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DESIGN SENSITIVITY ANALYSIS OF ZWICKER'S LOUDNESS USING THE ADJOINT VARIABLE METHOD

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Abstract

A design sensitivity formulation of Zwicker's loudness is presented using the adjoint variable method (AVM). The dynamic behaviour of a structure is analyzed by the frequency-response method. Acoustic pressure radiated from the vibrating structure is obtained by the Boundary Element Method (BEM). The global acoustic sensitivity is calculated using AVM, and the sensitivity of main specific loudness with respect to the design variables is obtained using the chain rule. The proposed sensitivity result is compared to finite difference sensitivity. It turned out that the computational time for calculating sensitivity is extremely reduced, and the sensitivity result is similar to the result of the finite difference method.

1. INTRODUCTION

For the purpose of just noise reduction, A-weighted sound pressure level is an adequate parameter for designing products. However considering human's subjective feelings on the sound, the A-weighted level is not sufficient to present the relation of physical sound stimulus with the human perceptual judgement. Sound quality is based upon the idea that lower sound levels are not always better, and subjective judgement on the sound of the products is more important than the sound pressure level. Zwicker and Fastl defined sound quality metrics, such as loudness, sharpness, roughness, fluctuation strength, etc. [1]. The Loudness is a measure for how loud or how soft a sound is heard relative to a standard sound. It is the most important metric among the sound quality parameters. By using the loudness, engineers can improve the sound quality and reduce the sound pressure level. The loudness models of Stevens and Zwicker are adopted as international standard [2].

In order to develop and improve the products, optimization process should be carried out in systematic development. But there isn't sufficient research on reducing the loudness using design sensitivity analysis (DSA). In a gradient-based optimization, it is important to calculate the sensitivity of object function and constraints with respect to the design variables [3]. As increase of model size, the number of the design variable is also increased, which is proportional to time consumption for calculating sensitivity in the optimization.

Some researches about DSA of a structural-acoustic problem have been reported. Wang [4] proposed continuum DSA of a coupled vibro-acoustic system using finite element method

(FEM). Wang and Lee [5] and Kim and Choi [6] calculated global acoustic design sensitivity using structural sensitivity in the FEM and acoustic sensitivity in the BEM for semi-coupled problems. Wang and Kang [7] calculated the sensitivity of the main specific loudness using semi-analytical method. The derivative of the main specific loudness is formulated using the loudness proposed by Zwicker et al.[8]. In this paper, the DSA of Zwicker's loudness is derived by employing the AVM in the vibro-acoustic system.

EVALUATION OF ZWICKER'S LOUDNESS

The evaluation model of the Zwicker's loudness makes a start with the concept of the specific loudness. The specific loudness comes from Stevens' law that a sensation belonging to the category of intensity sensation grows with physical intensity according to a power law. The formula of the main specific loudness is in ISO 532 B [2]:

$$NM = \left(0.0635 \cdot 10^{0.025 L_{TQ}}\right) \cdot \left[\left[1 + 0.25 \cdot 10^{0.1(L_E - L_{TQ})}\right]^{0.25} - 1\right] (sone_G / Bark) \tag{1}$$

NM is the main specific loudness defined in 1/3-oct band, L_{TQ} is excitation level at threshold in quite and L_E is excitation level. To calculate excitation level, some corrections are added. For 1/3-oct band filters, low frequency range is added and correction factor is used at all bands. And the logarithmic transmission factor to represent the transmission between free field and our hearing system is incorporated.

$$L_E = P_{band} - a_0 - c_1 \tag{2}$$

where

$$P_{band} = 20\, log \left[\left(\int_{\Delta\omega} p^2 d\omega / \Delta\omega \right)^{0.5} / p_{ref} \right] \; dB \tag{3}$$

where $\Delta\omega$ is frequency bandwidth, p_{ref} is reference sound pressure, 20e-6 pa.

GLOBAL ACOUSTIC DESIGN SENSIVITY ANALYSIS

The purpose of DSA is to compute the dependency of performance measures on the design variable. Assume that $\psi(\mathbf{u})$ is continuously differentiable with respect to design $\mathbf{u}$. The perturbation of the design is $\delta\mathbf{u}$ (arbitrary), and τ is a parameter that controls the perturbation size, then the variation of $\psi(\mathbf{u})$ in the direction of $\delta\mathbf{u}$ is defined as [9]

$$\psi'_{\delta\mathbf{u}} \equiv \frac{d}{d\tau} \psi(\mathbf{u} + \tau\delta\mathbf{u}) \Big|_{\tau=0} = \frac{\partial \psi^T}{\partial \mathbf{u}} \delta\mathbf{u} \tag{4}$$

If the variation of a function is continuous and linear with respect to $\delta\mathbf{u}$, the function is differentiable.

$$\mathbf{v}' = \frac{d}{d\tau}\psi[v(x,\mathbf{u}+\tau\delta\mathbf{u})]\bigg|_{\tau=0} = \frac{\partial \mathbf{v}^T}{\partial \mathbf{u}}\delta\mathbf{u} \tag{5}$$

and

$$p' = \frac{d}{d\tau}\left[p\left(x,\mathrm{u}+\tau\delta\mathrm{u}\right)\right]\bigg|_{\tau=0} = \frac{\partial p^T}{\partial \mathrm{u}}\delta\mathrm{u} \tag{6}$$

where v' is structural sensitivity and p' is acoustic sensitivity. In the vibro-acoustic system, the structural velocity is the boundary information of acoustic boundary element model. So the structural behaviour must first be computed, and then structural analysis results can be used as boundary conditions to compute radiated sound pressure p. If x_0 is a point on the acoustic boundary surface then the boundary integral equation can be expressed as follows from the Neumann boundary condition $\partial \mathrm{p}/\partial \mathbf{n} = -\mathrm{j}\omega\rho\mathrm{v}_n$ [5]

$$c(x_0)p(x_0) = \int_{\Omega^S}\left[p(x)\cdot\frac{\partial G}{\partial n} + j\omega\rho\mathrm{v}_n\cdot G\right]d\Omega \tag{7}$$

where ρ is the fluid density, $c(x_0)$ is the coefficient with respect to field point, and v_n is the normal velocity. For derivational convenience, Eq. (7) can be written as

$$b(\mathbf{x}_0;\mathbf{v}) + e(\mathbf{x}_0;\mathbf{p}_s) = c(\mathbf{x}_0)p(\mathbf{x}_0) \tag{8}$$

where $b\left(\mathrm{x}_0;\bullet\right)$ and $e\left(\mathrm{x}_0;\bullet\right)$ are linear integral forms that correspond to the right-hand side of Eq. (7) and . If the field point x_0 is positioned at the boundary, then the Eq. (8) can be represented as following linear equation:

$$\left[\mathrm{A}\right]\left\{\mathrm{P}_s\right\} = \left[\mathrm{B}\right]\left\{\mathrm{v}\right\} \tag{9}$$

where $\left[\mathrm{A}\right]$ and $\left[\mathrm{B}\right]$ are the coefficient matrix composed by complex numbers. Once $\left\{\mathrm{p}_s\right\}$ has been computed, Eq. (8) can be used to compute the acoustic sensitivity at any field point in the acoustic domain using Eq. (6) and (9).

$$\begin{aligned} p'\left(\mathrm{x}_0\right) &= \left\{\mathrm{b}\left(\mathrm{x}_0\right)\right\}^\dagger\left\{\mathrm{v}\right\}' + \left\{\mathrm{e}\left(\mathrm{x}_0\right)\right\}^\dagger\left\{\mathrm{p}_s\right\}' \\ &= \left[\left\{\mathrm{b}\left(\mathrm{x}_0\right)\right\}^\dagger + \left\{\mathrm{e}\left(\mathrm{x}_0\right)\right\}^\dagger\left[\mathrm{A}\right]^{-1}\left[\mathrm{B}\right]\right]\left\{\mathrm{v}\right\}' \end{aligned} \tag{10}$$

Let's consider the acoustic performance at the field point x_0

$$\psi = p\left(\mathrm{x}_0\right) \tag{11}$$

The variation of the performance measure w.r.t. the design variable becomes

$$\begin{aligned} \frac{d\psi}{d\mathrm{u}}^\dagger\delta\mathrm{u} &= \frac{\partial\psi}{\partial\mathrm{u}}^\dagger\delta\mathrm{u} + \frac{\partial\psi}{\partial p}\frac{\partial p}{\partial\mathrm{v}}^\dagger\frac{d\mathrm{v}}{d\mathrm{u}}^\dagger\delta\mathrm{u} \\ &= \psi'_{\delta\mathrm{u}} + \psi'_p\frac{\partial p}{\partial\mathrm{v}}^\dagger\mathrm{v}' = \psi'_{\delta\mathrm{u}} + \psi'_p p' \end{aligned} \tag{12}$$

To solve the Eq. (12) using adjoint variable, structural harmonic equation should be considered. The variational equation for the harmonic response problem can be expressed as

$$j\omega d_{\mathrm{u}}\left(\mathrm{v},\bar{\mathrm{z}}\right)+\kappa a_{\mathrm{u}}\left(\mathrm{v},\bar{\mathrm{z}}\right)=\ell_{\mathrm{u}}\left(\bar{\mathrm{z}}\right), \qquad \forall\bar{\mathrm{z}}\in Z \tag{13}$$

where $d_{\mathrm{u}}\left(\bullet,\bullet\right)$ is kinetic sesqui-linear form, $a_{\mathrm{u}}\left(\bullet,\bullet\right)$ is structural sesqui-linear form, $\ell_{\mathrm{u}}\left(\bullet\right)$ is semi-load linear form, and $\kappa=1+\varphi$ with structural damping coefficient φ [6].
After the structure is approximated using finite elements, the following system of matrix equation is obtained.

$$\left[j\omega\mathrm{M}+\kappa K\right]\left\{\mathrm{v}\left(\omega\right)\right\}=\left\{\mathrm{f}\left(\omega\right)\right\} \tag{14}$$

By taking variation of both sides of Eq. (14) w.r.t. the design variable, and by moving explicit terms to the right side, the structural sensitivity equation can be obtained:

$$\mathrm{v}'=\left[j\omega\left[\mathrm{M}\right]+\kappa\left[\mathrm{K}\right]\right]^{-1}\frac{\partial}{\partial\mathrm{u}}\left[\mathrm{f}-j\omega\left[\mathrm{M}\right]\left\{\tilde{\mathrm{v}}\right\}+\kappa\left[\mathrm{K}\right]\left\{\tilde{\mathrm{v}}\right\}\right]\delta\mathrm{u} \tag{15}$$

where superposed tilde $(\sim)$ indicates a variable held constant during the partial differentiation. After substituting the Eq. (15) for v' of Eq. (12), the global acoustic sensitivity equation can be obtained:

$$\begin{aligned}\frac{d\psi}{d\mathrm{u}}^{\dagger}\delta\mathrm{u}&=\frac{\partial\psi}{\partial\mathrm{u}}^{\dagger}\delta\mathrm{u}+\frac{\partial\psi}{\partial p}\frac{\partial p}{\partial\mathrm{v}}^{\dagger}\frac{d\mathrm{v}}{d\mathrm{u}}^{\dagger}\delta\mathrm{u}\\&=\frac{\partial\psi}{\partial\mathrm{u}}^{\dagger}\delta\mathrm{u}+\frac{\partial\psi}{\partial p}\frac{\partial p}{\partial\mathrm{v}}^{\dagger}\left[j\omega\left[\mathrm{M}\right]+\kappa\left[\mathrm{K}\right]\right]^{-1}\frac{\partial}{\partial\mathrm{u}}\left[\mathrm{f}-j\omega\left[\mathrm{M}\right]\left\{\tilde{\mathrm{v}}\right\}-\kappa\left[\mathrm{K}\right]\left\{\tilde{\mathrm{v}}\right\}\right]\delta\mathrm{u}\end{aligned} \tag{16}$$

From Eq. (16), the constant terms about design derivative can be substituted by adjoint variable λ as

$$\left\{\lambda\right\}^{\dagger}=\frac{\partial\psi}{\partial\mathrm{p}}\frac{\partial\mathrm{p}}{\partial\mathrm{v}}^{\dagger}\left[j\omega\left[\mathrm{M}\right]+\kappa\left[\mathrm{K}\right]\right]^{-1} \tag{17}$$

At the Eq. (17), by moving the inverse term to the left side and multiplying the virtual adjoint displacement, structural adjoint equation can be obtained:

$$\left\{\lambda\right\}^{\dagger}\left[j\omega\left[\mathrm{M}\right]+\kappa\left[\mathrm{K}\right]\right]\left\{\bar{\lambda}\right\}=\frac{\partial\psi}{\partial p}\frac{\partial p}{\partial\mathrm{v}}^{\dagger}\left\{\bar{\lambda}\right\} \tag{18}$$

In the Eq. (18), adjoint variable λ is the displacement at the original model when adjoint load $\frac{\partial\psi}{\partial p}\frac{\partial p}{\partial\mathrm{v}}^{\dagger}$ is applied on the structure. Adjoint load can be calculated by the acoustic adjoint equation using Eq. (10). Then we obtain the structural adjoint problem as

$$\left[j\omega[\mathrm{M}]+\kappa[\mathrm{K}]\right]\{\lambda\}^{*}=\{\mathrm{b}\}+[\mathrm{B}]^{\dagger}\{\eta\} \tag{19}$$

where $\{\eta\}$ is the acoustic adjoint solution derived from $[\mathrm{A}]^{\dagger}\{\eta\}=\{\mathrm{e}\}$.

DSA OF ZWICKER'S LOUDNESS

In many cases loudness pattern diagram clearly shows which partial area is dominant. It is efficient to reduce the dominant part of the noise that produces the largest area in the loudness pattern. So reducing the main specific loudness contributes largely reducing total loudness. This procedure is efficient especially because of making effect. In Eq. (1), the main specific loudness has not structural design variables. To calculate sensitivity with respect to structural design variable, chain rule is used as below.

$$\frac{\partial NM}{\partial \mathrm{u}}=\frac{\partial NM}{\partial L_E}\cdot\frac{\partial L_E}{\partial \mathrm{u}} \tag{20}$$

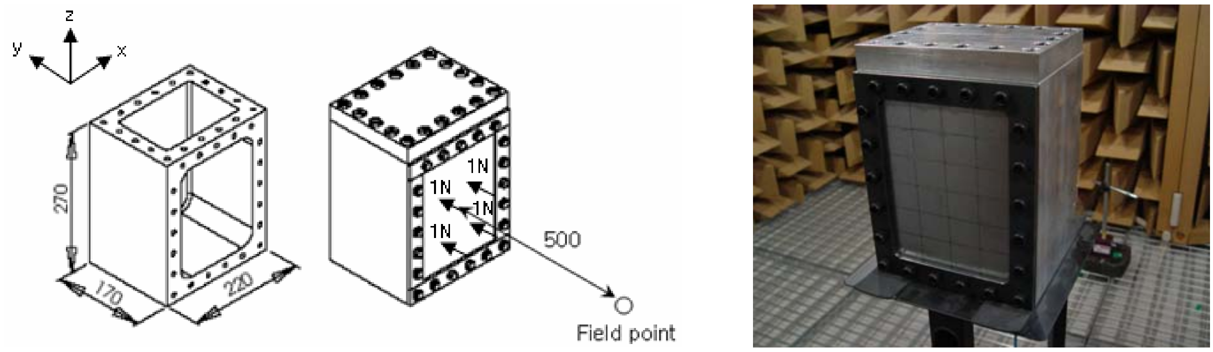
The first derivative can be derived directly from Eq. (1). Because L_{TQ} is constant in the specific octave band, $\partial NM/\partial L_E$ is constituted by the excitation level. And the second term is equal to $\partial P_{band}/\partial u$ because of a_0 and c_1 which are constant in Eq. (2). By chain rule, $\partial P_{band}/\partial u$ can be obtained as

$$\frac{\partial P_{band}}{\partial u}=\frac{10}{\ln 10}\cdot\frac{\left(\int_{\Delta\omega}p^2 d\omega/\Delta\omega\right)^{-1}}{\Delta\omega}\cdot\int_{\Delta\omega}p\cdot p' d\omega \tag{21}$$

where p' is global acoustic design sensitivity.

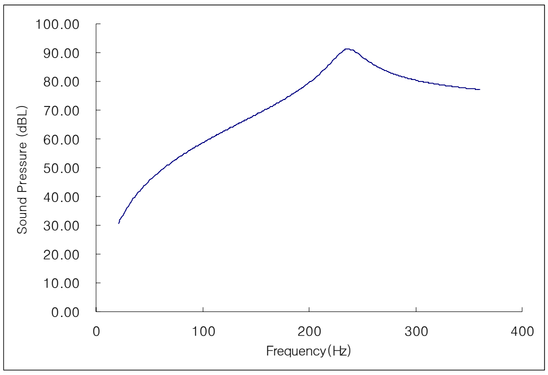
NUMERICAL EXAMPLE

Numerical example is a simple aluminium box model with 30mm thick walls, considered as rigid wall, and 1 flexible steel plate (1mm) [10]. Acoustic medium is air, and wave propagation velocity c is $344\mathrm{m/s}$. Structural damping coefficient φ is 0.11, and a harmonic force $f=1.0\mathrm{N}$ in the y direction is applied at four points on the flexible plate as shown in Figure 1(a). Figure 1(b) shows real model of the aluminium box. In this paper, only the sensitivity of analysis model was considered. The acoustic field point is 0.5m apart from the centre of the flexible plate. The structural and acoustic response of the model is calculated by using commercial tools, ANSYS and COMET. Figure 2 demonstrates the narrow band graph (a), 1/3 octave band graph (b), and Loudness pattern (c) w.r.t. same frequency responses. The highest value of the acoustic pressure of analysis in the Figure 2(a) is 91.17dB at 236Hz which is the resonance frequency of the box. This frequency is only used to evaluate the harmonic and acoustic sensitivity as target. And the 3rd critical band level is biggest one indicating 7.458 sone/bark. The specific loudness of the 3rd critical band is chosen as objective function. In the sensitivity analysis, the derivative of object function w.r.t. the design variable is equal to the sensitivity.

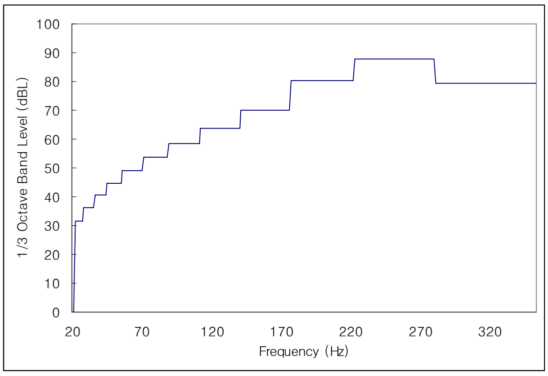


(a) Geometry and analysis model of box (b) Real model

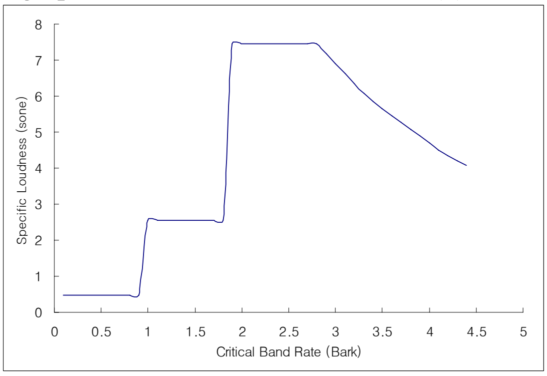
Figure1. Aluminum box model for sensitivity



(a) Narrow band graph



(b) 1/3-oct band graph



(c) Loudness pattern

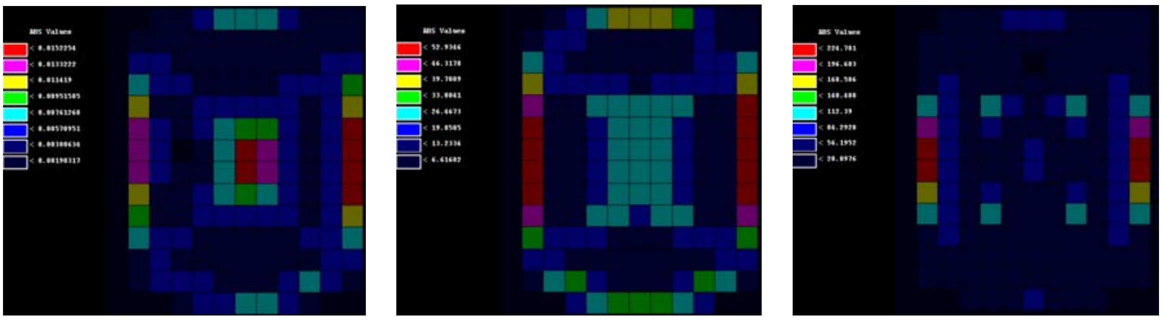
Figure2. Analysis results of the aluminum box

DESIGN SENSITIVITY ANALYSIS RESULTS

To confirm the loudness sensitivity, the structural and the acoustic sensitivity at the resonance frequency of the box should be validated. All processes for calculating the design sensitivity are operated by In-house code, ANSENS. Figure 3(a), (b), and (c) represents the plot of the harmonic sensitivity, the acoustic sensitivity, and the loudness sensitivity result respectively. Central finite difference method (FDM) is used to validate the AVM results.

In the table 1, 2, and 3, $\Delta\psi / 2\delta u$ is obtained by the FDM, and ψ' is the predicted design sensitivity using the AVM. The design perturbation $\delta u = 1.0\times10^{-5}\,\mathrm{m}$ is corresponding to 1% perturbation. Harmonic sensitivity is same with the result by FDM as shown in Table 1. In the case of the acoustic sensitivity, the results of the FDM and the AVM are not precisely identical. But the distribution of the sensitivity has similar pattern in Table 2. By using the chain rule in

Eq. (20), the loudness sensitivity is compared with the results by FDM in Table 3. In spite of the some errors in the acoustic sensitivity, the loudness sensitivity of AVM and FDM are almost same. It may be caused by averaging process of errors in the calculation of P_{band} in Eq. (3). If the FDM is used for the calculation of sensitivity on the aluminium box model, 154 design sensitivity equations must be solved. However with the AVM, only one adjoint equation is needed to calculate the sensitivity of all design variables. Therefore the calculation time for sensitivity of this model can be reduced about 154 times compared to FDM.



(a) Harmonic sensitivity (b) Acoustic sensitivity (c) Loudness sensitivity

Figure3. Sensitivity plot at the resonance frequency and 3rd critical band

Table1. Harmonic sensitivity verification for aluminium box model

DV Number	Perturbation δu [%]	$\psi(d+\delta u)$ m	$\psi(d-\delta u)$ m	$\Delta\psi/2\delta u$ m/m	ψ' m/m	Accuracy $\psi'/\Delta\psi$ (%)
67	1	1.39924E-04	1.40168E-04	-1.21838E-02	-1.21807E-02	99.97455
77		1.39894E-04	1.40198E-04	-1.52283E-02	-1.52254E-02	99.98080

Table2. Acoustic sensitivity verification for aluminium box model

DV Number	Perturbation δu [%]	$\psi(d+\delta u)$ dBL	$\psi(d-\delta u)$ dBL	$\Delta\psi/2\delta u$ dBL/m	ψ' dBL/m	Accuracy $\psi'/\Delta\psi$ (%)
67	1	7.17770E-01	7.19023E-01	-6.26500E+01	-5.26738E+01	84.07636
77		7.17767E-01	7.19025E-01	-6.29000E+01	-5.29346E+01	84.15675

Table3. Sensitivity verification for aluminium box model

DV Number	Perturbation δu [%]	$\psi(d+\delta u)$ sone/Bark	$\psi(d-\delta u)$ sone/Bark	$\Delta\psi/2\delta u$ sone/(Bark*m)	ψ' sone/(Bark*m)	Accuracy $\psi'/\Delta\psi$ (%)
67	10	7.43988E+00	7.39642E+00	2.17322E+02	2.24359E+02	103.23804
	1	7.42015E+00	7.41577E+00	2.18998E+02		102.44814
	0.1	7.41818E+00	7.41774E+00	2.19190E+02		102.35861
77	10	7.43991E+00	7.39637E+00	2.17694E+02	2.24781E+02	103.25536
	1	7.42015E+00	7.41577E+00	2.19387E+02		102.45873
	0.1	7.41818E+00	7.41774E+00	2.19254E+02		102.52054

CONCLUSIONS

In this paper, the design sensitivity analysis of the Zwicker's loudness is developed using the AVM. The loudness is more appropriate to express the human hearing than the sound pressure level. Structural FEM, acoustic BEM and ISO 532 B method are used to calculate the main specific loudness. Calculation of the loudness sensitivity also requires three stages composed of the structural sensitivity, acoustic sensitivity and the derivative of loudness formula. Structural and acoustic sensitivities are combined into global acoustic sensitivity.

The design sensitivity proposed in this paper is formulated especially considering sizing design variable, i.e. thickness. Using the AVM, adjoint variable is calculated from a structural FEM with adjoint load obtained from acoustic BEM. The sensitivity of the main specific loudness includes the derivative of the loudness with respect to excitation level and the derivative of excitation level with respect to the design variable. The derivative of excitation level is calculated with global acoustic sensitivities. ANSENS, In-house code developed in this research controls all procedures for DSA of loudness by calling ANSYS and COMET/ACOUSTICS. A simple aluminum box model is used for numerical example to verify loudness sensitivity formulation. The loudness sensitivities of AVM and FDM are almost same. These sensitivity results give engineers the guideline to reduce loudness efficiently. Also sensitivity information plays the gradient role of optimization. By the proposed method, the analysis time can be reduced very much.

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