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APPLICATION OF MODAL PARAMETER DERIVATION IN ACTIVE SUPPRESSION OF THERMO ACOUSTIC INSTABILITIES

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Abstract

Low NO_x premixed surface burners are often applied in modern central heating systems. Systems equipped with these burners often experience noise problems. In order to make any further advance in the development of these systems these problems have to be solved. Therefore, the feasibility of model-based control strategies in suppressing the (thermo acoustic) instabilities is investigated. Based on an acoustic network approach, a model for the gas-fired boiler is derived. Special attention is paid to the flame behavior, since a modal analysis identification method has been used to describe the dynamics. A $\mathcal{H}_\infty$ controller is synthesized utilizing this model. The performance of this controller is validated using simulations and compared with a previously derived physical model-based controller.

1. INTRODUCTION

In combustors using 'lean premixed flames', the suppression of thermo acoustic instabilities by applying several control strategies is an opportunity to broaden their range of operation. The first passive control approaches to this problem consisted of installing acoustic dampers and baffles, and adjusting the flame anchoring, burning mechanism and fuel line piping to change the feedback loop involving fluctuations in pressure, velocity and heat release.

Due to the evolution of sensor and actuator technology, active control is introduced recently. Available sensors are OH$*$/CH$*$ chemiluminescence sensors (to measure heat release rates) and pressure transducers. Actuators will induce perturbations in acoustic pressure, velocity and mass flow. The control algorithms expanded from simple phase-shift control to more robust LQG/LTR and (model-based) $\mathcal{H}_\infty$ control.

In the current project a $\mathcal{H}_\infty$ controller is designed for gas-fired household boilers. Such boilers are often equipped with surface burners, where the interaction of the system with the burner-stabilized premixed flat flames can lead to acoustic instabilities. The behavior of the flame can be based on physical modeling, but in this paper a model is derived using a circle fit, a single degree of freedom modal analysis technique. The advantage of this approach is that a flame transfer function can be derived with the same behavior as the physical based model, yet in absence of any time-delay. This also simplifies the control design.

The paper is organized as follows: a model for the household boiler is derived. Then a model for the flame behavior is determined based on a modal analysis technique. The models are combined, resulting in a complete model for the gas-fired household boiler, making use of an acoustic network approach. Thereupon, a controller is synthesized. In this part, the mixed sensitivity problem in the frame of $\mathcal{H}_\infty$ control is presented and a short summary of the design methodology is given. Finally, a simulation is performed in order to check if the performance is fulfilled. Also a comparison is made with earlier synthesized controllers.

2. ACOUSTIC DESIGN OF THE SYSTEM

A modified Rijke tube, Figure 1, is used as a start for this research in order to focus on the most important part, being the burner and to reduce the complexity of the rest of the gas-fired boiler system. The complete model include a network of 1-dimensional acoustics (ducts, end conditions, etc.) and (non)linear flame dynamics.

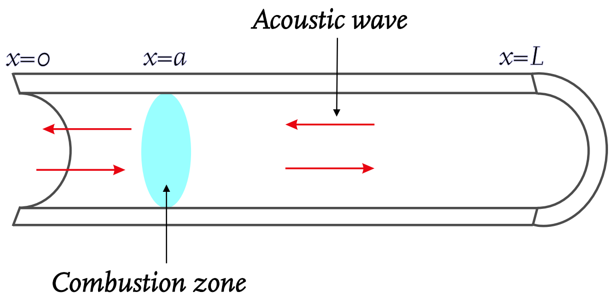

Figure 1. General control configuration

Only plane wave propagation is considered, since the geometrical size of boiler tubes is small compared to the acoustic wavelengths present in the system. Therefore, the use of the acoustic network approach is limited to the low frequency regime, see Campos-Delgado *et al.* [2] for more details. Every element of the system can be treated as a two-port element (Munjal [4]), where acoustic pressure p and acoustic velocity u upstream and downstream are often an element linearly linked, in matrix form:

$$\begin{bmatrix} p'_2 \\ u'_2 \end{bmatrix} = T_{\text{pu}}(s) \begin{bmatrix} p'_1 \\ u'_1 \end{bmatrix}, \tag{1}$$

where $T_{\text{pu}}(s)$ is a 2x2 matrix, and the subscripts 1 and 2 denote conditions up- and downstream of the element.

2.1. The ducts

The tube elements can easily be presented in a four-element transfer matrix, however, a state-space representation is preferable for control design. This representation is based on the method explained in the paper by Schuermans *et al.* [9].

The eigenfrequencies(ω_{n}) and eigenvectors(ψ_{n}), for a duct with length L and closed at both ends, for a given mode n are given by:

$$\omega_{\mathrm{n}} = \frac{\pi n c_{\mathrm{u}}}{L} \tag{2}$$

$$\psi_{\mathrm{n}} = \cos(\frac{\pi n x}{L}) \tag{3}$$

$$\Lambda = \frac{L}{2 - \delta_{\mathrm{kron}}(n)} \tag{4}$$

with $\delta_{\mathrm{kron}}(n)$ the Kronecker delta. The state-space representation is then expressed as follows:

$$\dot{x}(t) = A_{\mathrm{n}} x(t) + B_{\mathrm{n}} u(t) \tag{5}$$

$$\frac{p_{\mathrm{n}}(t)}{\rho c_{\mathrm{u}}} = C_{\mathrm{n}} x(t), \tag{6}$$

where

$$A_{\mathrm{n}} = \begin{bmatrix} -\alpha_{\mathrm{n}} & -\omega_{\mathrm{n}} \\ \omega_{\mathrm{n}} & -\alpha_{\mathrm{n}} \end{bmatrix}, \qquad B_{\mathrm{n}} = \begin{bmatrix} 0 \\ \psi_{\mathrm{n}}(x) \end{bmatrix} \tag{7}$$

$$C_{\mathrm{n}} = \begin{bmatrix} 0 & \frac{c_{\mathrm{u}} S_{\mathrm{e}}}{\Lambda} \psi_{\mathrm{n}}(x_{\mathrm{e}}) \end{bmatrix}, \tag{8}$$

and ρ the density of the fluid, c_{u} the velocity of sound, and S_{e} the area of the input of the element. The state of the system, represented by the 2x1 vector $x(t)$, is according to the notation adopted in control theory. The first element of this vector can be interpreted as the modal value of the velocity potential, the second element corresponds to the modal value. The vector $u(t)$ represent velocity inputs to the system.

2.2. Flame Dynamics

The flame transfer function is the key quantity in conversance of the acoustic behavior of flames in a closed system (and fighting the noise problem). To be more distinct, the transfer function between an upstream acoustic variable (i.e. pressure or velocity) and an acoustic variable downstream should be known in front. Acoustic transfer models for 1D premixed flat flames in terms of velocity have been developed for instance by Rook *et al.* [6], and have been validated by Schreel *et al.* [7], [8]. Another approach to physically model these flames is given in van den Boom [1], which is represented by the following relation:

$$\begin{bmatrix} p'_2 \\ u'_2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 + \frac{(\gamma-1)}{c_{\mathrm{u}}^2 c_{\mathrm{p}} (\overline{T}_{\mathrm{b}} - \overline{T}_{\mathrm{u}})(s \exp(s\psi_{\mathrm{f}}/\overline{u}_{\mathrm{u}} + 1))} \end{bmatrix} \begin{bmatrix} p'_1 \\ u'_1 \end{bmatrix}, \tag{9}$$

with γ the ratio of the specific heats, c_{p} the specific heat capacity at constant pressure, $\overline{u}_{\mathrm{u}}$ the mean gas velocity, ψ_{f} the standoff distance , i.e. the distance between the burner deck and the flame front, T_{b} the flame temperature and T_{u} the temperature of the burner. This relation is derived based on the coupling between the heat loss of the flame to the burner and the gas velocity.

This transfer function can be used for an analysis of a gas-fired boiler system is less practicable when the interest is the design of a linear active controller for a particulary frequency range, since it is a non-linear or dead-time system. In these circumstances, one method is to remove the modes which correspond to frequencies that lie outside the bandwidth of interest and only keep the low frequency modes. This last approach will result in a low order model for the system which is preferable, because it simplifies the control synthesis.

The development of the flame transfer function is founded on a single degree of freedom modal analysis technique: the circle fit. In Figure 2 the Nyquist plot of the transfer function of a premixed flat flame on a brass burner deck is shown for experiments and numerical computations. The experimental data is acquired by quantifying the velocity amplitudes on the unburnt and burnt side of the surface burner with respect to a single reference wave. In the unburnt region time-correlated measurement of the velocity are performed using a two-microphone method, since the medium has constant properties in this part. The measurement on the burnt side are achieved by Laser Doppler Velocimetry, see Schreel [7] for more details on the experimental set-up.

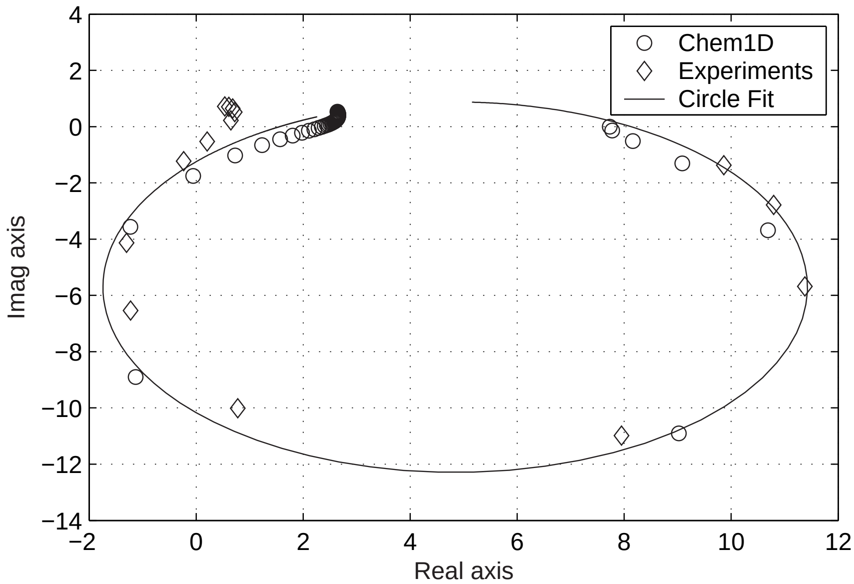

Figure 2. Nyquist plot for the flame transfer function ($\phi = 0.8$ and $\overline{u}_{\mathrm{u}} = 14(cm/s)$)

Numerical evaluation of the transfer function for a planar burner-stabilized flame is transposed using the flame code Chem1D [11] in combination with the detailed chemistry of GRI 2.11 [12]. The velocity response of the flame is determined, via a time-dependent simulation, by applying a step to the inlet velocity. From the Fourier transformation of the signal of the inlet velocity and the response of the flame velocity the transfer function is constructed.

Another approach for deriving a reliable model to represent the dynamics of the premixed flat flame is presented here. In Figure 2, it can be seen that both frequency response functions (FRF) resemble a circle for one resonance. Therefore, a single degree of freedom analysis, circle-fit (see Ewins [3]), is applied to find a description for the model.

The circle-fit is a simple technique to determine the modal parameters. However, the following assumptions have to be made:

- Only one mode is relevant ($k = 1$), so there is no interaction with higher modes. The contribution of these modes is approximated by the complex constant $R_\mathrm{k} + jI_\mathrm{k}$;

- System is weakly damped, $\mu_\mathrm{k}/\nu_\mathrm{k}$ is small. For $\omega \approx \nu_\mathrm{k}$: $\frac{A_\mathrm{kI}[i,l]}{-\mu_\mathrm{k}+j(\omega+\nu_\mathrm{k})}$ is negligible compared to $\frac{A_\mathrm{k}[i,l]}{-\mu_\mathrm{k}+j(\omega-\nu_\mathrm{k})}$.

The FRF for the burner-stabilized flames is then given by (for $\omega \approx \nu_\mathrm{k}$):

$$H[i](\omega) = \frac{A_\mathrm{kR}[i] + jA_\mathrm{kI}[i]}{-\mu_\mathrm{k} + j(\omega - \nu_\mathrm{k})} + R_\mathrm{k} + jI_\mathrm{k}, \tag{10}$$

with the eigenvalues $\lambda_\mathrm{k} = \mu_\mathrm{k} + j\nu_\mathrm{k}$ that appear in complex conjugated pairs and the residu matrices $A_\mathrm{k} = A_\mathrm{kR}[i] + jA_\mathrm{kI}$. Note that no time-delay is present in this model. Only six parameters have to be determined in successive order: ν_k, μ_k, A_kR and A_kI, and R_k and I_k, which can be done using the following sequence ([3]):

1. select frequency points to be used;
2. locate frequency of maximum response, obtain an estimate for the damping;
3. fit circle, using a least squares approach;
4. determine modal constants.

This model is transformed into a state-space notation and interconnected with the state-space system model of the ducts, using the Redheffer star product. This gives a transfer matrix, H for the modified Rijke tube. Schuermans *et al.* [9] also used this product to make an interconnection. The same is done with equation (9), after linearization of the model, for an equivalence ratio $\phi = 0.8$ and a mean gas velocity of $\overline{u}_\mathrm{u} = 14(cm/s)$, and applying a sixth-order Padé approximation for the time delay. Stability of the transfer matrix can easily checked, because it is satisfied when the real parts of all the eigenvalues of this matrix are negative. Computing the eigenvalues, using standard methods, shows that for both transfer matrices the first unstable mode can be found at 227 (Hz).

3. CONTROLLER DESIGN

The feedback control design is expressed as an $\mathcal{H}_\infty$ optimization problem. This is presented in a general configuration shown in Figure 3 (Skogestad [10], Ortega *et al* [5]). In this figure, $P(s)$ is the generalized plant, $C(s)$ is the controller, w are the exogenous signals such as disturbances, u are the control signals, v the measured variables and z the so-called error variables.

In this case a controller is synthesized which suppresses the thermo acoustic instabilities in the system and is able to endure parametric uncertainties. A common design method for this type of specifications is used, namely the mixed sensitivity design (Skogestad [10]).

In solving the mixed sensitivity problem, techniques are used to minimize the weighted transfer function matrix $T_\mathrm{zw}(s)$:

$$T_\mathrm{zw}(s) = \begin{bmatrix} W_\mathrm{s}(s)S(s) \\ W_\mathrm{T}(s)T(s) \end{bmatrix}$$

This transfer function contains frequency dependent weights $W_\mathrm{S}(s)$ and $W_\mathrm{T}(s)$ have to meet performance requirements on the transfer function matrices, the sensitivity $S(s)$ and the complex

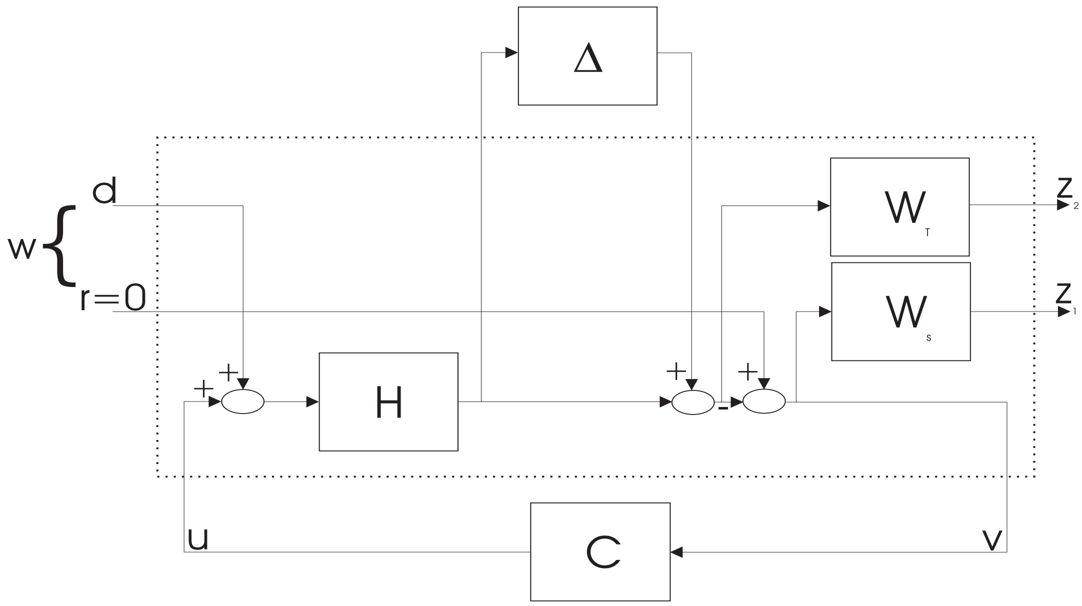

Figure 3. General control configuration

sensitivity $T(s)$. These weights are the control engineer's means for affecting the contribution of $S(s)$ and $T(s)$ over certain frequency ranges in the final design.

The controller is obtained from the generalized plant, so the synthesis problem with this configuration is reduced to the design of a nominal model H and some appropriate weighting matrices which impose the control specifications and knowledge of the system uncertainty. The output multiplicative uncertainty description, $H(s) = H(s)(1 + \Delta(s))$, is used. However, despite the importance of the design of the weighting functions, no unique methodology can be extracted from the literature. Furthermore, most of them confirm that these functions are tuned through a trial and error process.

The steps to design the $\mathcal{H}_\infty$ controller are taken as follows (Ortega *et al.* [5] and Skogestad [10]):

1. choose a nominal model with low order, in this case the transfer matrix H derived in the previous section;
2. estimate the multiplicative output uncertainty(= the percentage of ignorance at each frequency) of the system with respect to the chosen nominal model, using:

$$E(s) = \frac{(P_{\mathrm{j}}(s) - P(s))}{P(s)} \qquad j = 1, 2, ..$$

   with $P_{\mathrm{j}}(s)$ the non-nominal system at the operating point where the controller needs to work properly;
3. design the weighting matrix $W_{\mathrm{T}}(s)$;
4. design the weighting matrix $W_{\mathrm{s}}(s)$;
5. build up the augmented plant $G(s)$ and synthesize a controller;
6. repeat step 4 and 5 until obtaining the desired behavior.

This synthesis of the control has been implemented in Matlab using the μ-analysis toolbox.

4. RESULTS

Closed loop simulations were evaluated with two controller designs, namely the one based on a physical model for the flame behavior (an approximation of equation(9)) and the one derived using the circle fit. The main focus in the control design was disturbance attenuation, which requires that the maximum singular value of $S(s)$ is small. There was also interest in robustness to system parametric variations, so the maximum singular value of $T(s)$ should be small as well. It is well known that achieving simultaneously a small $S(s)$ and $T(s)$ is not possible, since $T(s) + S(s) = 1$. However, disturbances occur at low frequency whereas the effects of model uncertainties become significant at higher frequencies.

The envelopes of the burnt velocity responses are shown for both control configurations in Figure 4. A thermo acoustic instability at the first unstable mode at 227 (Hz) is suppressed by the two controllers. The performance of the controller based on the circle fit is better than for the controller founded on the physical model. This is also demonstrated by calculation of the integral squared error (ISE) of both systems. This error is a measure of the systems performance by integrating the square of the error signal over a fixed interval of time. The ISE value of the controller founded on a circle fit method is lower (0.35%) compared to the physical model-based controller (0.38%).

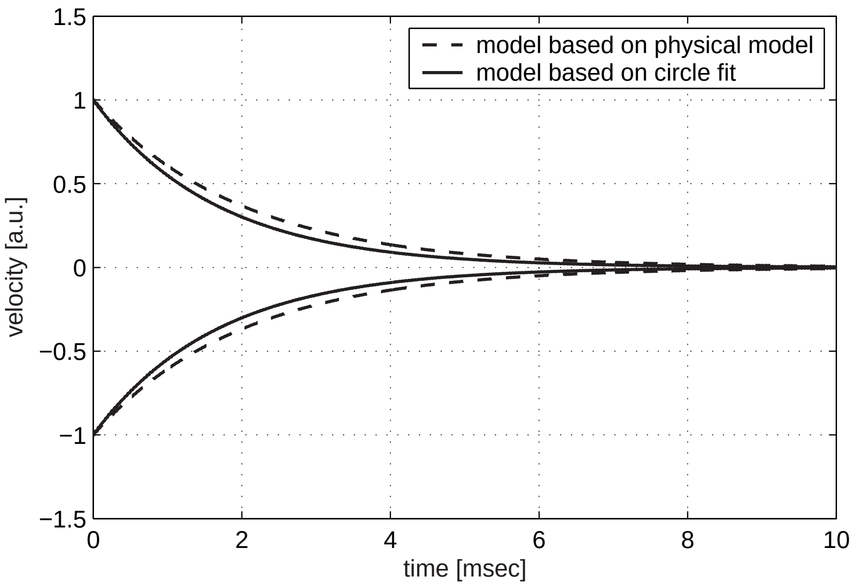

Figure 4. Envelope of the simulation results for both controllers

Note that the two control designs are based on a linear model and are accurate in a small frequency band (in this case around the first mode).

5. CONCLUSIONS

A flame transfer function is derived by making using of a circle fit of the physical models of the flame transfer function. Coupling of this relation, with a representation for the other acoustic elements, resulted in the model for a modified Rijke tube. This transfer matrix is the starting point of the $\mathcal{H}_\infty$ control design. This synthesis is shortly explained. The performance of the controller, that is designed based on the modal analysis, is judged in simulation. Also a comparison has been made with a physical model-based controller based on the integral squared error. It is shown that both controllers are able to suppress the thermo acoustic instability, but the ISE of the model-based controller is higher.

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