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STUDY ON NUMERICAL VIBRO-ACOUSTIC ANALYSIS OF SPACECRAFTS

Takashi Takahashi, Keiichi Murakami, Takashi Aoyama and Hideaki Aiso

Institute of Space Technology and Aeronautics, Japan Aerospace Exploration Agency
7-44-1 Jindaiji-Higashimachi, Chofu, Tokyo, 182-8522, Japan
takahashi.takashi@jaxa.jp

Abstract

The objective of this paper is to examine some limitations in vibro-acoustic analysis using the finite element method (FEM), and to investigate the applicability of a novel deterministic approach called the wave base method (WBM) to steady-state vibro-acoustic analysis of spacecraft inside a payload fairing during the lift-off. It is pointed out that the limitations in vibro-acoustic analysis using the FEM are mainly on numerical dispersion error and on model size. A structural FEM simulation using the different sound field models shows that a deterministic approach as an alternative to the FEM is necessary for the coupled vibro-acoustic analysis with the wide frequency range. Next, in order to overcome the limitations of the FEM, the applicability of the WBM to steady-state vibro-acoustic analysis of spacecraft is investigated. From a simulation example, local structural responses can be obtained due to its deterministic characteristic, and the coupled vibro-acoustic analysis of spacecrafts can be performed easily and properly. From the WBM formulation and numerical simulation, it can be stated that the WBM is a quite practical approach, and has high potential for the vibro-acoustic analysis with the wide frequency range.

1. INTRODUCTION

Spacecraft are mounted on top of expendable launch vehicles and interact with the environment as depicted in figure 1. These are excited with mechanical vibrations via adapters between spacecraft and launch vehicles during the lift-off. It can be classified as sinusoidal vibrations with 5-100 Hz and random vibrations with 20-2kHz in spacecraft ground tests. In addition to such mechanical vibrations, the spacecraft are also exposed to acoustic pressure transmitted through the air and payload fairings with the wide frequency range (typically 20-10kHz). The sound pressure level (SPL) is generally maximum at the lift-off though it also becomes large at transonic speeds due to boundary layer separation and impact sounds. Lightweight and large area structures, such as solar array panels and antenna dishes, respond to acoustic pressure. Some components with relatively high resonant frequencies, such as actuator and sensor units, are also sensitive to the acoustic environment. For large spacecraft (>1000 kg), acoustic pressure generates responses greater than those of mechanical random vibrations [1]. Therefore, it is quite important to predict the acoustic environment in order to develop reliable spacecraft.

This paper focuses on the spacecraft vibro-acoustic analysis due to sound waves inside a payload fairing.

The statistical energy analysis (SEA) [2] has been applied to predict vibro-acoustics of spacecraft and the International Space Station [3][4]. The SEA is preferred in the high frequency range where individual vibration modes can no longer be distinguished and structural responses are quite sensitive to variations of material properties and dimensions. A simplified prediction equation for the changes in the SPL inside a fairing called 'fill effect' or 'fill factor', has been given based on the SEA [5]. However, since the SEA deals with only spatially and frequency averaged quantities, it can not predict local responses such as resonance peaks in structural analysis. Furthermore, the lower the analysis frequency is, the worse the prediction accuracy becomes. On the other hand, the finite element method (FEM) is an almost only prediction method in the low frequency range typically less than 100 Hz in spacecraft mechanical vibration analysis at the moment [6]. The FEM is a deterministic method which is capable of analyzing local acoustic and structural responses. In spacecraft vibro-acoustic analysis, this method has also been applied to some simple structures [7][8]. However, the model size becomes larger and numerical dispersion error becomes more dominant in the higher frequency range. In general, it is known that there exists mid-frequency range where it is difficult to obtain accurate prediction results using methods for the higher frequency range (such as the SEA) and lower frequency range (such as the FEM) [9]. Several studies have been conducted to fill the gap by extending the existing approaches and by proposing new ideas. Note that the mid-frequency range may include resonance frequencies of spacecraft components which are quite critical for the spacecraft design.

The objective of this paper is to examine some limitations in vibro-acoustic analysis using the FEM, and to investigate the applicability of an alternative deterministic approach called the wave base method (WBM) [9] to steady-state vibro-acoustic analysis of spacecraft with the wide frequency range. Because of the novel approach, it has been applied to a few practical engineering problems. Therefore, it is investigated how this approach can be applied to steady-state vibro-acoustic analysis of spacecraft inside a payload fairing by overcoming the limitations of the FEM.

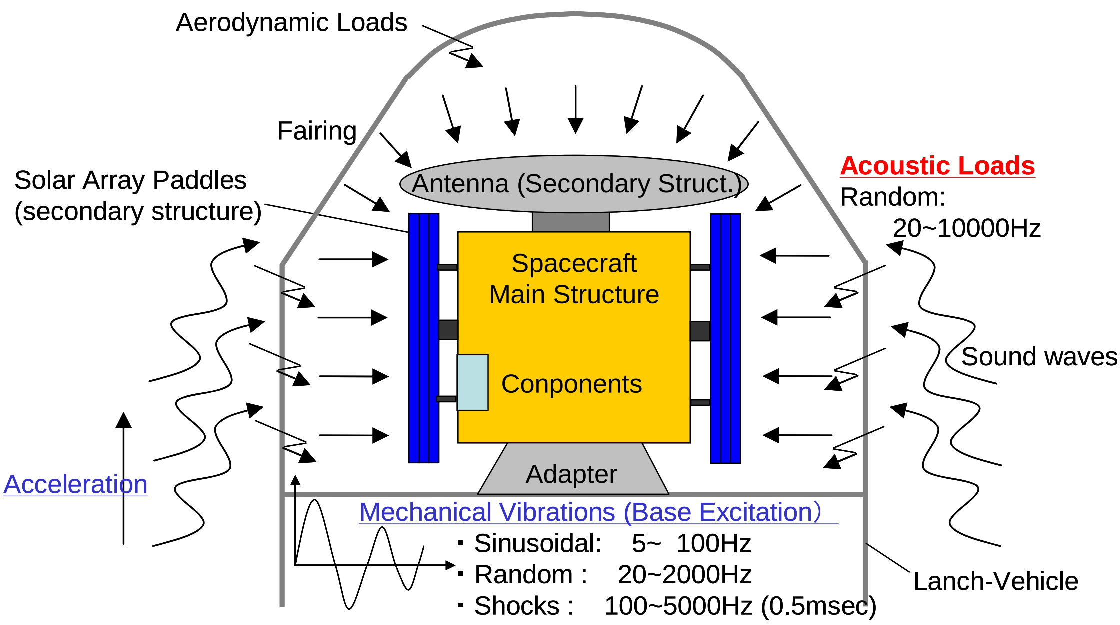


Figure 1. Vibrational environment for spacecraft.

2. LIMITATIONS IN VIBRO-ACOUSTIC ANALYSIS USING FEM

2.1 Limitations on Numerical Errors

The FEM is one of the most widely used methods to solve plenty of engineering problems. Its theoretical description is out of the scope of this paper, so this section will discuss some numerical errors of the FEM. The error can be briefly classified into the interpolation and dispersion errors. It is known that the former error, which is generally dominant in lower frequency, can be kept within the user-defined acceptable tolerance. That is, there is a simple criterion which can give us how many elements should be used per wavelength [10]. On the other hand, the latter error, which generates the difference between the exact and numerical wave numbers, has no such simple criteria [11]. Moreover, the error is generally dominant in the higher frequency range, and also has directional dependency in 2 and 3-D cases. Therefore, it is difficult to control the dispersion error completely. And even if the error can be kept within the acceptable level, the numerical model size can be immense especially for the analysis in the higher frequency range.

2.2 Modelling Limitations in Random Response Analysis

Although structural random acceleration RMS values can be estimated simply using Miles' equation in the spacecraft design phase [6][12], this section will discuss the modelling limitations of the FEM in vibro-acoustic analysis for more general complex structures. In case of using only structural FEM models, the correlation among element pressures must be taken into account to simulate the interactions between structures and sound fields properly. The correlation can be expressed as cross power spectral density (PSD) function $\mathbf{S}_{\mathrm{pp}}(\omega)$ as

$$\mathbf{S}_{\mathrm{pp}}(\omega)=\frac{\mathbf{W}_{\mathrm{pp}}(f)}{2}=\frac{W_{\mathrm{p}}(f)\mathbf{C}_{\mathrm{pp}}(f)}{2}, \tag{1}$$

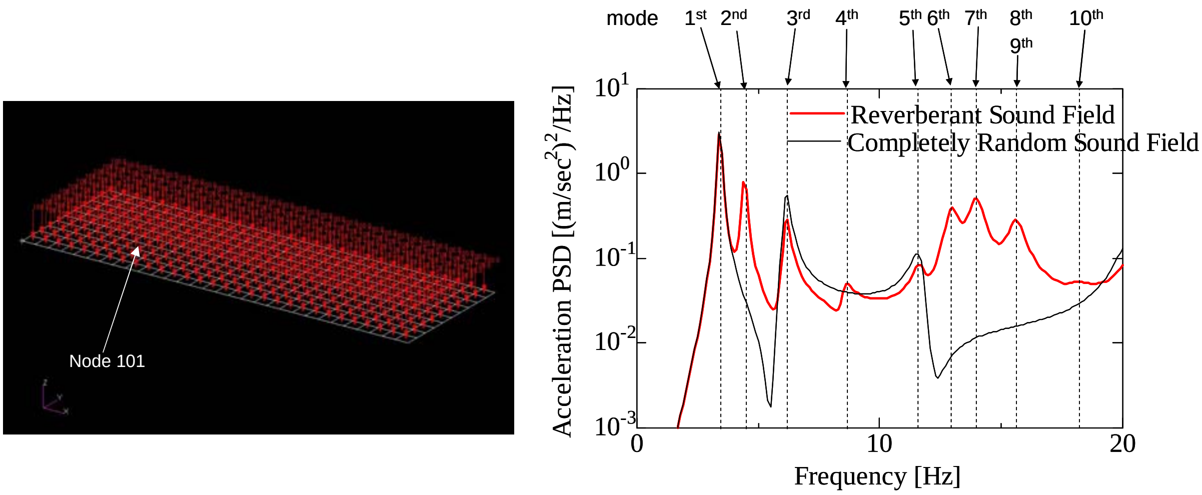
where $\mathbf{W}_{\mathrm{pp}}(f)$ is the one-sided cross PSD function. The function $W_{\mathrm{p}}(f)$ is the one-sided reference pressure PSD function, and is generally given by the SPL of the ambient sound field surrounding the structure. The matrix $\mathbf{C}_{\mathrm{pp}}(f)$ has a coherence function $C_{p_a p_b}(f)\in[-1,1]$ between pressures p_a and p_b at node a and b as (a, b) element. When the field is reverberant, $C_{p_a p_b}(f)$ is given as follows [13]:

$$C_{p_a p_b}(f)=\frac{\sin\left(k\left|\mathbf{x}_a-\mathbf{x}_b\right|\right)}{k\left|\mathbf{x}_a-\mathbf{x}_b\right|}, \tag{2}$$

where $\left|\mathbf{x}_a-\mathbf{x}_b\right|$ is the distance between the node a and b, and k is the acoustic wave number. When the field has completely random characteristic, then $\mathbf{C}_{\mathrm{pp}}(f)$ is an identity matrix.

The acoustic pressure is generally given as the completely random sound field in structural random analysis using the FEM. On the other hand, the reverberant sound field is approximately generated in ground acoustic tests. Then, we will compare the random structural responses using an aluminium plate FE model (CQuad4, 341 nodes, 300 elements, damping ratio 0.05) excited by the completely random and reverberant sound field as shown in figure 2. As a result of the eigenvalue analysis, 10 natural frequencies are obtained in the range less than 20 Hz. Assuming that $W_{\mathrm{p}}(f)$ is constant not depending on frequencies, then, figure 2 (b) illustrates the results of random analysis. We can see that all expected resonant peaks are appeared in the analysis using the reverberant sound field, while all peaks are not observed using the completely random sound field. Moreover, since to define the correlation among element pressures is the combinatorial problem, it is practically impossible to apply the approach presented here to complex structure FEM models with numerous degrees of freedom.

By the limitations on numerical errors and on vibro-acoustic modelling in the FEM, a deterministic approach as an alternative to the FEM is necessary for the coupled vibro-acoustic analysis with the wide frequency range.



(a) Thin plate model with acoustic pressure. (b) Structural acceleration PSD (at node 101).

Figure 2. Comparison of random responses using different sound field models.

3. VIBRO-ACOUSTIC ANALYSIS USING WBM

3.1 Wave Based Method

The WBM [9] is a deterministic method for steady-state coupled vibro-acoustic analysis. It is based on an indirect Trefftz approach, and has overcome an ill-conditioned problem in a Trefftz formulation by defining a complete wave function set. The main feature of the method is that there are no numerical dispersion errors since the wave function set exactly satisfies the governing equations. Therefore, the method has high potential to predict vibro-acoustic responses with the wide frequency range.

3.1.1 Problem Definitions

This section discusses a 2-D steady-state coupled interior vibro-acoustic problem using the WBM. Figure 3 shows a cavity filled with a fluid and surrounded with some boundary surfaces. First, the acoustic cavity must be decomposed into some convex subdomains due to a feature of the WBM described later. For simplicity, the acoustic cavity is composed of only two convex subdomains V_e (e = 1, 2). Each subdomain V_e is assumed to have a circumscribing minimum rectangle with dimension L_{x_e} and L_{y_e}. An acoustic boundary surface Ω_{ae} in V_e may consist of six kinds of surfaces, i.e. $\Omega_{ae} = \Omega_{pe} \bigcup \Omega_{ve} \bigcup \Omega_{Ze} \bigcup_{s=1}^{n_{es}} \Omega_{ses} \bigcup_{s_c=1}^{n_{12sc}} \Omega_{sc12s_c} \bigcup_{c=1}^{n_{12c}} \Omega_{c12c}$. Surfaces Ω_{pe}, Ω_{ve} and Ω_{Ze} are imposed pressure, normal velocity, and normal impedance BCs, respectively. Surface Ω_{se} consist of n_{es} flat thin plates, and each surface Ω_{ses} (s = 1,...,n_{es}) is assumed to be a plate (length L_{es} and infinite wideness perpendicular to the paper) imposed some BCs at both edges. Surfaces Ω_{c12} and Ω_{sc12} are boundary surfaces between subdomain V_1 and V_2, and consist of n_{12c} surfaces and n_{12sc} flat thin plates, respectively. Surface Ω_{c12c} (c = 1,...,n_{12c}) is imposed both pressure and normal velocity continuity conditions. For simplicity, surface Ω_{sc12s_c} (s_c = 1,...,n_{12sc}) is ignored in the derivation below. Moreover, several external normal line forces f_{esi} (i = 1,...,n_{efs}) can be applied at local position x'_{efsi} on Ω_{ses}. Furthermore, several external point sound sources q_{ei} (i =1,...,n_{eq}) can be placed at position $\mathbf{r}_{eqi}$ inside V_e. Both external excitations are assumed to be time-harmonic functions with angular frequency ω.

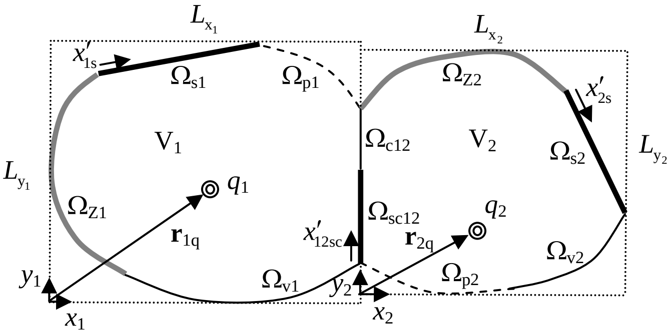


Figure 3. 2-D steady-state coupled interior vibro-acoustic model.

3.1.2 Governing Equations and Variable Expansions

The steady-state acoustic pressure p_e at position $\mathbf{r}_e$ in V_e is governed by the Helmholtz equation:

$$\nabla^2 p_e(\mathbf{r}_e) + k_e^2 p_e(\mathbf{r}_e) = -j\rho_e\omega\sum_{i=1}^{n_{eq}} q_{ei}\delta(\mathbf{r}_e, \mathbf{r}_{eqi}) \qquad \mathbf{r}_e \in V_e, \tag{3}$$

where k_e is the acoustic wave number and δ is the Dirac delta function. In the WBM formulation, the acoustic pressure $p_e(\mathbf{r}_e)$ is approximated by using an acoustic wave function set and a particular solution as follows.

$$p_e(\mathbf{r}_e) \approx \hat{p}_e(\mathbf{r}_e) = \sum_{i=0}^{n_{ea}^{(1)}} p_{eai}^{(1)}\phi_{eai}^{(1)}(\mathbf{r}_e) + \sum_{i=0}^{n_{ea}^{(2)}} p_{eai}^{(2)}\phi_{eai}^{(2)}(\mathbf{r}_e) + \hat{p}_{eq}(\mathbf{r}_e), \tag{4}$$

where $\phi_{eai}^{(1)}(\mathbf{r}_e)$ and $\phi_{eai}^{(2)}(\mathbf{r}_e)$ are the wave functions which exactly satisfy a homogeneous equation, and are normalized to relax the numerical poor condition. In order to guarantee the convergence, cavity domains must be convex due to the feature of the wave functions, and the function must be truncated depending on the physical wave numbers and the dimensions of the bounding boxes. The function $\hat{p}_{eq}(\mathbf{r}_e)$ is the particular solution of the inhomogeneous equation (3).

On the other hand, steady-state plate normal displacement w_{es} at local position x'_{es} on Ω_{ses} is governed by the plate bending equation, Kirchhoff equation:

$$\frac{d^4 w_{es}(x'_{es})}{d{x'_{es}}^4} - k_{ebs}^4 w_{es}(x'_{es}) = \frac{1}{D_{es}}\sum_{i=1}^{n_{efs}} f_{esi}\delta(x'_{es}, x'_{efsi}) + \frac{p_e(\mathbf{r}_{ess}(x'_{es}))}{D_{es}}, \tag{5}$$

where k_{ebs} is the structural wave number, and D_{es} is bending stiffness of the plate. Then, the structural displacement $w_{es}\left(x'_{es}\right)$ is approximated in the WBM as follows:

$$w_{es}\left(x'_{es}\right) \approx \hat{w}_{es}\left(x'_{es}\right) = \boldsymbol{\psi}^{\mathrm{T}}_{ess}\left(x'_{es}\right)\mathbf{w}_{ess} + \hat{w}_{efs}\left(x'_{es}\right) + \hat{\mathbf{w}}^{\mathrm{T}}_{eas}\left(x'_{es}\right)\mathbf{p}_{ea} + \hat{w}_{eq}\left(x'_{es}\right), \tag{6}$$

where the vector $\boldsymbol{\psi}_{ess}\left(x'_{es}\right)$ consists of four structural wave functions $\psi_{essi}\left(x'_{es}\right)$ which exactly satisfy a homogeneous equation as follows:

$$\psi_{essi}\left(x'_{es}\right) = e^{-j^{i}k_{ebs}x'_{es}} \qquad (i = 1,...,4). \tag{7}$$

These functions are also normalized like acoustic ones. The function $\hat{w}_{efs}\left(x'_{es}\right)$ is a particular solution due to some external force terms in the inhomogeneous equation (5). The vector $\hat{\mathbf{w}}_{eas}\left(x'_{es}\right)$ has an element $\hat{w}_{easi}\left(x'_{es}\right)$ which is also a particular solution due to an acoustic pressure term in equation (5) associated with the acoustic wave functions. The function $\hat{w}_{eqs}\left(x'_{es}\right)$ is also a particular solution due to the acoustic pressure term in equation (5) related to the external sound sources.

From above definitions, we can see that all variable expansions (4) and (6) exactly satisfy the governing equations. This feature is quite important to predict vibro-acoustic responses accurately with the wide frequency range due to no numerical dispersion errors.

3.1.3 Weighted Residual Formulation and WBM model

In order to solve contribution coefficients p_{eai} and w_{esi} in expansions (4) and (6), the weighted residual method can be applied to acoustic BCs. That is, the BCs are satisfied approximately while the governing equations are satisfied exactly. As the Galerkin method in FEM, a weighting function $\tilde{p}_e$ can be expanded using the acoustic wave function set.

$$\tilde{p}_e\left(\mathbf{r}_e\right) = \sum_{i=1}^{n_{ea}} \tilde{p}_{eai}\phi_{eai}\left(\mathbf{r}_e\right) = \boldsymbol{\phi}^{\mathrm{T}}_{ea}\left(\mathbf{r}_e\right)\tilde{\mathbf{p}}_{ea} = \tilde{\mathbf{p}}^{\mathrm{T}}_{ea}\boldsymbol{\phi}_{ea}\left(\mathbf{r}_e\right). \tag{8}$$

Using the function $\tilde{p}_e$, the weighted residual formulation can be derived as follows:

$$\int_{\Omega_{p1}} \frac{-j}{\rho_1\omega}\frac{\partial\tilde{p}_1}{\partial n}R_{1p}d\Omega + \int_{\Omega_{v1}} \tilde{p}_1R_{1v}d\Omega + \int_{\Omega_{Z1}} \tilde{p}_1R_{1Z}d\Omega + \sum_{c=1}^{n_{12c}}\int_{\Omega_{c12}} \tilde{p}_1R_{1cvc}d\Omega + \sum_{s=1}^{n_{1s}}\int_{\Omega_{s1s}} \tilde{p}_1R_{1ss}d\Omega = 0 \tag{9}$$

$$\int_{\Omega_{p2}} \frac{-j}{\rho_2\omega}\frac{\partial\tilde{p}_2}{\partial n}R_{2p}d\Omega + \int_{\Omega_{v2}} \tilde{p}_2R_{2v}d\Omega + \int_{\Omega_{Z2}} \tilde{p}_2R_{2Z}d\Omega + \sum_{c=1}^{n_{12c}}\int_{\Omega_{c12}} \frac{-j}{\rho_2\omega}\frac{\partial\tilde{p}_2}{\partial n}R_{2cpc}d\Omega + \sum_{s=1}^{n_{2s}}\int_{\Omega_{s2s}} \tilde{p}_2R_{2ss}d\Omega = 0 \tag{10}$$

where R_{ep}, R_{ev}, R_{eZ}, R_{1cvc}, R_{2cpc} and R_{ess} are residual error functions of the BCs. The structural BCs on Ω_{ses} such as clamped, simply supported, free and symmetric BCs can also be derived using expansions (6).

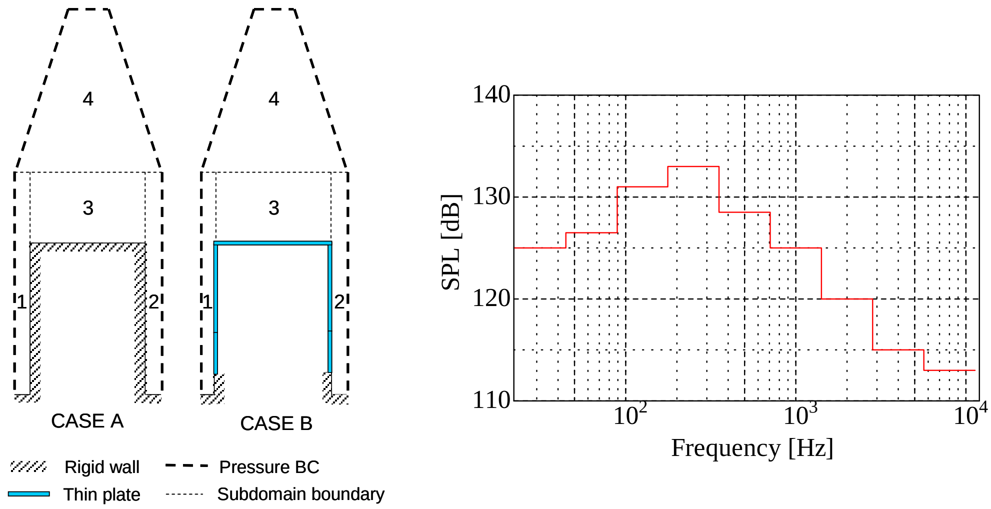
Finally, the WBM model can be expressed as the matrix form:

$$\left[\begin{array}{cc|cc} \mathbf{A}_{1ss} & \mathbf{C}_{1sa} & \mathbf{O} & \mathbf{O} \\ \mathbf{C}_{1as} & \mathbf{M}_{1aa} & \mathbf{O} & \mathbf{N}_{12} \\ \hline \mathbf{O} & \mathbf{O} & \mathbf{A}_{2ss} & \mathbf{C}_{2sa} \\ \mathbf{O} & \mathbf{N}_{21} & \mathbf{C}_{2as} & \mathbf{M}_{2aa} \end{array}\right] \left[\begin{array}{c} \mathbf{w}_{1s} \\ \mathbf{p}_{1a} \\ \hline \mathbf{w}_{2s} \\ \mathbf{p}_{2a} \end{array}\right] = \left[\begin{array}{c} \mathbf{f}_{1s} \\ \mathbf{f}_{1a} \\ \hline \mathbf{f}_{2s} \\ \mathbf{f}_{2a} \end{array}\right], \tag{11}$$

where $\mathbf{C}_{esa}$ and $\mathbf{C}_{eas}$ explicitly present structural-acoustic coupling effects. The matrices $\mathbf{N}_{12}$ and $\mathbf{N}_{21}$ show subdomain coupling effects.

3.2 Numerical Example

Based on the theoretical description in the previous section, a 2-D WBM code was implemented using MATLAB® [14]. In order to examine structural-acoustic coupling effects in spacecraft vibro-acoustic analysis, a simple rigid spacecraft model (case A) and a flexible spacecraft model (case B) are built as shown in figure 4 (a). In case B, the model is composed of 5 clamped flat plates. A SPL illustrated in figure 4 (b) is actually the envelope level during launch and flight of the H-IIA launch vehicle, and is assumed to be uniform around the spacecraft. However, the acoustic pressure in the SPL is inputted on inside surfaces of a faring as pressure BCs in this numerical example. Figure 5 illustrates some results of the steady-state (frequency domain) vibro-acoustic analysis using the WBM code. Figure 5 (a) shows variations of the structural acceleration PSD functions which are generally the final outputs of the acoustic ground tests. Some resonant peaks can be observed because of the deterministic approach. Compared with the acoustic pressure fields in case A and B in figure 5 (b), it is clearly shown that the acoustic field is affected by structural vibrations, and the coupled vibro-acoustic problem of spacecraft can be solved by the WBM.

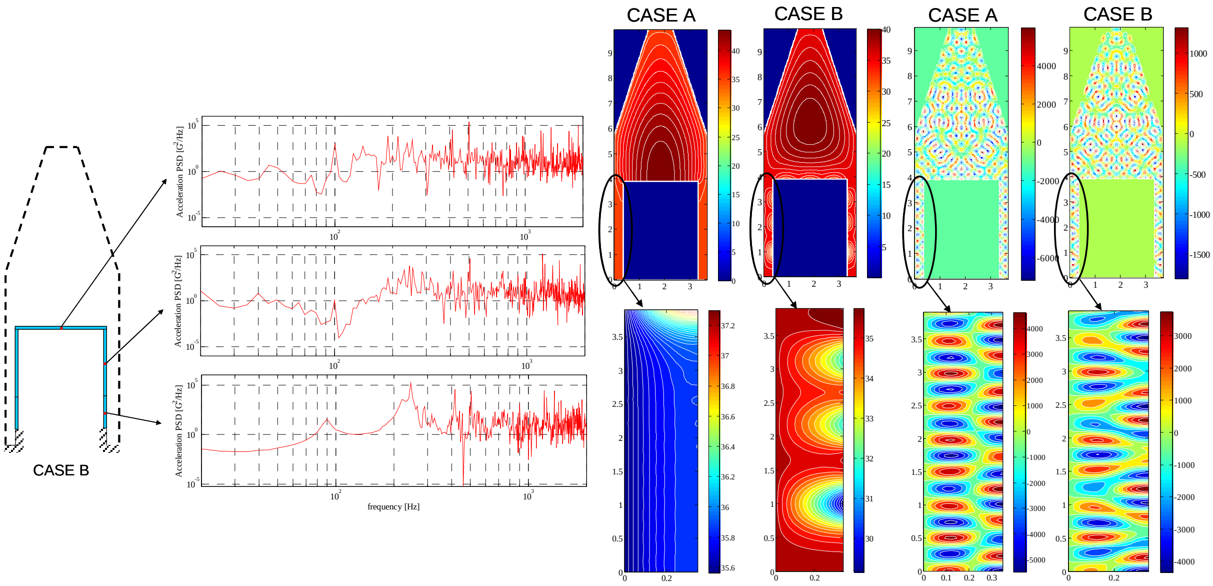


(a) (b)

Figure 4. Spacecraft model inside a fairing (a), and input sound pressure level (b)

4. CONCLUSIONS

In this paper, it is pointed out that there are some limitations on numerical dispersion error and on vibro-acoustic modelling in the FEM analysis. The different resonant peaks are observed in random structural FEM analysis using a simple plate model excited by different sound fields. Therefore, it follows from the results that a deterministic approach as an alternative to the FEM is necessary for the coupled vibro-acoustic analysis with the wide frequency range. Next, in order to overcome the limitations of the FEM, the applicability of the WBM to steady-state vibro-acoustic analysis inside a fairing is investigated. In a simulation example, local structural responses can be obtained due to its deterministic characteristic, and it shows that the coupled vibro-acoustic analysis of spacecraft can be performed easily and properly. All numerical results can be obtained by just setting all BCs without using any meshes. Moreover, there is no dispersion error in the WBM formulation. Therefore, it can be stated that the WBM is a quite practical approach, and has high potential for the vibro-acoustic analysis with the wide frequency range.



(a) (b) (c)

Figure 5. Structural acceleration power spectral densities (a), and sound pressure inside fairing ((b): 20Hz and (c) 1kHz).

REFERENCES

[1] P. Fortescue, J. Stark and G. Swinerd, *Spacecraft System Engineering: 3rd ed.*, John Wiley and Sons, 2003.

[2] R.H. Lyon and R.G. De Jong, *Theory and Application of Statistical Energy Analysis*, Butterworth-Heinemann, New York, 1995.

[3] S. Xiang, J. Zhang and Z. Wang, Development of the DFH-3 satellite vibro-acoustic environment by SEA method, 50th International Astronautical Congress, Amsterdam, Netherlands, IAF-99-1.2.09, 1999.

[4] M.E. Wang, Modeling of acoustic environment of Node 1 of International Space Station by statistical energy analysis, 29th International Conference on Environmental Systems, Denver, Colorado 12-15 July 1999, 1999-01-2197.

[5] Payload vibroacoustic test criteria, NASA Technical Standard, NASA-STD-7001, 1996.

[6] J. Wijker, Mechanical vibrations in spacecraft design, Springer, 2004.

[7] P.E. Dupuis, Vibroacoustic test prediction dedicated to flat structures, AIAA-98-2306, 1998.

[8] F.W. Grosveld, R.D. Buehrle and J.H. Robinson, Structural and acoustic numerical modeling of a curved composite honeycomb panel, AIAA-2001-2277, 2001.

[9] W. Desmet, *A wave based prediction technique for coupled vibro-acoustic analysis*, Ph.D. dissertation, K.U. Leuven, 1998.

[10] F. Ihlenburg and I. Babuška, Finite element solution of the Helmholtz equation with high wave number Part I: The h-version of the FEM, *Computers and Mathematics with Applications* **30**, pp.9-37 (1995).

[11] P. Bouillard and F. Ihlenburg, Error estimation and adaptivity for the finite element method in acoustics: 2D and 3D applications. *Computer Methods in Applied Mechanics and Engineering* **176**, pp.147-163 (1999).

[12] J.W. Miles, On structural fatigue under random loading, *Journal of Aeronautical Sciences*, pp.753-762 (1954).

[13] R.K. Cook, R.V. Waterhouse, R.D. Berendt, S. Edelman and M.C. Thompson, Jr., Measurement of correlation coefficients in reverberant sound fields, *Journal of Acoustical Society of America* **27**, pp.1071-1077 (1955).

[14] http://www.mathworks.com/products/matlab/