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FREQUENCY DEPENDENT TORTUOSITY MEASUREMENT BY MEANS OF ULTRASONIC TESTS

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Abstract

The tortuosity is a fundamental parameter which gets involved in the description of the complexity of the path of the sound wave propagating within a porous material. Several methods have been developed to measure the tortuosity, based on electrical resistivity measurements, high frequency acoustic transmission and reflection measurement and ultrasonic tests.
In the present work it has been developed an equipment for determining tortuosity based on the determination of the high frequency limit for the ratio between the sound speed in air and the real part of complex phase velocity within the material.
The above mentioned ratio can be evaluated from the phase difference, in the frequency domain, of an ultrasonic pulses between two transducers, with and without material to be tested. The results is a frequency dependent tortuosity curve, that has to be handled in order to get the value of the parameter when frequency tends to infinity. In this work some considerations about these manipulations are reported and discussed.
Furthermore, when highly dissipative materials are tested, the signal may be corrupted because of the low S/N ratio. In this paper different receiving transducers are used and different stimuli are applied in order to improve the accuracy of the measurement.

1. INTRODUCTION

The determination of physical parameters of porous media has became of great significance for predicting the acoustical behaviour of these materials.

Among these quantities the tortuosity is an important parameter which intervenes in the description of the complexity of the path of the sound wave propagating within a material.

Several methods have been developed to measure tortuosity. In 1980 Brown [1] showed that it is possible to determine this parameter with an electrical resistivity measurement, saturating the material with a conducting fluid. Successively Allard *et al.* [2] proposed a method based on determination of the high frequency limit for the complex phase velocity. The transmitted ultrasonic signal from a porous layer is measured in frequency domain. The phase velocity is calculated from the phase difference of the signals obtained with and without sample. Recently, Attenborough *et al.* [3] and Fellah *et al.*[4] have proposed techniques based on high frequency acoustic transmission and reflection measurements and ultrasonic tests.

In the present work an equipment has been developed for determining tortuosity based on Allard's method. In the paper the experimental set-up will be shown and the optimization of the technique will be presented and discussed.

2. MEASUREMENT PRINCIPLE

The method is based on the high frequency limit of the complex phase velocity in the rigid frame approximation. At high frequencies the relation between the complex wave number k_c and the tortuosity α_∞ is:

$$k_c = \frac{\omega}{c_0}\sqrt{\alpha_\infty}\left[1+\frac{\sqrt{2\eta/\omega\rho_0}\,(1-j)}{2}\left(\frac{1}{\Lambda}+\frac{\kappa-1}{\sqrt{N_{pr}}\Lambda'}\right)\right] \quad (1)$$

where c_0 [m/s] is the sound speed in air, ρ_0 [m/s] is the density of the air, η [Ns/m^2] the viscosity of the air, ω [rad/sec] the angular frequency, κ the specific heat ratio, N_{pr} the Prandtl number and Λ and Λ' [m] the characteristic viscous and thermal lengths.

The sound velocity within the porous material is demonstrated to be:

$$c = \frac{c_0}{\sqrt{\alpha_\infty}}(1-\varphi) \quad (2)$$

with the loss angle defined as:

$$\varphi = \frac{\delta}{2}\left(\frac{1}{\Lambda}+\frac{\kappa-1}{\sqrt{N_{pr}}\Lambda'}\right) \quad (3)$$

From equation (2) it is possible to write the following expression for a "frequency dependent" tortuosity:

$$\alpha_\infty\big|_\omega = \left(\frac{c_0}{c}\right)^2(1-\varphi)^2 \quad (4)$$

When the angular frequency tends to infinity (or equivalently $\omega^{-1/2}$ tends to zero) then φ tends to zero. Consequently, from equation (4):

$$\alpha_\infty = \left\{\lim_{\omega^{-1/2}\to 0}\left(\frac{c_0}{c}\right)\right\}^2 \quad (5)$$

Equation (5) represents a straight line as a function of $\omega^{-1/2}$. Thus the limit in equation (5) can be calculated from the intercept of a linear regression on the experimental data.

In Figure 1 a sketch of the measurement configuration is shown. The ratio between (c_0/c), named "refraction index" n, can be evaluated from the increase of the time of flight, (i.e. a phase difference in the frequency domain) of an ultrasonic pulse between two transducers, with and without material to be tested. The expression for the phase velocity is demonstrated to be:

$$c = \left(\frac{1}{c_0} + \frac{\Delta\phi}{2\pi f \cdot d} \right)^{-1} \tag{6}$$

where $\Delta\phi$ is the above mentioned phase difference, f the frequency and d the thickness of the tested material. The phase difference is calculated with a Fast Fourier Transform analysis.

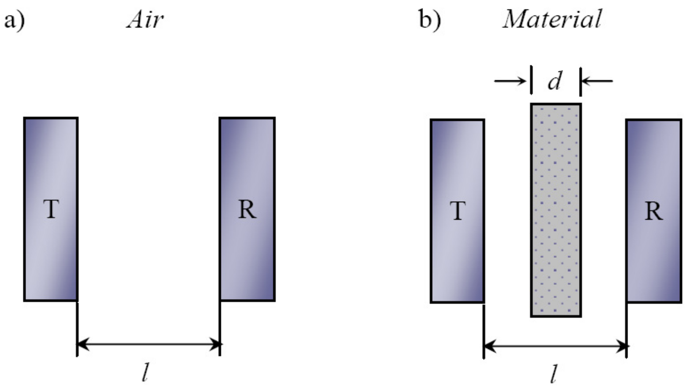


Figure 1 - Scheme of measurement configuration for measuring tortuosity

In Figure 2 an example of impulse response measured in both configurations is reported.

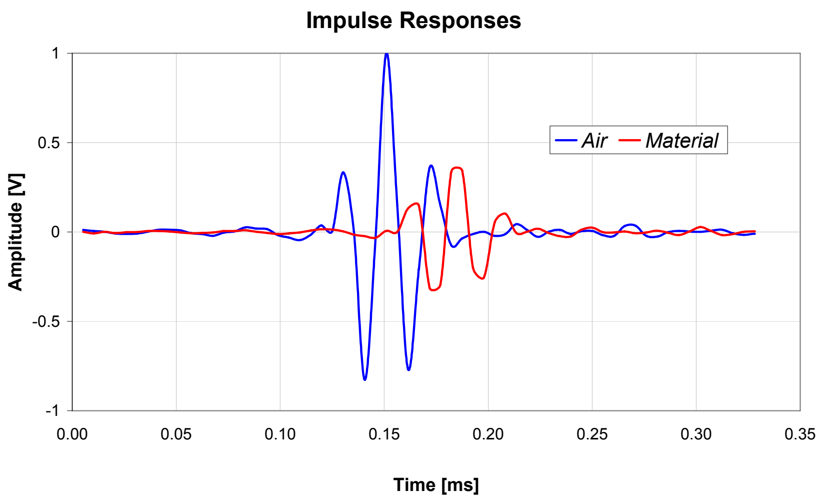


Figure 2 - Example of impulse response measured with and without material

In this paper the loss angle φ has been determined still by means of the linear regression of the quantity n as a function of $\omega^{-1/2}$. In fact, it can be proved that the slope of this straight line is proportional to the value φ.

Summarizing, by calculating the phase velocity from a measurement of the phase difference between signals measured with and without material, it is possible to determine the refraction index. The limit of this quantity as the frequency tends towards infinity allows calculation of the tortuosity value α_∞ with equation (5).

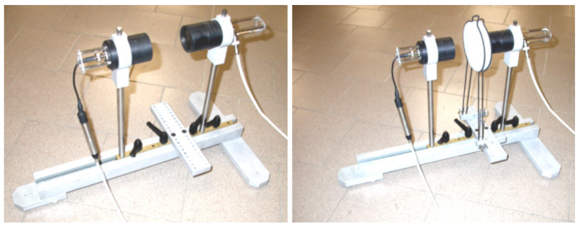
3. MATERIALS AND METHODS

The experimental set-up consists of:

- power amplifier Gras mod. 14AA;
- a broadband electrostatic ultrasonic transducer (S.Square Enterprise mod. 500ES430, 20-100KHz Bandwidth, 119/-42dB) used as source and receiver. The diameter of the transducer is 45 mm;
- signal conditioner MESA MUX10A;
- moveable system to change the position of the source and/or the receiver and the sample holder;
- personal computer equipped with sound card ESI Wami Rack 192X.

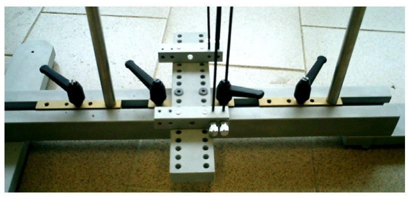
The pulses are generated by means of the exponential sine sweep technique [5], in the frequency range between 20 and 90 KHz. Adobe Audition® and Aurora® plug-in are used for the generation and acquisition of the signals, while the post-processing operations are implemented on a Labview® platform.

Figures 3 shows the set-up and some details.



Experimental set-up for measuring tortuosity

Electrostatic trans-receiver



Moveable system

Figure 3 – Experimental set-up and some details

4. EXPERIMENTAL RESULTS

4.1 Optimization of the technique

In order to discard multiple reflections between source and receiver and the effect of cross talk between input channels, the acquired pulses must be selected in time domain and a window must be applied to avoid discontinuities at the ends. For the tests presented in this work, 128 samples around the peak of the impulse responses have been selected and a 7 Term Blackman-Harris time window has been applied.

In addition the high frequency limit of the technique has been investigated, since it depends on the tortuosity and the internal losses in the material. Five different fibrous and porous materials were tested and the *Insertion Loss* (calculated through the amplitudes of the signals with and without material) and the refraction index were calculated in frequency domain. The materials are listed in Table 1 and the comparisons between materials for the above mentioned parameters are shown in figures 4 and 5.

Table 1 - Description of the materials

	Sample	*Thickness [m]*	*Density [Kg/m³]*	*Typology*
1	**Armafoam Sound**	0.024	64	Cellular
2	**Polyurethane Foam B**	0.02	25	Cellular
3	**Polyurethane Foam E**	0.02	25	Cellular
4	**Glass Wool**	0.025	20	Fibrous
5	**Nomex Felt**	0.014	90	Felt

The analysis shows a correlation between the losses within the material and the reliability in determining the tortuosity. It can be observed that when the *Insertion Loss* is higher than around 50-60 dB, then the behaviour of the refraction index does not follow the expected trend. For example, materials 4 and 5 are characterized by values of *IL* lower then 50 dB and the refraction index decreases with frequency. For material 2 the curve is reliable for frequencies lower than 80 KHz, where the value of the *Insertion Loss* is lower than the above mentioned limit. The same considerations can be applied to material 3 for frequencies lower than 60 KHz. The *Insertion Loss* for material 1 is higher than the limit over almost the entire frequency range and the curve of the refraction index is totally unreliable.

Figure 4 - Insertion Loss curves for the tested materials

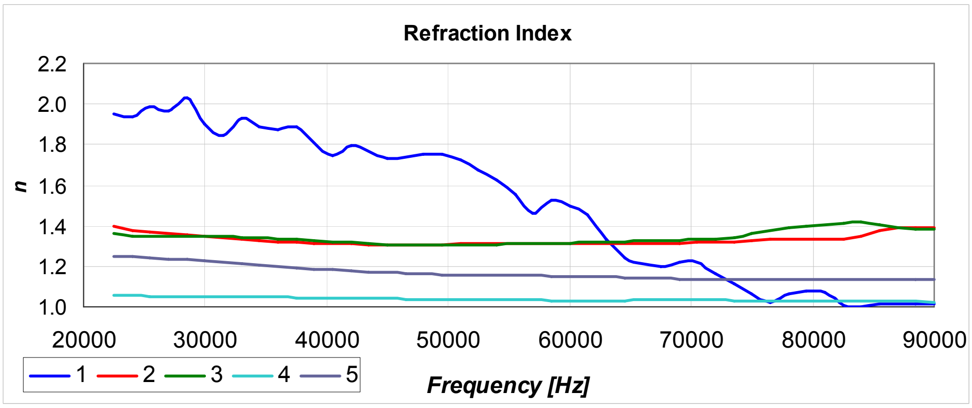


Figure 5 - Refraction index curves for the tested materials

4.2 Determination of tortuosity

In Paragraph 2 it has been underlined that the value of tortuosity is the limit of the refraction index as the frequency tends to infinity. At high frequencies a linear behaviour is obtained, the intercept of which leads to the tortuosity.

The need to select an opportune frequency range for interpolating data is obvious. The simple rule described in previous section was applied in order to choose a range with an adequate S/N ratio.

With the application of a linear best-fit, the uncertainty in tortuosity, which is related to the intercept of the regression line, can be determined by way of the error propagation on the linear regression.

Many authors calculate the tortuosity in time domain by determining the time of flight with:

$$c = \left(\frac{1}{c_0} + \frac{\Delta t}{d} \right)^{-1} \tag{7}$$

where Δt is the difference of the time of flight (i.e. the transit time through the material) in air and in the sample. Thus the tortuosity is given by:

$$\alpha_\infty \big|_{t.o.f} = \left(1 + \frac{c_0 \Delta t}{d} \right)^2 \tag{8}$$

Here this time distance is calculated by measuring the distance between the maxima of the signals. The signals are filtered (4th order Butterworth) within the range used for the calculation of tortuosity using regression. The error is calculated by applying the error propagation rules to the equation (8).

Thirteen porous materials have been tested and the tortuosity was determined using the linear interpolation in a reliable frequency range with equation (5). For almost all the materials the frequency range used for linear regression is between 30 and 70 KHz.

The results are reported in Figure 6 with uncertainties as described above. In the same figure the values of the tortuosity determined in the time domain (*t.o.f*) and the mean values with standard deviation of the frequency dependent tortuosity (*Avg*), calculated in the frequency range used for the interpolation with equation (4), are also depicted.

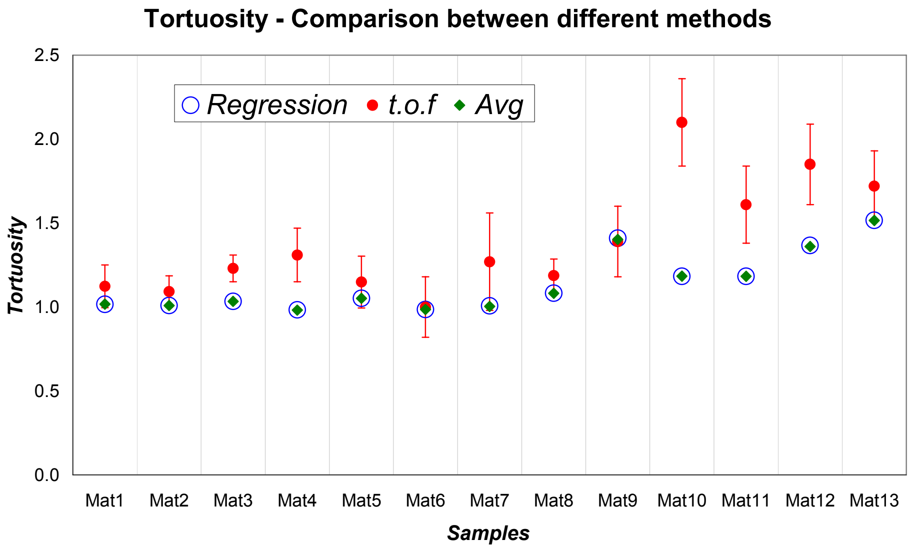


Figure 6 - Comparison between different methods

It is interesting to notice that for material with tortuosity close to 1 (i.e. melamine foam) the regression could lead to values of tortuosity lower than 1. The reason could be due to a not sufficient high frequency range for the test; higher frequencies could be needed to reach the asymptotic limit.

Moreover for all the tested material the mean value of the frequency dependent tortuosity is close to its value at infinity.

In all cases the values obtained with the time of flight are higher than the results determined using linear regression. This makes sense since the distance in time domain should correspond to the values at the frequency that is predominating in the impulse response. Consequently it would be more correct to compare this value to n^2 (i.e. the squared ratio between sound speeds in air and within the material), not dependent on the loss angle, in the frequency domain. In Figure 7 a comparison is shown between tortuosity obtained in the time domain, the refraction index and frequency dependent tortuosity for the Polyurethane Foam.

From the curve it is seen that the value of the tortuosity obtained with the time of flight, calculated directly from the impulse response, is close to the value of n^2 at 50-55 KHz, when the transducer has maximum efficiency. The frequency dependent tortuosity, depending on the losses due to the viscosity of the air filling the pores, is characterized by lower values. In agreement with theoretical predictions, as the frequency tends to infinity the values of both curves approach the same value (1.08).

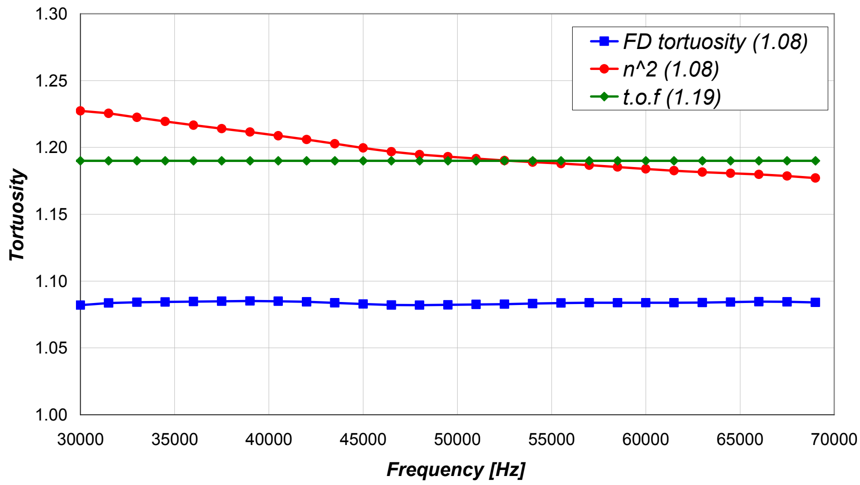


Figure 7 - Comparison between tortuosity obtained in the time domain (*t.o.f*) , the refraction index (*n^2*) and frequency dependent tortuosity (*FD*) for a Polyurethane Foam

4.3 REPEATABILITY AND ERRORS

In order to determine the repeatability of the technique, the tortuosity for samples of Melamine Foam and Polyurethane Foam sample was measured five times. The values of the tortuosity, with mean value, standard deviation and relative percentage error, are reported in Table 2. In the same table results calculated by using the time of flight approach are also shown.

Table 2 - Summary of the repeatability tests

Material		1	2	3	4	5	α_∞	σ	%
Melamine foam	*Regr*	0.99	0.98	0.97	0.96	0.98	**0.97**	0.01	1%
	t.o.f	1.39	1.19	1.00	1.00	1.39	**1.19**	0.20	16%
Polyurethane Foam	*Regr*	1.41	1.35	1.34	1.36	1.39	**1.37**	0.03	2%
	t.o.f	1.85	1.19	1.61	1.85	1.85	**1.67**	0.29	17%

From the table it is possible to notice that the relative percentage error in tortuosity, calculated as the high frequency limit of refraction index is lower than 2%, while the error in tortuosity determined in time domain is lower than 17%.

Of course the reason for the low accuracy of the method based on time of flight is mainly due to the precision in determining the maxima of the pulses; signals may be corrupted because of external noise and then the peaks could be shifted in time.

5. CONCLUSIONS

In this paper a device for measuring tortuosity was presented. The method is based on the determination of the high frequency limit for the complex phase velocity. The ultrasonic transmitted signal from a porous layer is measured in frequency domain. The phase velocity is calculated from the phase difference of the signals obtained with and without sample and the refraction index is determined. The limit of this quantity as the frequency tends to infinity is proportional to the tortuosity.

An optimized procedure in terms of hardware and signal processing was determined and a simple empirical rule was established for setting the high frequency limit of the technique. Moreover the repeatability of the method was investigated, finding values for the relative error in tortuosity lower than 2 %.

Some considerations regarding the values of the tortuosity in time and frequency domain were presented and discussed, underling differences due to the diverse definitions of this parameter.

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