ROBUST CONTROL DESIGN FOR ACTIVE NOISE CONTROL SYSTEMS OF DUCTS WITH A VENTILATION SYSTEM USING A PAIR OF LOUDSPEAKERS

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Abstract

In this paper we deal with design problems of active noise control systems with a pair of loudspeakers, in order to improve the system performance achieved by a single loudspeaker presented in our previous study where advantages of robust control (sampled-data $H_\infty$ control) design have been shown by comparing to the existing adaptive control based design. Firstly, as a pair of loudspeakers, the Swinbanks’ source is composed of an appropriate delay and two loudspeakers whose dynamic characteristics are equivalent, then similar advantages of robust control design are shown by experimental results with the ventilation system. Secondly, the pair of loudspeakers is considered as two independent actuators to meet dynamic characteristic difference of the loudspeakers, then less-conservative SIMO controller is designed by robust control design to improve system performance.

1. INTRODUCTION

In the previous study, we have shown that robust control (sampled-data $H_\infty$ control) design is applicable to achieve inexpensive active noise control system for ducts of ventilation systems, rather than by using adaptive based control method, when temperature variation is small as like in recent energy-efficient houses [1]. However, further improvement of the sound attenuation is desired. The sound attenuation level reported in [1] was not enough.

The method originally proposed by Swinbanks [2] is well-known as an effective one for the improvement of the system performance, where an additional loudspeaker is attached to cancel out the upstream sound generated by a control source [3]. The method has been examined in detail under adaptive control setup [4]. However, the effect of the Swinbanks’ source under robust control setup has not been studied. Moreover, no experimental results applied to actual...
ventilation systems installed in houses have not been reported. In this paper, we examine robust control design of active noise control systems with a pair of loudspeakers in order to improve the system performance. By regarding the loudspeakers as two independent sources, a single input multiple output (SIMO) controller is also designed to be compared with the Swinbanks’ source. The validity of robust control design will be shown by experimental results using a ventilation system installed in a house.

2. EXPERIMENTAL APPARATUS

Fig.1 and Table 1 show the block diagram and instalments of the experimental apparatus which are the same that were used in [1] except that SPK3 and the corresponding D/A channel are attached so that an directional source is composed of SPK2 and SPK3. In addition, for simplicity of robust control design in this paper, SPK1 is used as a noise source to examine frequency response of the plant model.

Fig. 2 shows the configuration of the ventilation system installed to a two-storied real house which is also the same as in [1]. The grilles are attached on the ceiling of each floor, and the ANC system is connected between fresh-air grilles and the ventilation fan.

In this paper, we examine the following cases to drive the control source SPK2 and SPK3:

- **Case(a)** a single loudspeaker: by setting \( v(t) = 0 \), only SPK2 is used to generate control sound
- **Case(b)** the Swinbanks’ source[4]: by setting

  \[
  v(t) = -u(t - \tau), \quad \tau = \frac{d}{c_0},
  \]

![Figure 1. Experimental apparatus](image)

<table>
<thead>
<tr>
<th>Table 1. Experimental instruments</th>
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<tbody>
<tr>
<td><strong>Ventilation fan</strong></td>
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<tr>
<td><strong>Loudspeaker (SPK1)</strong></td>
</tr>
<tr>
<td><strong>Loudspeaker (SPK2 &amp; SPK3)</strong></td>
</tr>
<tr>
<td><strong>Microphones</strong></td>
</tr>
<tr>
<td><strong>Sound level meter</strong></td>
</tr>
<tr>
<td><strong>Power amplifier</strong></td>
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<tr>
<td><strong>High Pass Filter</strong></td>
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<tr>
<td><strong>Low Pass Filter</strong></td>
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<tr>
<td><strong>PC</strong></td>
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<tr>
<td><strong>A/D, D/A</strong></td>
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</tbody>
</table>
SPK3 is driven to cancel out the upstream sound generated by SPK2, where \( d \) is the distance between SPK2 and SPK3, and \( c_0 \) is the sound speed.

**Case (c)** an array of two loudspeakers: by setting \( v(t) \) free to \( u(t) \), SPK2 and SPK3 are driven as independent sources.

In the experiments for the case (b) below, (1) is approximately implemented as a real-time module of RTLinux that updates the signal \( v(t) \) at every 0.1 msec which is considered to be short enough to avoid aliasing effect. In addition, by letting \( d = 0.34 \) m from Fig. 1 and \( c_0 = 344 \) m/sec from normal temperature environment, we use \( \tau = 1 \) msec which exactly corresponds to 10 times of the period of the real-time module mentioned above. Moreover, the cut-off frequency of HPF is determined by considering the frequency range of the Swinbanks’ source given as \([f_0, 5f_0]\) where \( f_0 := \frac{d}{12c_0} \) [4]. The sampling period of controller is 1 msec throughout this paper.

### 3. ROBUST CONTROL DESIGN

The design procedure for case (a) is the same as in [5]. The detail of the design procedure for case (c) is omitted but it is done by simply replacing the signal \([u \ v]^T\) to \( u \) to apply the design process of case (a).

#### 3.1. Modeling

The plant models for Fig. 1 are determined by frequency response experiment. The system from \([w \ u]^T\) to \([z \ y]^T\) is considered as the plant transfer function \(G(s)\) as

\[
G(s) := \begin{bmatrix} G_{zw}(s) & G_{zu}(s) \\ G_{yw}(s) & G_{yu}(s) \end{bmatrix},
\]

where \(G_{ab}(s)\) means the transfer function from the signal \(b\) to the signal \(a\).

Fig. 3 shows the frequency response of \(G(s)\) and corresponding nominal plant obtained by subspace-based method where the order is 85: In the figures for \(G_{zw}(s)\) and \(G_{yu}(s)\), two frequency response results are shown in blue and yellow curves corresponding with case (a).
In experimental results of case (a), the phase lag becomes larger in the order of $G_{yw}$, $G_{zu}$, $G_{yu}$, and $G_{zw}$, of which order coincides with that of the distance from corresponding microphone to speaker. In case (b), remarkable change on $G_{yu}(s)$ is observed compared with case (a): the gain is smaller in the whole frequency range, and the phase lag becomes larger, which can be considered as the result that the distance for sound traveling from the control source to reference microphone becomes larger. This implies that the separation of control input and measured output is improved so that the better performance is expected [6]. Although such remarkable change is not observed in $G_{zu}(s)$, the gain is slightly larger in the middle frequency range of the Swinbanks' source, which is the nature of the source reported in [4].

In addition, in order to guarantee the closed-loop system stability against the modeling error of the nominal plant, additive uncertainty model is introduced for feedback-path transfer function, $G_{yu}(s)$, by

$$G_{yu}(s) = \bar{G}_{yu}(s) + W(s)\delta(s),$$  \hspace{1cm} (3)

where $\bar{G}_{yu}(s)$ is the nominal plant for $G_{yu}(s)$, and $W(s)$ is a weighting function which is determined to cover the modeling error as shown in Fig. 4.

### 3.2. Controller design

According to the preparation above, sampled-data $H_\infty$ control synthesis [7] is applied to the following digital controller design problem: find a discrete-time controller $K_d(z)$ which maximizes positive scalar $\alpha$ so that the following conditions hold:

- the closed-loop system of Fig. 5 is internally stable;
- there exists positive scalar $d$ such that $\mathcal{L}_2$ induced norm of the closed-loop system is less than 1,

where $S$ is the sampler with sampling period $h = 1$ msec, $H$ is the zero-th order hold, and $W_p(s)$ is a bandpass filter given by

$$W_p(s) = \left(\frac{s}{s + \omega_{p_1}}\right)^2 \left(\frac{s}{s + \omega_{p_2}}\right)^2, \quad \omega_{p_1} = 2\pi \times 80, \quad \omega_{p_2} = 2\pi \times 400.$$  \hspace{1cm} (4)

Note that the closed-loop system gain is robustly minimized by maximizing $\alpha$ to improve control performance to attenuate fan noise.

The design results are as follows: The maximal $\alpha = 4.64$ was achieved for $d = 1.07$ for case (a), and the maximal $\alpha = 5.87$ was achieved for $d = 1.56$ for case (b), which implies that the closed-loop performance will be improved by the Swinbanks’ source. Furthermore $\alpha$ was further improved by case (c) as $\alpha = 6.10$ for $d = 1.23$, because of the less conservative design. The order of $K_d(z)$ is 93.

### 4. COMPARISON OF CONTROLLERS

In this section, both adaptive and robust controllers are examined, where adaptive controllers are determined as fixed IIR filter of 100th order by using the Filtered-U RLMS method [1].
Figure 3. Frequency response of plant

Figure 4. Additive uncertainty and weight

Figure 5. Robust performance problem with scalings
Fig. 6 shows adaptive and robust controllers for case (a) and (b). It can be seen that for the adaptive controllers, the peak gain at about 60, 90 and 100 Hz become smaller when the Swinbanks’ source is used, while the similar phenomena has been reported in [4]. The effect of the Swinbanks’ source is also observed for the robust controllers shown as the flat gain characteristic within the frequency range from 80 to 400 Hz, while for the case with a single loudspeaker, relatively large peak at about 180 Hz is appeared.

Fig. 7 shows the controller for case (c) compared with case (b). It can be seen that for the controller of case (c), the Swinbanks’ source characteristic is automatically obtained by robust control design, since the gain characteristics of both channel of the controller are similar and the phase difference is around 180 deg. Furthermore, by comparing the characteristic in detail, advantages of robust control design are shown: Firstly, from the gain characteristic, the controller for case (c) has relatively large peaks at about 120 and 170 Hz comparing with case (b), which suggests that the flat gain characteristic of the Swinbanks’ source is not essential for performance improvement for actual ventilation system. Secondary, in the frequency range around 60 ∼ 400, the gain from $y$ to $v$ is slightly smaller than from $y$ to $u$, which can be interpreted as the result of robust control design to compensate the attenuation due to sound propagation.

It should be noted that such single input multiple output (SIMO) controller in case (c) might be obtained by adaptive control method [8], however, faster hardware to implement adaptive filters might be needed in controller design stage, since the required calculations effort for SIMO controller is about twice the size of SISO ones. On the other hand, robust control design does not need such faster hardware since controller design is done in off-line.

5. CONTROL EXPERIMENTS

In this section, actual performance of robust controllers designed in the previous section are examined by control experiments.

Fig. 8 shows time response of error microphone signal $z$, where the first 12.5 second is without control and the following 12.5 second is with control. The smaller sampling period (0.5 msec) is used for measurement to observe inter-sample behaviour within the sampling period of the controller. It can be seen that case (b) and (c) show better performance than case (a), while in [4] it has been reported by experimental result that the performance of (a) and (b) are similar. This might be caused since directional microphones are used in [4] but not used in this paper.

Fig. 9 shows the FFT analysis result of Fig. 8. It can be seen that the amplitude of $z$ is reduced within 80 to 400 Hz.
Figure 8. Time response of $z$

Figure 9. FFT analysis result of $z$

Table 2. Sound pressure level at each grille

<table>
<thead>
<tr>
<th>Grille</th>
<th>$L_{Aeq,10\sec}$ [dB]</th>
<th>Without Control</th>
<th>With Control</th>
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<tbody>
<tr>
<td></td>
<td></td>
<td>Case (a)</td>
<td>Case (b)</td>
</tr>
<tr>
<td>#1</td>
<td>34.2</td>
<td>33.0 (-1.2)</td>
<td>32.5 (-1.7)</td>
</tr>
<tr>
<td>#2</td>
<td>40.4</td>
<td>38.8 (-1.6)</td>
<td>37.4 (-3.0)</td>
</tr>
<tr>
<td>#3</td>
<td>31.9</td>
<td>30.5 (-1.4)</td>
<td>28.9 (-3.0)</td>
</tr>
<tr>
<td>#4</td>
<td>41.2</td>
<td>39.1 (-2.1)</td>
<td>37.1 (-4.1)</td>
</tr>
</tbody>
</table>

It should be noted that the main frequency component of noise occurs around 100 Hz whose noise shape very differs from the open-loop frequency response of $G_{zw}$ shown in Fig. 3. Therefore, it is expected that the system performance will be improved by setting the weight function $W_p(s)$ to consider the noise shape.

Table 2 shows sound pressure level measured below each grille. It can be seen that the
attenuation level of case (b) is about twice the level of case (a), which shows the availability of robust control design for the Swinbanks’ source. On the other hand, advantage of case (c) could not be shown i.e. the attenuation level is similar to case (b). It is not what we expect from the design results with larger $\alpha$. We would need to improve the modeling for utilizing the potential advantage of the design setup in case (c).

6. CONCLUSIONS

In this paper, we have examined robust control design of active noise control systems with a pair of loudspeakers, and the validity of robust control design have been shown experimentally by using a ventilation system installed in a real house. The results are summarized as follows:

- As a pair of loudspeakers, the Swinbanks’ source was examined, and the similar advantage of the Swinbanks’ source reported for adaptive control, i.e. the flat gain characteristic of controller, was observed.
- A less conservative SIMO controller was designed by considering the pair of loudspeakers as two independent actuators, and better design result was achieved.
- The sound attenuation level achieved by the robust control systems with a pair of loudspeakers were up to 4 dB which is about twice the level of the system with a single loudspeaker.

Therefore, we conclude together with the result of [1] that the robust control design is useful to implement inexpensive active noise control systems with a pair of loudspeakers for ducts of ventilation systems.

REFERENCES