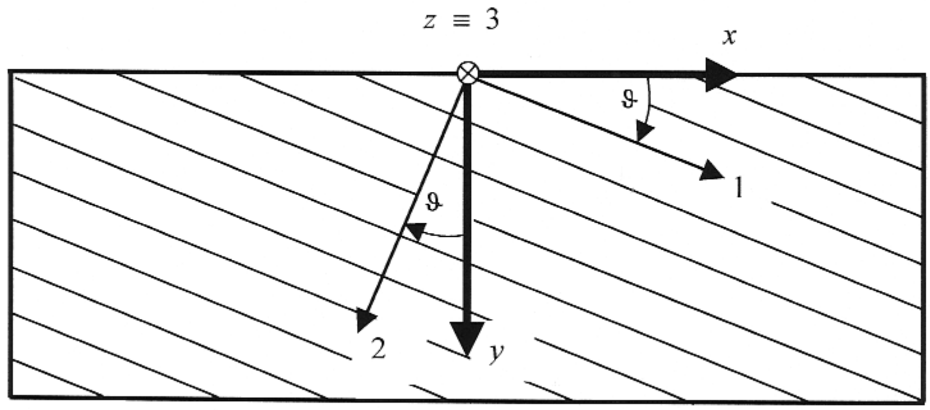

Figure 1. A thin unidirectional laminate strip

Due to a plane stress state it may be written

$$\sigma_{33} = \sigma_{32} = \sigma_{31} = 0\,. \tag{1}$$

The stress-strain relation in principal material axes 1,2 has the following form

$$\begin{Bmatrix} \sigma_{11} \\ \sigma_{22} \\ \sigma_{12} \end{Bmatrix} = \begin{bmatrix} B_{11} & B_{12} & 0 \\ B_{12} & B_{22} & 0 \\ 0 & 0 & B_{66} \end{bmatrix} \bullet \begin{Bmatrix} \varepsilon_{11} \\ \varepsilon_{22} \\ 2\varepsilon_{12} \end{Bmatrix} \quad , \tag{2}$$

where four independent material (stiffness) coefficients B_{ij} can be expressed in terms of the engineering constants,

$$\begin{aligned} B_{11} &= \frac{E_1}{1-\nu_{12}\cdot\nu_{21}} \quad , \quad B_{22} = \frac{E_2}{1-\nu_{12}\cdot\nu_{21}} \quad , \\ B_{12} &= \frac{\nu_{21}E_1}{1-\nu_{12}\cdot\nu_{21}} = \frac{\nu_{12}E_2}{1-\nu_{12}\cdot\nu_{21}} \quad , \quad B_{66} = G_{12}\,. \end{aligned} \tag{3}$$

In the x,y coordinate system it holds

$$\begin{Bmatrix} \sigma_{xx} \\ \sigma_{yy} \\ \sigma_{xy} \end{Bmatrix} = \begin{bmatrix} Q_{11} & Q_{12} & Q_{16} \\ Q_{12} & Q_{22} & Q_{26} \\ Q_{16} & Q_{26} & Q_{66} \end{bmatrix} \bullet \begin{Bmatrix} \varepsilon_{xx} \\ \varepsilon_{yy} \\ 2\varepsilon_{xy} \end{Bmatrix} . \tag{4}$$

The matrix components Q_{ij} are dependent on the coefficients B_{ij} and the angular position ϑ. Their somewhat complicated forms can be found in [1]. The displacement equations of motion in the absence of body forces are

$$\rho \cdot \frac{\partial^2 u}{\partial t^2} = Q_{11} \cdot u,_{xx} + 2Q_{16} \cdot u,_{xy} + Q_{66} \cdot u,_{yy} + Q_{16} \cdot v,_{xx} + (Q_{12} + Q_{66}) \cdot v,_{xy} + Q_{26} \cdot v,_{yy} \; , \tag{5}$$

$$\rho \cdot \frac{\partial^2 v}{\partial t^2} = Q_{66} \cdot v,_{xx} + 2Q_{26} \cdot v,_{xy} + Q_{22} \cdot v,_{yy} + Q_{16} \cdot u,_{xx} + (Q_{12} + Q_{66}) \cdot u,_{xy} + Q_{26} \cdot u,_{yy} \; , \tag{6}$$

where u and v are displacements in the axes x and y respectively. The in-plane loading is applied to the upper edge (y=0) of the strip 2L x H. The lower (y = H) and both lateral strip edges (x = ± L) are traction-free. The time and space dependence of the loading may be given by an arbitrary function. At the beginning the strip is at rest (zero initial conditions) and without any stress. Material density is given by ρ.

3. FINITE ELEMENT SOLUTION

Modelling of wave motion in the strip was carried out by using the finite element code MSC MARC. The thin strip of length 2L = 200 mm and width H = 50 mm (optimal dimensions of the strip with regard to gridsize used) was considered as an orthotropic elastic medium in the state of plane stress. It is assumed that strip is made of orthotropic composite SE84LV with following material constants: E_1 =129.9 GPa, E_2 = 13.9 GPa, G_{12} = 4.5 GPa, ν_{12} = 0.28, ρ = 1540 kg/m^3 . The loading of the strip was distributed along a small part of its upper edge (y=0) in the vicinity of x = 0, see Fig.1. The spatial discretization was performed using the bilinear square elements. The meshsize a = 0.1-0.2 mm was used so that grid dispersion might be suppressed [2]. It is known that for the time integration the lumped mass matrix and the central difference scheme represent the best combination because errors in the temporal and spatial approximations tend to cancel each other [3]. The numerical methods are very time-consuming. To obtain results faster some computing had to run as parallel jobs in cluster of workstations. Due to constricted software in case of cluster computing also Newmark and Houbolt time integration methods have been considered. For the same reason as above (i.e. reduction of approximation errors) at application of implicit methods is good to take (contrary to explicit methods) the full mass matrix. The mutual comparison of the above mentioned methods may be seen in Fig.2. In the figure the transient velocity response $\partial v/\partial t$ on the upper (loaded) edge of the strip at x=20 mm, y=0 and for ϑ=0 is shown. The spatial distribution of the loading pulse was given by halfcosine in the interval x∈<-1 mm, 1 mm> for y = 0. Time dependence of the loading was taken as half-period square-sine function in the interval t∈<0,2μs>. The results for Newmark and Houbolt methods are practically the same and therefore only velocities corresponding to central difference and Newmark methods are depicted. The first peak in the Fig.2 belongs to R-wavefront, the second wave packet represents the von-Schmidt and qT waves reflected from the bottom edge (y=H) of the strip. It is seen that the biggest differences in obtained velocities are in the vicinities of wavefronts. The explanation consists in the fact that high frequency components of pulse travel faster in

Newmark whereas in central diff. method they propagate slower [4]. It is a case of so-called grid dispersion mentioned above.

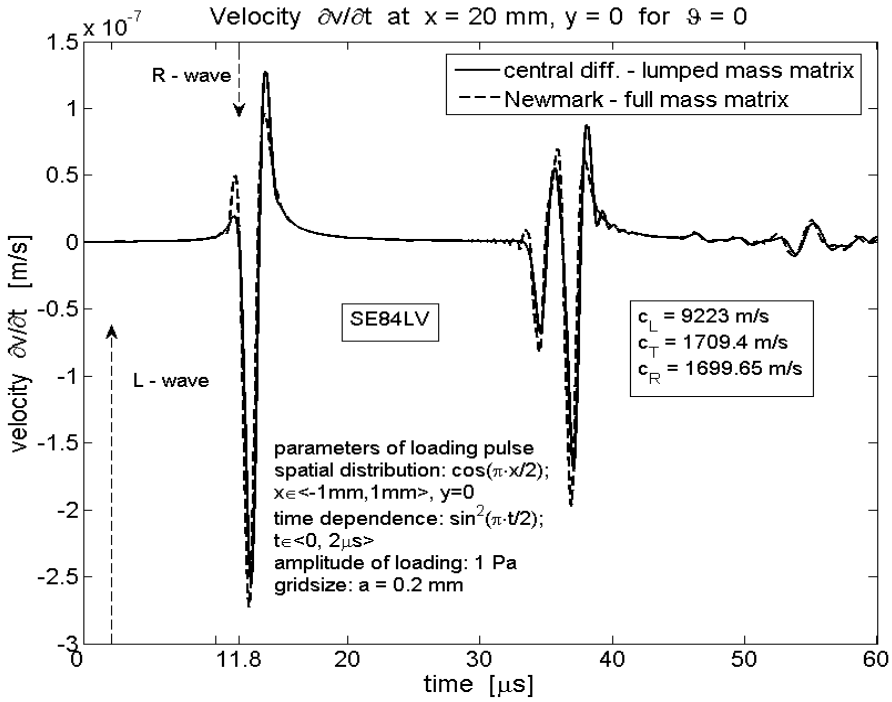

Figure 2. Time history of velocities at x = 20 mm, y = 0 for $\vartheta = 0$

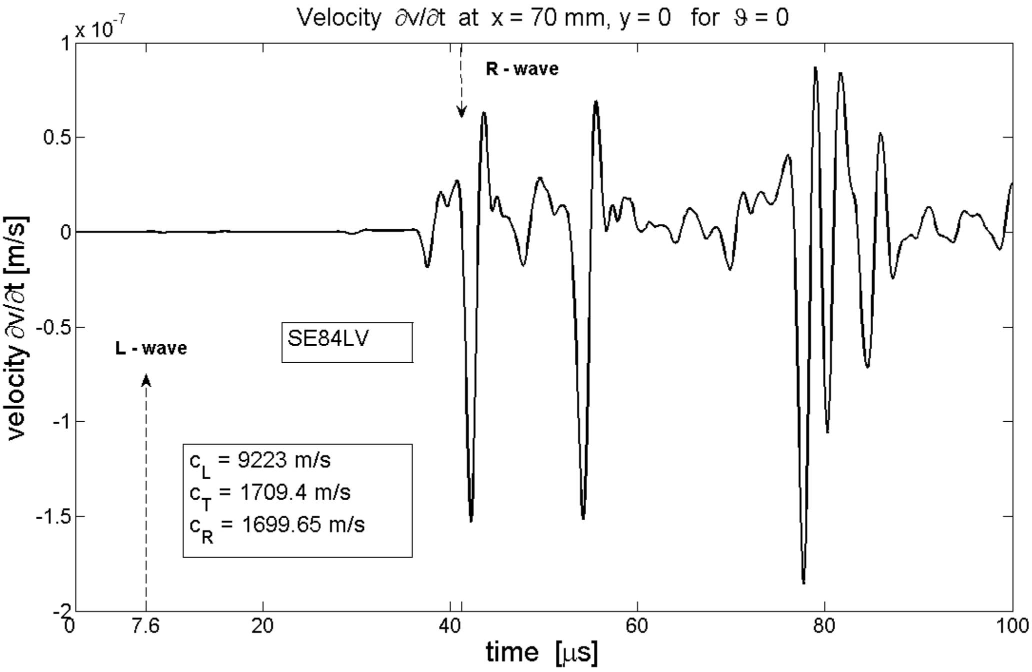

Figure 3: Time history of velocity at x = 70 mm, y = 0 for $\vartheta = 0$

In the Fig.3 the time history of $\partial v/\partial t$ on the upper edge of the strip at x=70 mm, y=0 and for ϑ=0 is displayed. The graph was obtained by means of central diff. method for the same parameters as above. It is seen that the velocity response is influenced by many wave reflections from bottom and lateral edges of the strip.

4. EXPERIMENT

The Compact Laser Vibrometer 2000 (CLV) was employed to measure velocities at selected points located on edges of the composite panel, which was in-plane loaded by the laser light pulse. The CLV system combined with Decoder Modul CLV-M030 is non-contact optical transducer whose output frequency ranges from 0.5Hz to 250kHz and its sensitivity used in experiments was 5mm/s/V. The dimension of the composite panel made of SE84LV (material constants are in the paragraph 3) was 500 x 500 x 2.2 mm and the orientation of the fibres has been, to date, $\vartheta=0°$and $\vartheta=90°$, see Fig 4. The experimental set up used for measurement of velocity $\partial v/\partial t$ at the points 40, 70 and 200 mm distant from the loading point x=0, y=0 is shown in the Fig.4. The output beam of the ruby laser HLS2 was reflected off the mirror 1 and divided into two parts by means of a glass plate. One of these parts, going directly through the glass plate, was directed by the mirror 2 and focused by a cylindrical lens onto the centre of the panel edge and across the edge to induce the plane state of stress. The other part of the laser beam, reflected from a front surface of the glass plate and attenuated by a neutral-density filter ND Filter, illuminated a PIN photodiode SGD-040A (rise-time 3ns). The signal from the photodiode was accepted as beginning of the loading process t=0.

The laser line focused on the panel edge was (due to cylindrical lens) 0.1 mm in width and 10 mm in length. The laser light pulse has maximum energy 1 J and primary pulse duration 15 ns. The power density was thus in order 10^9 Wcm^{-2}. Such concentration of irraditiation caused high-temperature heating followed by quick evaporation of the material in the close proximity of the loading point. As a consequence a mechanical impulse of very short duration (in the order of several 10^{-6} s) was propagated through the material.

To increase signal-to-noise ratio of an analog vibration output signal of the CLV system we stuck on the measuring points 3M Scotchlite Reflective Tapes (Scotchlite^{3M} 8850). The oscilloscope TDS 224 Tektronix (100MHz, 1GS/s) was used for recording output signals and the signal from the PIN diode was used for oscilloscope triggering.

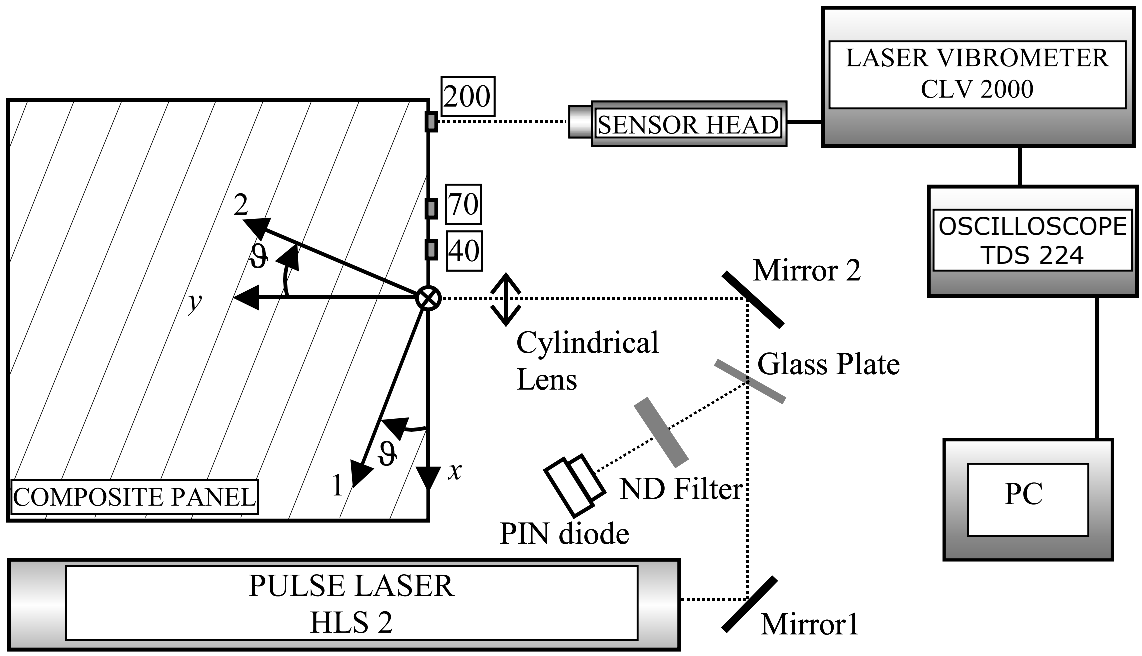

Figure 4: The experimental set-up

The panel dimensions were designed to exclude wave reflections from bottom and lateral edges for a sufficiently large time interval. This is also connected with the possibility to separate the Rayleigh from qT waves. It is known that differences between these waves are very small in the case of laminates [5]. For the considered material SE84LV we have c_R / c_T = 0.994. Results obtained by the experiment may be seen in the Figs.5 and 6. In the Fig. 5 the time record of velocity at x = 70 mm, y = 0 for ϑ = 90° is shown.

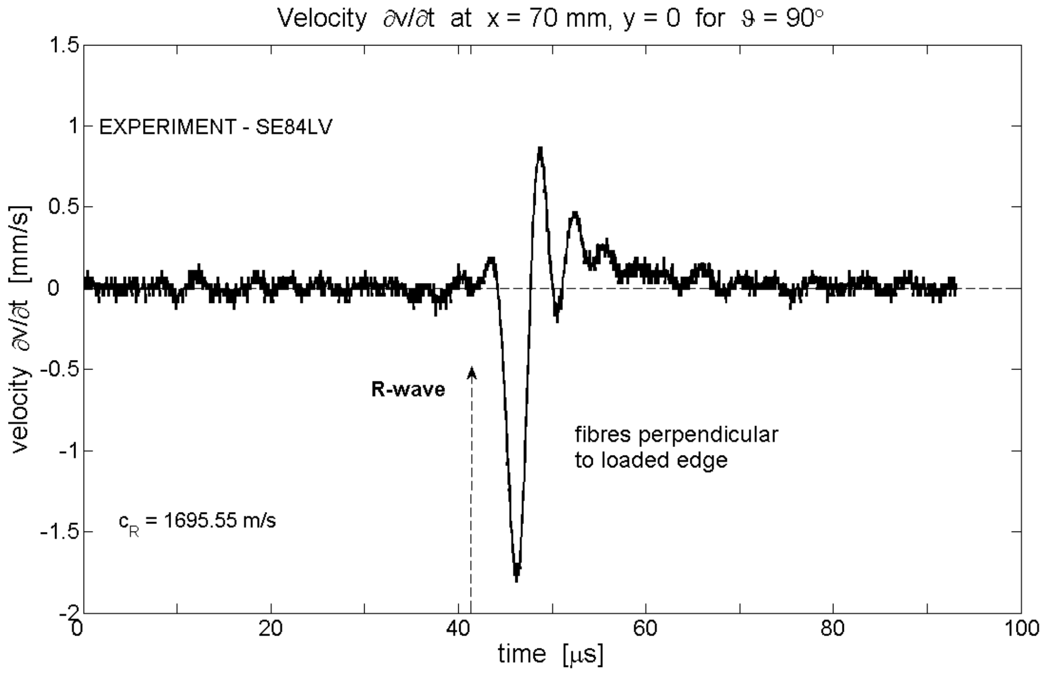

Figure 5: Time history of velocity at x = 70 mm, y = 0 for ϑ = 90° (experiment)

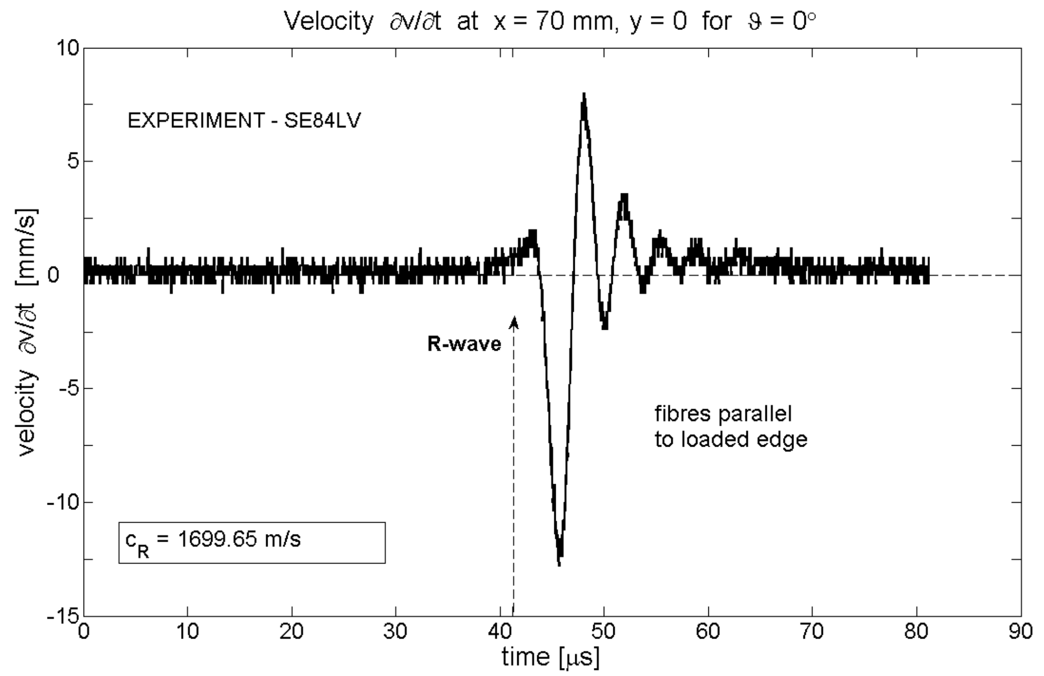

Figure 6: Time history of velocity at x = 70 mm, y = 0 for ϑ = 0° (experiment)

In the Fig.6 the time history of velocity ∂v/∂t at x = 70 mm, y = 0 for ϑ = 0° is depicted. It is seen that for ϑ = 90° (fibres perpendicular to loaded edge of the panel) we have at the same point (x = 70 mm, y = 0) much smaller velocity in the vicinity of R-wavefront than that obtained for $\vartheta = 0°$ (fibres parallel to loaded edge). It is due to energy leakage along fibres into the panel.

5. SOME OTHER RESULTS

Comparison of FEM results with experiment may be seen in the Fig.7. Because the composite panel for finite element computation is much smaller than that used in experiment the comparison is made for the time interval <0, 40 μs> only. For longer time the FEM data are influenced by reflections from bottom and lateral edges of the panel.

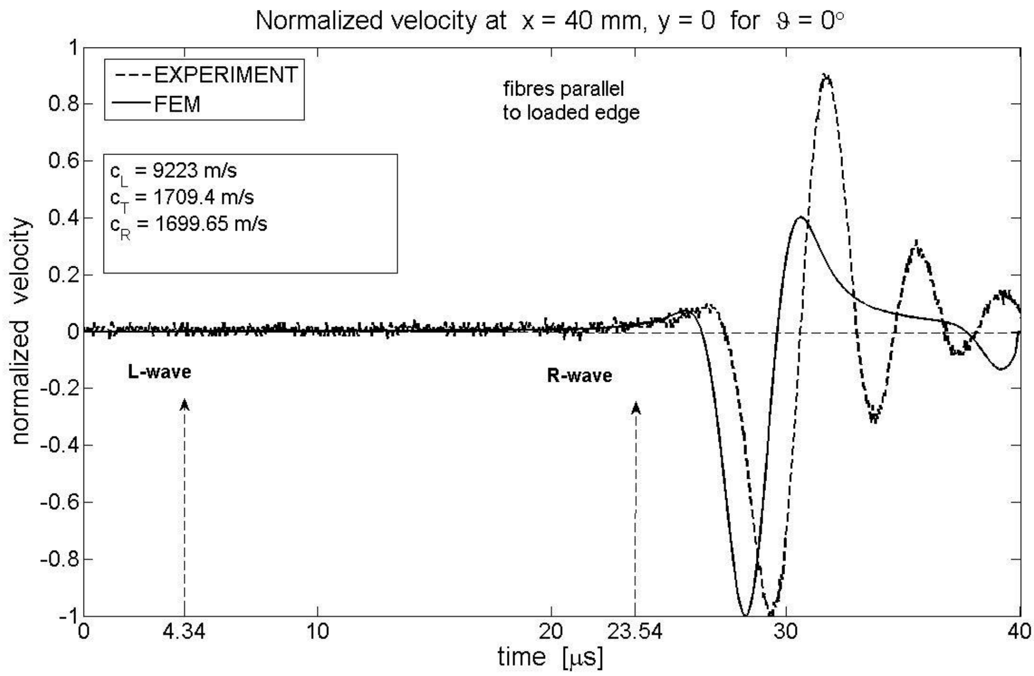

Figure 7: Normalized velocities versus time – theory and experiment

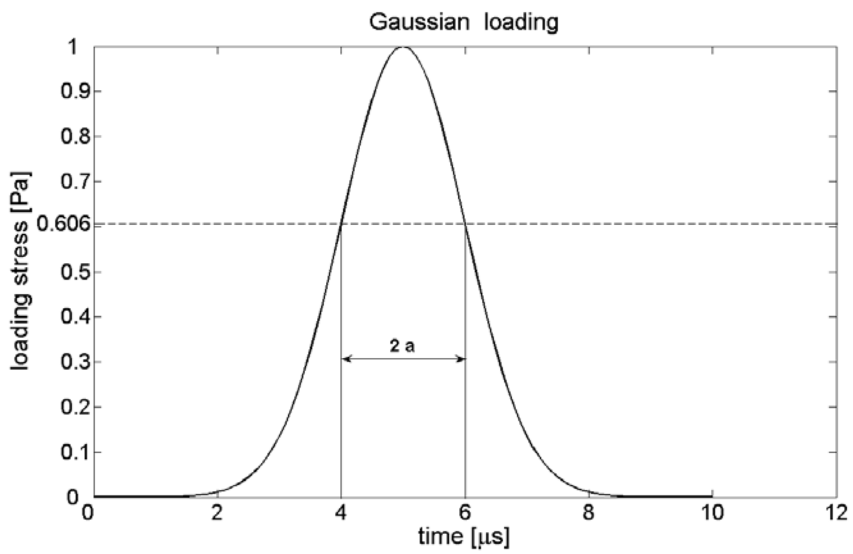

Figure 8: Time dependence of loading in finite element computation

Velocities in the Fig.7 are normalized to their maximal values in the considered time interval. The FEM loading was taken as edge load, spatially constant 1Pa for $x \in$ <-1mm, 1mm>, y=0. Time dependence of FEM loading is shown in Fig.8.

There is time shift about 1μs between theory and experiment in Fig.7 and also some discrepancies behind the R-wavefront. It can be caused by inaccurate time dependence of FEM loading and, probably, by incorrect material constants which have been given by the producer of the composite SE84LV.

6. CONCLUSIONS

The wave motion in fibre-reinforced thin composite is studied in the paper. The fibres are considered to be long compared to their diameters and the fibre spacing. Since the fibres are systematically orientated, a composite of this kind has strong directional properties, so macroscopically for sufficiently long wavelength it can be regarded as a homogeneous orthotropic material. The thin orthotropic panel made of laminate is then loaded in-plane by stress pulses. Theoretical solution is based on finite element approach. Experimental solution utilizes for noncontact measurements a laser vibrometer. It was found out that there are some discrepancies in the vicinity of R-wavefront between theory and experiment. It turns out that material constants given by the producer of the composite SE84LV are somewhat incorrect. Measurements performed during these days indicate e.g. difference between Young's moduli in the direction of fibres about 10%. Also the determination of correct mechanical loading of the panel (used in the experiment) is inevitable.

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