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INVESTIGATING THE EFFECTS OF THE STATIC CUTTING FORCE ON THE CHATTER VIBRATIONS OF MILLING PROCESS

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Abstract

A chatter phenomenon is heavy vibration between work piece and tool which causes both a reduced product quality and rapid tool wear. In this paper, a 2-DOF milling process is considered with the symmetric parameters in the feed and normal directions. Then stability lobes diagram (SLD) is derived by the method presented by Altitas [1] in order to find the specific combination of depth-of-cut and spindle-speed, which results in the maximum chatter-free material removal rate. Also the border between a stable cut (i.e. no chatter) and unstable cut (i.e. with chatter) is obtained from solving the time domain delay-differential equations (DDE) by means of the numerical method (4th order Runge-Kutta). The point of interest in this study is that the static part of the cutting force is not ignored and directional dynamic milling force coefficients are utilized in exact form. Finally, numerical results, experimental results from previous works and SLD have been compared. It is observed that the correlation between the numerical and experimental results is much better than the corresponding correlation between the SLD and experimental results.

1. INTRODUCTION

High-speed milling is one of the most preferred and efficient cutting processes nowadays. It is a challenging task for researchers to explore its special dynamical properties, including stability conditions of the cutting process and the nonlinear vibrations, which may occur locally, close to the stability boundaries, and the arising globally. A chatter phenomenon is heavy vibration between work piece and tool which is the most common difficulty in high-speed milling.

Altintas [1,5,8] obtained an analytical solution in frequency domain for 2-DOF milling operations that leads to stability lobes diagram. In the presented method by Altintas, the cutting forces excite the structure in the feed and normal directions, causing dynamic and static displacements. The dynamic displacement is carried to rotating tooth number in the chip thickness direction, whereas the static displacement is generated by work piece feed. He ignored “the static part of the cutting force" and approximated "the directional dynamic milling force coefficients". Also numerical and experimental investigations have confirmed that in cases of very small immersion milling, the true stability boundary differs significantly from the approximation one predicted by the Single Frequency Solution (SFS) method, which is widely used for prediction of chatter-free parameters in milling.

Insperger et al. [2,3,4], investigated the two degree of freedom model of milling process. The governing equation of motion is decomposed into two parts: an ordinary differential equation describing the stable periodic motion of the tool and a delay-differential equation describing chatter. Stability chart is derived by using semi-discretization method for the delay-differential equation corresponding to the chatter motion. The stable periodic motion of the tool and the associated surface location error are obtained by a conventional solution technique of ordinary differential equations. Fassen et al. [6], showed that both the material properties and the machine dynamics are dependent on the spindle speed by means of experimental results. Bayly et al. [7] implemented time finite element analysis to obtain a stability map of intermittent machining. This method was used form an approximation solution at each time increment in the cut.

In this paper, the point of interest is that the static part of the cutting force is not ignored and directional dynamic milling force coefficients are utilized in exact form. In the following sections, after presenting the mechanical model of milling process, the linear stability theory is presented. Before the conclusion section, the results of the numerical simulations have been investigated.

2. MECHANICAL MODEL

The 2-DOF mechanical model of end milling is shown in Figure 1. The tool is assumed to be flexible relative to the stiff work piece. The 2-DOF oscillator is excited by the cutting force $\mathbf{F}(t)$. The governing equation has the form

$$\mathbf{M}\ddot{\mathbf{x}}(t)+\mathbf{C}\dot{\mathbf{x}}(t)+\mathbf{K}\mathbf{x}(t)=\mathbf{F}(t) \tag{1}$$

where

$$\mathbf{x}(t)=\begin{Bmatrix} x(t) \\ y(t) \end{Bmatrix}, \quad \mathbf{F}(t)=\begin{Bmatrix} F_x(t) \\ F_y(t) \end{Bmatrix} \tag{2}$$

and $\mathbf{M},\mathbf{C}$ and $\mathbf{K}$ are the mass, damping and stiffness matrices, respectively. If the tool is modelled as a symmetric beam, its modal matrices are diagonal with the same diagonal values. The tangential and radial cutting forces acting on j^{th} tooth are proportional to the axial depth of cut (a) and chip thickness (h) [1]:

$$F_{tj}=g_j K_t a h(\varphi_j) \quad , \quad F_{rj}=K_r F_{tj} \tag{3}$$

Cutting coefficients K_t and K_r are constant and for a tool with N number of teeth the angular position of the j^{th} cutting edge is:

$$\varphi_j=(2\pi\Omega/60)t+(2\pi(j-1)/N) \tag{4}$$

where $\Omega[rpm]$ is the spindle speed. The function g_j is a unit step function that determines whether the tooth is in or out of cut; that is

$$g_j = \begin{cases} 1 & \text{if} \quad \varphi_{st} < \varphi_j < \varphi_{ex} \\ 0 & \text{if} \quad \varphi_j < \varphi_{st} \; or \; \varphi_j > \varphi_{ex} \end{cases} \tag{5}$$

The parameters φ_{st} and φ_{ex} are the angles where the j^{th} tooth enters and exists the cut, respectively. The chip thickness consists of a static part $\left(f_z \sin\varphi_j(t) \right)$ and a dynamic part $\left((x(t)-x(t-\tau))\sin\varphi_j(t) + (y(t)-y(t-\tau))\cos\varphi_j(t) \right)$ that can be expressed as:

$$h_j(t) = \Delta x \sin\varphi_j(t) + \Delta y \cos\varphi_j(t) + f_z \sin\varphi_j(t) \tag{6}$$

where $f_z = \left(60/N\Omega\right) v_f$ and v_f is the feed speed and $\Delta x = x(t) - x(t-\tau), \Delta y = y(t) - y(t-\tau)$.

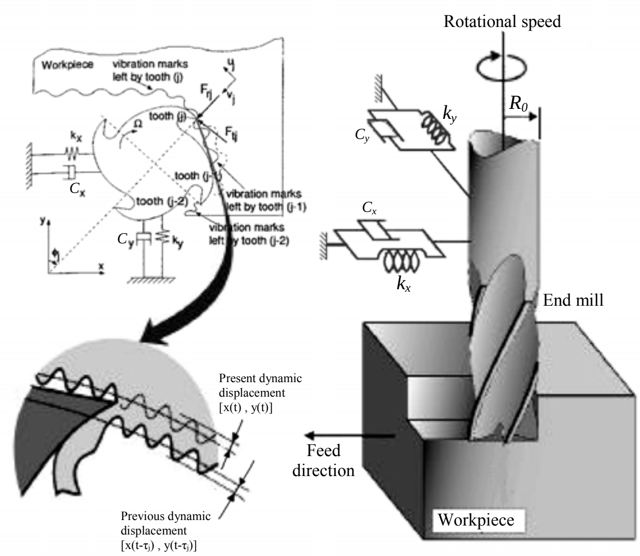

Figure 1. Schematic mechanical model of milling process.

The cutting forces contributed by all teeth in the *x* and *y* directions are:

$$\begin{Bmatrix} F_x \\ F_y \end{Bmatrix} = \sum_{j=0}^{N-1} \begin{bmatrix} -\cos\varphi_j & -\sin\varphi_j \\ \sin\varphi_j & -\cos\varphi_j \end{bmatrix} \begin{Bmatrix} F_{tj} \\ F_{rj} \end{Bmatrix} \tag{7}$$

Substituting the chip thickness (Eq.6) and the tooth forces (Eq.3) into (Eq.7) results:

$$\begin{Bmatrix} F_x \\ F_y \end{Bmatrix} = \frac{1}{2} a K_t \begin{bmatrix} \alpha_{xx} & \alpha_{xy} \\ \alpha_{yx} & \alpha_{yy} \end{bmatrix} \begin{Bmatrix} \Delta x \\ \Delta y \end{Bmatrix} + \begin{Bmatrix} G_x \\ G_y \end{Bmatrix} \tag{8}$$

where the time-varying “directional dynamic milling force coefficients (α)” and “static part of the cutting force (G)” are given by:

$$G_x = 0.5 a K_t f_z \alpha_{xx}, G_y = 0.5 a K_t f_z \alpha_{yx} \tag{9}$$

$$\begin{aligned}
\alpha_{xx} &= \sum_{j=0}^{N-1} -g_j\left[\sin 2\varphi_j + K_r\left(1-\cos 2\varphi_j\right)\right], \alpha_{xy} = \sum_{j=0}^{N-1} -g_j\left[\left(1+\cos 2\varphi_j\right) + K_r \sin 2\varphi_j\right] \\
\alpha_{yx} &= \sum_{j=0}^{N-1} g_j\left[\left(1-\cos 2\varphi_j\right) - K_r \sin 2\varphi_j\right], \alpha_{yy} = \sum_{j=0}^{N-1} g_j\left[\sin 2\varphi_j - K_r\left(1+\cos 2\varphi_j\right)\right]
\end{aligned} \tag{10}$$

If the static part of the chip thickness (Eq.6) is not considered, the last term of the cutting force (Eq. 8) will be omitted from this equation.

3. LINEAR STABILITY THEORY

In the method presented by Altintas [1], the first step to obtain the stability lobes diagram is to identify experimentally the transfer function matrix which relates the forces and displacements at the cutter-work piece contact zone:

$$\left[\phi(i\omega)\right] = \begin{bmatrix} \phi_{xx}(i\omega) & \phi_{xy}(i\omega) \\ \phi_{yx}(i\omega) & \phi_{yy}(i\omega) \end{bmatrix} \tag{11}$$

where $\phi_{xx}(i\omega)$ and $\phi_{yy}(i\omega)$ are the direct transfer functions in the x and y directions, and $\phi_{xy}(i\omega)$ and $\phi_{yx}(i\omega)$ are the cross transfer functions. The second step is to calculate the dynamic cutting coefficients from Eq.(10) for a specified cutter, work piece material and radial immersion of the cut. Then, a chatter frequency (ω_c) is selected from transfer function around a dominant mode and the characteristic quadratic equation must be solved [1]:

$$a_0\Lambda^2 + a_1\Lambda + 1 = 0 \tag{12}$$

where $a_0 = \phi_{xx}(i\omega_c)\phi_{yy}(i\omega_c)\left(\alpha_{xx}\alpha_{yy} - \alpha_{xy}\alpha_{yx}\right)$ and $a_1 = \alpha_{xx}\phi_{xx}(i\omega_c) + \alpha_{yy}\phi_{yy}(i\omega_c)$. The critical depth of cut is evaluated from the real and imaginary part of the eigenvalue $\Lambda = \Lambda_R + i\Lambda_I$

$$a_{\lim} = -\frac{2\pi\Lambda_R}{NK_t}\left(1+\left(\frac{\Lambda_I}{\Lambda_R}\right)^2\right) \tag{13}$$

Finally, the spindle speed $n = \frac{60}{NT}\,(rpm)$ can be calculated for each stability lobe. The parameter T is the tooth passing period. In the method presented by Altintas [1,8], "the static part of chip thickness" has been ignored and the estimated values of the directional dynamic milling force coefficients:

$$\begin{aligned}
\alpha_{xx} &= \frac{1}{2}\left[\cos 2\varphi - 2K_r\varphi + K_r \sin 2\varphi\right]_{\varphi_{st}}^{\varphi_{ex}}, \alpha_{xy} = \frac{1}{2}\left[-\sin 2\varphi - 2\varphi + K_r \cos 2\varphi\right]_{\varphi_{st}}^{\varphi_{ex}} \\
\alpha_{yx} &= \frac{1}{2}\left[-\sin 2\varphi + 2\varphi + K_r \cos 2\varphi\right]_{\varphi_{st}}^{\varphi_{ex}}, \alpha_{yy} = \frac{1}{2}\left[-\cos 2\varphi - 2K_r\varphi - K_r \sin 2\varphi\right]_{\varphi_{st}}^{\varphi_{ex}}
\end{aligned} \tag{14}$$

have been used instead of Eq. (10).

4. SIMULATIONS AND RESULTS

In this section, the time domain delay differential equations of the system have been solved by means of the 4th order Runge-Kutta numerical method. The main point of interest in this analysis is that the static part of the cutting force has not been ignored. In the other words, in contrast to the method presented by Altintas [1], the last term of the cutting force (Eq. 8) has been considered. In addition, the directional dynamic milling force coefficients have been utilized in exact form (Eq. 10). In the following, two different case studies have been considered in order to verifying the simulation results and investigating the effects of the static part in chatter phenomenon.

4.1 Case study 1

Altintas et al. [8] have been obtained empirical transfer functions for half immersion down milling of work material AL356 and Jack carbide cutter with 0.75[inch] diameter and four flutes as follows:

$$\phi_{xx} = \frac{0.09222 + 0.00001s}{s^2 + 159.4s + 0.78\times10^7} + \frac{0.66219 - 0.00001s}{s^2 + 395.2s + 0.1254\times10^8} + \frac{0.07246 - 0.00001s}{s^2 + 577.2s + 0.2393\times10^8},$$
$$\phi_{xy} = \phi_{yx} = 0, \phi_{yy} = \frac{0.83150 - 0.00002s}{s^2 + 162.2s + 0.1052\times10^8}$$

It can be seen that there are 3 modes in the " x " direction. Regarding to the 2 DOF mechanical model of the present work, only the first mode in the "x" direction has been considered. According to $\phi = (1/m)/\left(s^2 + 2\zeta\omega_n s + \omega_n^2\right)$, the required parameters for numerical simulations are

$$m_x = 11.125^{Kg},\ c_x = 1773.28^{N\cdot s/m},\ k_x = 85662500^{N/m}$$
$$m_y = 1.199^{Kg},\ c_y = 194.48^{N\cdot s/m},\ k_y = 12613480^{N/m}$$
$$N = 4,\ K_t = 697\times10^6[N/m^2],\ K_r = 0.367,\ f_z = 0.16[mm],\ \varphi_{st} = 90^{\deg},\ \varphi_{ex} = 180^{\deg}$$

Figure (2) indicates SLD obtained by the method presented in section 3. In this figure, comparison has been done between the two cases in which transfer function in the "x" direction contains three modes and one mode, respectively. It should be noted that the results obtained for the three modes case, are exactly the same as presented by Altintas et al. in [8]. It can be seen that, as expected, the effects of second and third modes are considerable especially for high spindle speeds.

By solving the time domain delay differential equations of the system, using the 4th order Runge-Kutta numerical method, the simulation results have been compared with the SLD. The obtained results have been indicated in Figures 3 and 4. In these figures, the effect of static part has been investigated. In addition, in the numerical method, in contrast to the method resulting SLD, all directional dynamic milling force coefficients have been used in their exact form.

Figure 3 shows that the SLD is very compatible with the numerical results, when the static part of the cutting force has been ignored. By considering the static part of the cutting force, the results of the numerical results differ with the SLD considerably. It can be seen from Fig. 4 that the static part of the cutting force increases the stability region of the milling process.

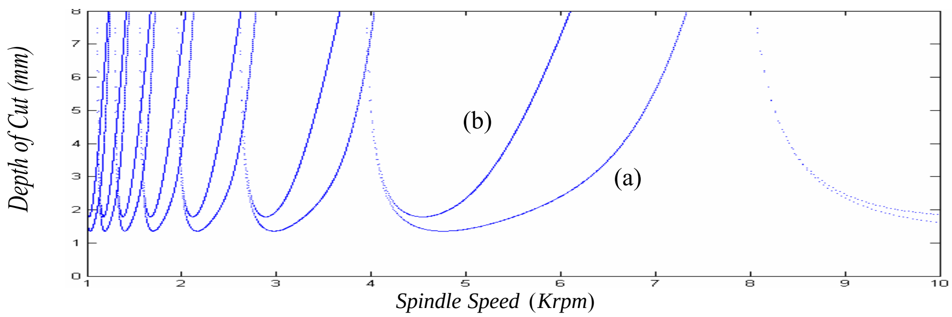

Figure 2. Stability lobes for the case study 1 with considering (a) three modes [8] and (b) first mode in transfer function in the “x” direction.

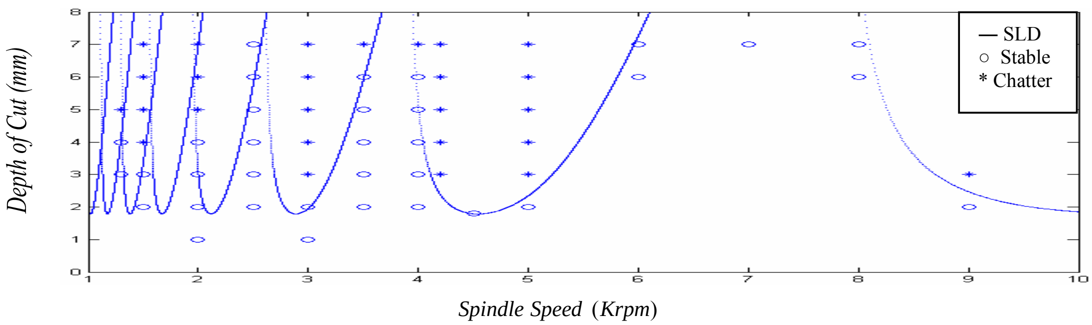

Figure 3. Stability lobes and numerical results for the case study 1-Without considering the static part of the cutting force in numerical simulations.

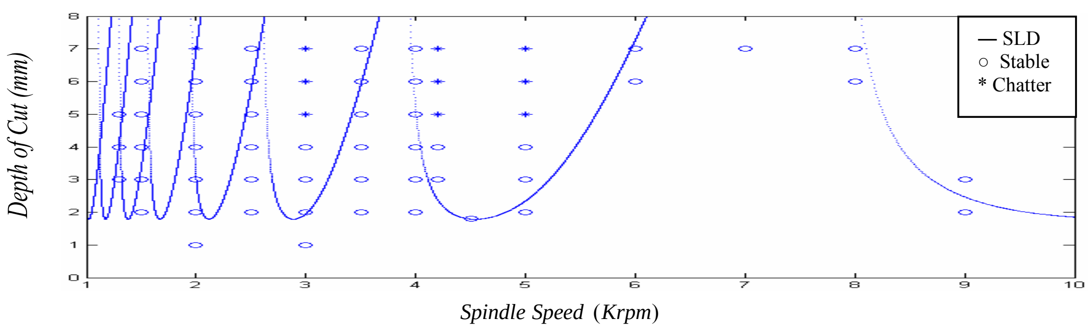

Figure 4. Stability lobes and numerical results for the case study 1-With considering the static part of cutting force in numerical simulations.

4.2 Case study 2

Insperger et al. [2] investigated milling process with the following conditions. These conditions have been considered in this case study.

$$N = 1,\ K_t = 644\times10^6[N/m^2],\ K_r = 0.368,\ f_z = 0.16[mm]$$

$$M = \begin{bmatrix} 0.0199 & 0 \\ 0 & 0.0201 \end{bmatrix}[Kg],\ C = \begin{bmatrix} 1.603 & 0 \\ 0 & 1.557 \end{bmatrix}[Ns/m],\ K = \begin{bmatrix} 409000 & 0 \\ 0 & 413000 \end{bmatrix}[N/m]$$

In Fig.5, based on the numerical solution, phasory diagrams for two different conditions have been shown. Constant amplitude of the diagram indicated in Fig.5a implies that the motion of tool is stable (i.e. no chatter), where as the rising amplitude of diagram indicated in Fig. 5b shows that the motion of tool is unstable (i.e. with chatter).

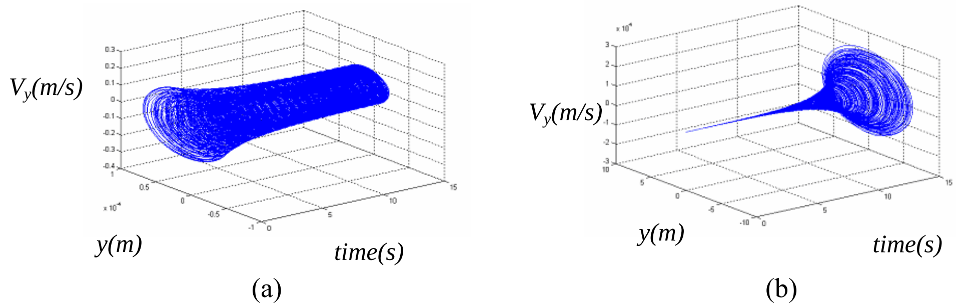

Figure 5. Phasory diagram- Case study 2- (a) no chatter (b) with chatter

Insperger et al. [2] compared SLD with experimental results, which are shown in Fig. 6. Figure (7) represents SLD together with simulation results, in which the static part of the cutting force has been neglected. In this case study, the same as the previous one, the numerical results are very close to SLD. Figures 6 and 7 show that the stability region of the experimental results are greater than the corresponding region of the SLD and numerical simulations.

Figure 8 represents the simulation results for the case in which the static part of the cutting force has been considered. The same as before, the static part increases the stability region. Comparison of Figs. 6-8 clears that not only the static part affects the chatter phenomena, but also with considering this part in motion equations, simulation results goes closer to experimental results. It should be noted that simulation results of both Figs. 8-9 have been obtained based on the exact form of directional dynamic milling force coefficients. Therefore, this improvement in prediction of stability region is only due to the static part of the cutting force.

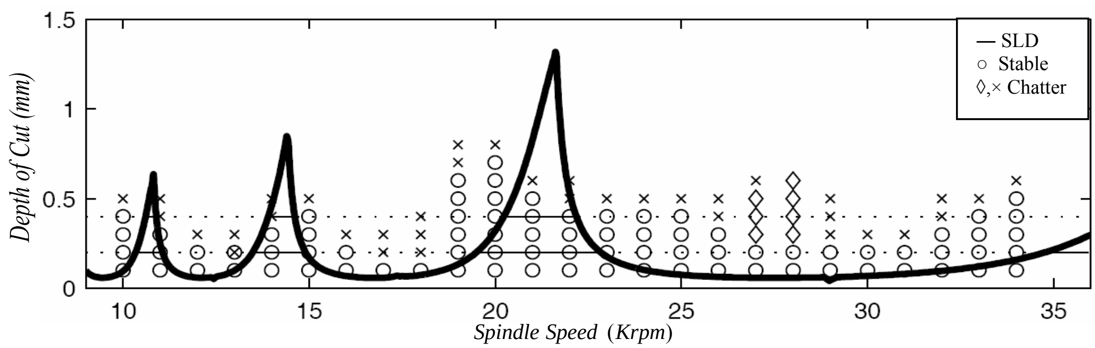

Figure 6. Stability lobes and experimental results for half immersion down milling [2].

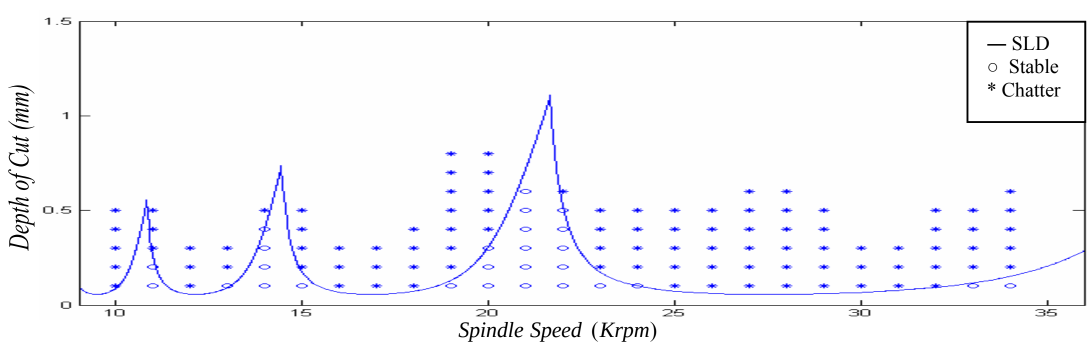

Figure 7. Stability lobes and numerical results for the case study 2-Without considering the static part of the cutting force in numerical simulations.

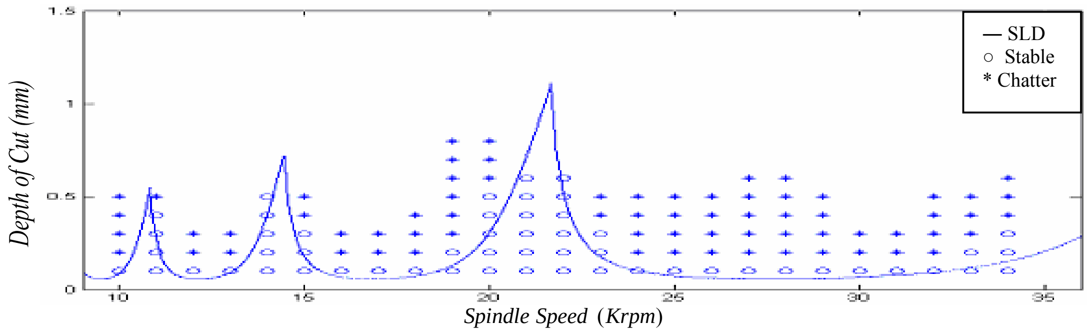

Figure 8. Stability lobes and numerical results for the case study 2-With considering the static part of the cutting force in numerical simulations.

5. CONCLUSIONS

A 2-DOF mechanical model has been used to simulate the chatter phenomenon in milling process. For two different case studies the stability lobes diagrams (SLD) are derived by the method presented by Altitas [1] in order to find the specific combination of depth-of-cut and spindle-speed, which results in the maximum chatter-free material removal rate. Also the border between a stable cut (i.e. no chatter) and unstable cut (i.e. with chatter) has been obtained from solving the time domain delay-differential equations (DDE). In numerical simulations, the static part of the cutting force has been considered and directional dynamic milling force coefficients are utilized in exact form. The obtained results clears that not only the static part of the cutting force affects the chatter phenomena and increases the stability region, but also with considering this part in motion equations, simulation results goes closer to experimental results.

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