ICSV14
Cairns • Australia
9-12 July, 2007

14th International Congress on Sound & Vibration

NONLINEAR SECONDARY PATH MODEL IDENTIFICATION

Jarosław Figwer

Institute of Automatic Control, Silesian University of Technology
Gliwice, Poland
Jaroslaw.Figwer@polsl.pl

Abstract

In the paper a new approach to nonlinear secondary path model identification for feedforward active noise control systems is presented. In the proposed approach the nonlinear secondary path is modeled as Wiener system. The model of Wiener system is identified using multisine random excitation. This excitation used in a specially designed identification experiment allows to decompose Wiener model identification problem into linear dynamic part and static nonlinearity identification problems. Properties of the presented nonlinear secondary path identification method in the case of *off- and on-line* model identification are discussed.

1. INTRODUCTION

In the last two decades, fast development of microprocessor control systems and necessity to control plants in which nonlinearities have a substantial influence on a quality of control contributed to development of nonlinear dynamic model identification methods. Identification methods based on signal processing in the time- and frequency-domain were developed [4], [21]. Special interest has been given to identification of simple nonlinear models, in which nonlinearity of plant is modeled by a static nonlinearity connected in series with a linear dynamic at input (Hammerstein model [2]), at output (Wiener model [3], [15]) or at input and output (Hammerstein-Wiener model [1], [6]). This gives new possibilities of secondary and feedback path modeling for active noise control systems (ANC) that should take into account nonlinear acoustic effects or in which nonlinear actuators are used [5], [7], [8]. Such situation is often met in applications of active noise control techniques in radar, sonar, telecommunication and cryptography signal processing. Additionally, in many cases the secondary path may be time-varying and its model should be identified and updated under operation of ANC system [13], [16].

In the paper a new approach to *off-* and *on-line* nonlinear secondary path model identification based on Wiener system is presented. The proposed approach is based on multisine random excitation [10], [11]. Properties of this excitation and specially designed identification experiment [9] allowed to decompose overall Wiener model identification problem into dynamic part and static nonlinearity identification problems. To identify model of the linear dynamic part static nonlinearity is interpreted as a random disturbance with specific properties. This interpretation allows to identify model of the linear dynamic part using classical methods [19], [22]. A feature of the proposed secondary path identification method that differ its from the literature methods [3], [21] is an easy detection of nonlinearity and its model identification as well as low computational complexity.

2. IDENTIFICATION PROBLEM

The block diagram of an adaptive feedforward ANC system [16], [17], [20] creating for example a local zone of quiet surrounding a single (error) microphone in an enclosure is shown in Fig. 1. The ANC system is working with the sampling interval T. The enclosure is disturbed by a noise that should be reduced using a control loud-speaker. It is assumed that the noise is a zero-mean random process. A reference microphone placed near to the noise source is used to measure the reference signal $x(i)$. The primary path represents an acoustic space between the reference and error microphones. The secondary path is composed of D/A converter, reconstruction filter, amplifier, control loud-speaker and an acoustic space between the loud-speaker and error microphone. It is assumed that the secondary path exhibits nonlinear behaviour. A digital linear filter is used as the compensator $W(z^{-1})$. Its coefficients are tuned on the basis of the error signal *e(i)* and signal $x(i)$ filtered through a nonlinear model of the secondary path. The goal of the adaptation algorithm is to calculate the coefficients of digital filter $W(z^{-1})$ that minimise the mean square value of error signal $e(i)$.

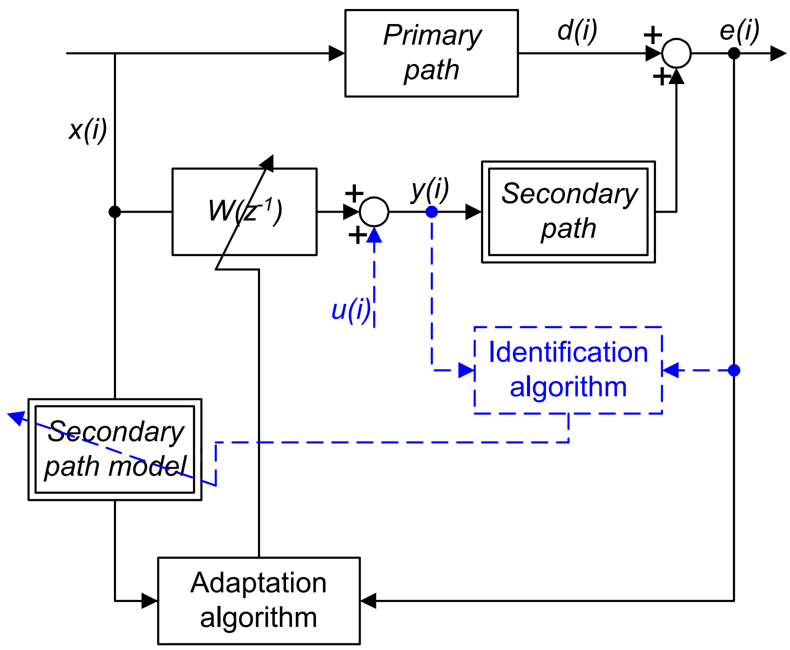

Figure 1. Adaptive feedforward ANC system.

The structure of ANC system implies that there is a need to identify secondary path model *off-line* before activating the ANC system. This model has a great influence on performance of the ANC system and in many cases it should be also identified and updated *on-line* during ANC system operation [16], [18]. The problem of *on-line* secondary path model identification (based on measurements of signals *y(i)* and *e(i)*) with active adaptation algorithm is a closed-loop identification problem with low signal to noise ratio if the ANC system works well [12], [13], [14]. It implies that for *on-line* secondary path model identification an external excitation signal added to the control signal should be used. Its variance must be chosen so as not to decrease radically noise attenuation obtained by the operating ANC system.

In the proposed approach the nonlinear secondary path is approximated by the Wiener system (see Fig. 2), i.e.:

$$e(i) = f(e_l(i)) + d(i), \tag{1}$$

where: i denotes consecutive time instants, $f(.)$ is a real-valued nonlinear function, $e_l(i)$ is the nonmesarueable discrete-time output of linear dynamic part of the Wiener system, and *d(i)* is a zero-mean output disturbance. The linear dynamic part of the Wiener system is described by the following relation:

$$e_l(i) = z^{-k}\frac{B(z^{-1})}{A(z^{-1})}y(i), \tag{2}$$

where: k is the discrete time delay, z^{-1} is a one step shift backward operator, $A(z^{-1})$ and $B(z^{-1})$ are polynomials of orders dA and dB, respectively. In the sequel the discrete-time signal $f(e_l(i))$ is called the noise free Wiener system output and is denoted by $e_n(i)$ (i.e. $e_n(i) = f(y_l(i))$).

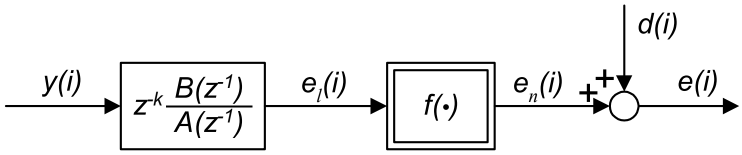

Figure 2. Wiener system.

The aim of secondary path model identification is to determine the structure (k, dA, dB) of the linear dynamic part as well as the corresponding parameter estimates of polynomials $A(z^{-1})$ and $B(z^{-1})$ and estimate of the nonlinear function $f(.)$ based on measurements of signals $y(i)$ and $e(i)$ taken during specially designed identification experiments with multisine random excitations.

It is also worth to note that an inherent feature of the Wiener system is a nonuniqueness: there is no possibility to distinguish amplification of the linear dynamic part and static nonlinearity. To overcome this problem it is additionally assumed that variance of the nonmesarueable output $e_l(i)$ of linear dynamic part is equal to 1.

3. MULTISINE RANDOM EXCITATION

The *N*-sample (*N* even) multisine random excitation [10], [11] is defined in the time-domain by a sum of harmonic sines including a constant component:

$$u(i) = \sum_{n=0}^{N/2} A_n \sin(\Omega n i + \varphi_n), \tag{3}$$

where: $\Omega = \frac{2\pi}{N}$ denotes the relative fundamental frequency, $i = 0,1,..., N-1$ denotes consecutive discrete time instants, A_n are deterministic amplitudes of the sine components, φ_n are phase shifts, of which φ_0 is deterministic ($\varphi_0 = \frac{\pi}{2}$) and the remaining phase shifts are random, independent and:

- uniformly distributed on $[0,2\pi)$ for $n = 1,2,..., \frac{N}{2} - 1$,
- Bernoulli distributed on the set of random events $\{\frac{\pi}{2}, \frac{3\pi}{2}\}$ with probability 0.5 for $n = \frac{N}{2}$.

Spectral properties of the multisine random excitation are uniquely defined by amplitudes A_n ($n = 0,1,..., \frac{N}{2}$) of sine components. Let $\Phi_{ww}(\omega T)$ ($\Phi_{ww}(\omega T) < \infty$) be a function of the relative frequency ωT ($\omega T \in [0,2\pi)$) corresponding to the power spectral density of a wide-sense stationary random process $w(i)$. The choice of amplitudes as the following

- $A_n = 2\sqrt{\frac{\Phi_{ww}(\Omega n)}{NT}}$ for $n = 1,2,..., \frac{N}{2} - 1$,
- $A_n = 2\sqrt{\frac{\Phi_{ww}(\Omega n)}{NT}}$ for $n = 0$ and $n = \frac{N}{2}$

implies that the periodogram of N-sample multisine random excitation is equal to the power spectral density $\Phi_{ww}(\omega T)$ for N equally spaced frequencies from the range $[0,2\pi)$. Multisine random excitations are nongaussian random processes that asymptotically for $N \to \infty$ turn into Gaussian ones.

Lets look at properties of the n-th ($n = 1,2,\ldots, \frac{N}{2}$) sine components

$$u_r^n(i) = A_{r,n} \sin(\Omega n i + \varphi_{r,n}) \tag{4}$$

for two N-sample multisine random excitations $u_r(i)$ ($r = 1,2$) with spectral properties defined by the functions $\Phi^1_{ww}(\omega T)$, $\Phi^2_{ww}(\omega T)$, respectively. Taking into account ensemble averaging it can be noticed that for every N these components are uncorrelated:

$$E\{u_1^n(i) u_2^n(i-\tau)\} = 0 \tag{5}$$

for $\tau = -\infty,...,0,1,...,\infty$. Time-domain averaging for any particular multisine random excitation realisation results in the following crosscorrelation function:

$$R_{u_1^n u_2^n}(\tau) = A_{1,n} A_{2,n} \cos(\Omega n \tau + \varphi_{1,n} - \varphi_{2,n}). \tag{6}$$

It is worth to note that for every *N*:

$$E\left\{R_{u_1^n u_2^n}(\tau)\right\} = E\left\{u_1^n(i)u_2^n(i-\tau)\right\} = 0\,. \tag{7}$$

Additionally, assymptotically for $N \to \infty$:

$$\lim_{N\to\infty} R_{u_1^n u_2^n}(\tau) = 0 \quad a.s. \tag{8}$$

The above properties of the multisine random excitation are the basis of the presented nonlinear secondary path model identification method.

4. MODEL IDENTIFICATION

During identification experiment the *N*-sample multisine random excitation $u(i)$ is periodically repeated. Start of the data acquisition is delayed to the time instant of putting the external excitation into the secondary path (*off-line* model identification) or ANC system (*on-line* model identification) so as all transients caused by inputting excitation become extinct. In the case of *on-line* model identification the excitation $u(i)$ is added to the control signal (see Fig. 1).

In the proposed approach, the secondary path is identified using measurements of signals $e(i)$ and $y(i)$ (in the case of *off-line* model identification $y(i)=u(i)$). The corresponding model is estimated on the basis of *mN*-sample secondary path input data sequence $\{y(0), y(1),..., y(mN-1)\}$ and *mN*-sample output data sequence $\{e(0), e(1),..., e(mN-1)\}$. It is worth to mention that to estimate the secondary path model *on-line* measurements of the external multisine random excitation are not used.

The method of data processing used in the proposed approach is based on the averaged $\bar{e}(i)$ for time instants $i=0,1,...,N-1$ values of secondary path output signal:

$$\bar{e}(i) = \frac{1}{m}\sum_{s=0}^{m-1} e(i+sN) \tag{9}$$

and obtained in the same way averaged values $\bar{y}(i)$ of secondary path input signal in the case of *on-line* model identification or values $u(i)$ of the multisine random excitation in the case *off-line* model identification. The averaged *N*-sample data sequences $\bar{y}(i)$ and $\bar{e}(i)$ ($i=0,1,...,N-1$) may be calculated recursively using an algorithm of *on-line* mean value calculation.

In the case of *off-line* secondary path model identification properties of the multisine random excitation $u(i)$ and equation (1) imply that for linear dynamic part of the Wiener system the corresponding noise free output $e_n(i)$ may be written as:

$$e_n(i) = e_l(i) + v_1(i) + a = z^{-k}\frac{B(z^{-1})}{A(z^{-1})} y(i) + v_1(i) + a \tag{10}$$

where a is a constant value, $v_1(i)$ is a zero-mean random disturbance. This representation of the noise free system output $e_n(i)$ follows from the fact that under steady state conditions the nonmesarueable discrete-time output $e_l(i)$ of the linear dynamic part of the Wiener system is the sum of a multisine random signal and random noise. Its transformation by the nonlinear function $f(.)$ may result in a constant component and additional sine components that are not present in the multisine random excitation $u(i)$. It means that for all frequencies Ωn ($n = 1,2,\ldots,\frac{N}{2}$) additional sine components with phase shifts being a function of the random phase shifts φ_t ($t = 1,2,\ldots,\frac{N}{2}$ and $t \neq n$) may appear. They are represented in the model (10) by the disturbance $v_1(i)$. Independence of the random phase shifts φ_t ($t = 1,2,\ldots,\frac{N}{2}$) implies that the disturbance $v_1(i)$ and multisine random excitation $u(i)$ are uncorrelated.

It follows from the above Wiener system interpretation that estimates of its linear dynamic part model parameters may be calculated using *N*-sample averaged values $\bar{e}(i)$ of secondary path discrete-time output and input $\bar{y}(i) = u(i)$ ($i = 0,1,\ldots,N-1$) using instrumental variable identification method [19], [22]. Taking into account properties (7) and (8) preciseness of the obtained parameter estimates may be increased by increasing the period *N* of multisine random excitation or (and) by averaging estimates obtained for different realizations of *N*-sample multisine random excitation.

In the next step estimates of secondary path linear dynamic part model parameters are used to calculate estimates of the output $e_l(i)$ of this part for the time instants $i = 0,1,\ldots,N-1$. These values together with averaged values $\bar{e}(i)$ $i = 0,1,\ldots,N-1$ values of the secondary path discrete-time output signal can be used to identify the nonlinear function $f(.)$. A plot of $\bar{e}(i)$ versus $\hat{e}_l(i)$ is a good tool that may be used to detect nonlinearities in the secondary path.

Calculation of secondary path model estimates *on-line* using the averaged signals $\bar{y}(i)$ and $\bar{e}(i)$ and its update may be done after each repetition of *N*-sample multisine random excitation. It follows from statistical properties of the noise that the averaged signals $\bar{y}(i)$ and $\bar{e}(i)$ are unbiased and consistent estimators of the noise free steady state secondary path input and output responses for one period of the external multisine random excitation. Their variances decline with the increase of the number *m* of processed data segments.

5. EXAMPLE

The secondary path of an adaptive ANC system creating a local zone of quiet in an enclosure and working with the sampling interval 0.001s was simulated as a Wiener system with the linear dynamic part being a FIR filter (*250* coefficients) and static nonlinearity of the form of a sine function. To identify the secondary path model the *1024*-sample multisine random excitation *u*(*i*) with spectral properties defined by the function $\Phi_{ww}(\omega T) = 1$ ($\omega T \in [0,2\pi)$) was used. The secondary path model was estimated using the proposed approach and *m=1* data segment. In Figs 4 and 5 estimated frequency response magnitude of linear dynamic part of the Wiener system and static nonlinearity are compared with the corresponding characteristics of the simulated secondary path, respectively.

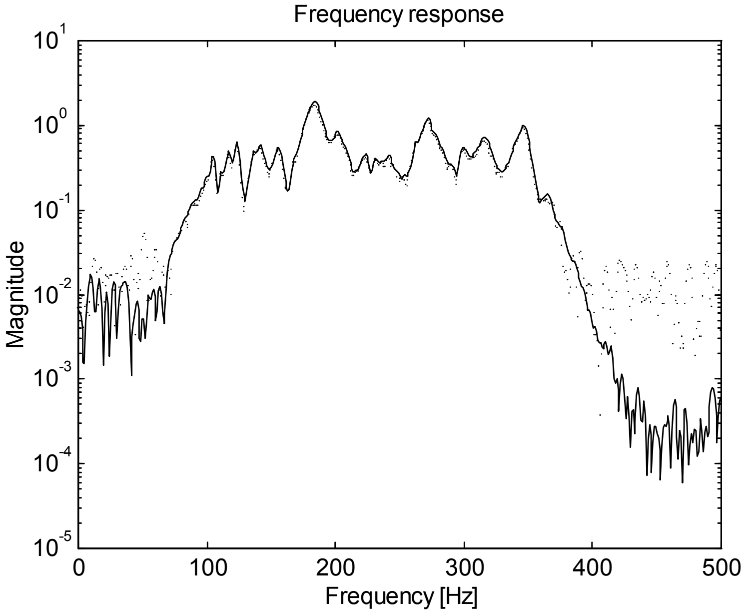

Figure 3. Frequency response magnitude of linear dynamic part of the Wiener system (plant - solid line; identified model - dotted line).

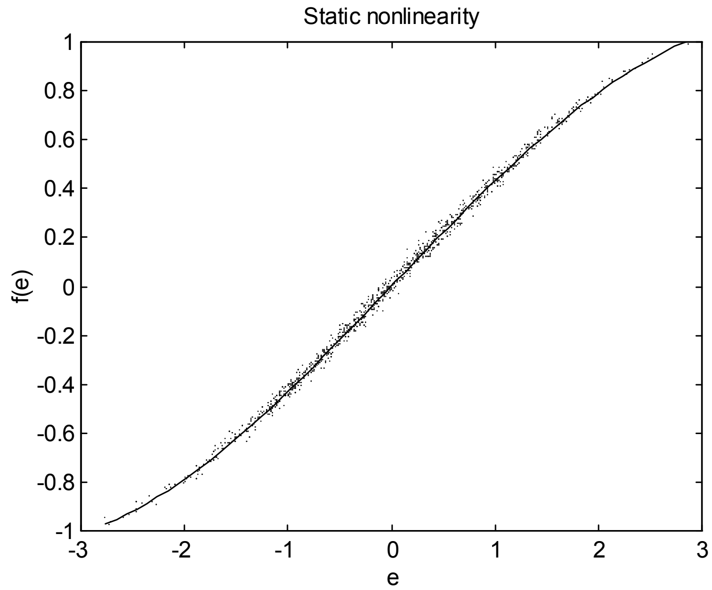

Figure 4. Static nonlinearity (plant - solid line; identified model - dotted line).

6. CONCLUSIONS

In the paper a new approach to *off*- and *on-line* nonlinear secondary path model identification and update under operation of ANC system was proposed. The method uses *N*-sample external random multisine excitation that is periodically repeated. In the proposed approach the nonlinear secondary path is modeled as Wiener system. The multisine random excitation used in a specially designed identification experiment allows to decompose Wiener model identification problem into linear dynamic part and static nonlinearity identification problems.

REFERENCES

[1] E.W. Bai, "A blind approach to the Hammerstein- Wiener model identification", Automatica, **38**, 967-979 (2002).

[2] E. W. Bai, "Identification of linear systems with hard input nonlinearities of known structure", Automatica, **38**, 853-860 (2002).

[3] E.W. Bai, "Frequency domain identification of Wiener models", Automatica, **39**, 1521-1530 (2003).

[4] J. S. Bendat, *Nonlinear Systems Techniques and Applications*, John Wiley & Sons, New York, 1998.

[5] M.H. Costa, J.C.M. Bermudez and N.J. Bershad, "Stochastic Analysis of the Filtered-X LMS Algorithm in Systems With Nonlinear Secondary Paths", IEEE Transactions on Signal Processing, **50,** 1327-1342 (2002).

[6] P. Crama, J. Schoukens, "Hammerstein-Wiener system estimator initialization", Automatica, **40**, 1543-1550 (2004).

[7] V.E. DeBrunner, D. Zhou, "Active Noise Control with Weak Nonlinearities in The Secondary Path", Proceedings 45th Meadwest Symposium on Circuits and Systems, Tulsa (2002).

[8] V.E. DeBrunner, D. Zhou, "Active Noise Control with Certain Nonlinearities in The Secondary Path", Proceedings of the Asilomar Conference on Signal, System and Computer, Asilomar, 2053-2058 (2002).

[9] J. Figwer, "Multisine Excitation for Process Identification", *Archives of Control Sciences,* **5**, 279-295 (1996).

[10] J. Figwer, "A new method of random time-series simulation", *Simulation Practice and Theory*, **5**, 217-234 (1997).

[11] J. Figwer, *Synthesis and Simulation of Random Processes,* Zeszyty Naukowe Politechniki Śląskiej, Seria: Automatyka 126, 1999.

[12] J. Figwer, "Closed-Loop System Identification with Multisine Excitation", Proceedings of the Eighth IEEE International Conference on Methods and Models in Automation and Robotics, Międzyzdroje, Poland, 2-5 September 2002, pp. 477-48.

[13] J. Figwer: "A new method of on-line model identification and update for multichannel active noise control systems", Archives of Control Sciences, **13**, 141-155 (2003).

[14] T. Główka, J. Figwer: "Higher-order spectra for on-line identification of ANC plants", Proceedings of the 13th International Congress on Sound and Vibration, Vienna (2006).

[15] W. Gręblicki, "Nonparametric identification of Wiener systems", IEEE Transactions on Information Theory, **38**, 1487-1493 (1992).

[16] C.H. Hansen and S.D. Snyder, *Active Control of Noise and Vibration*, Cambridge University Press, 1997.

[17] S.M. Kuo, D. R. Morgan, *Active Noise Control Systems. Algorithms and DSP Implementations*. J. Wiley & Sons, Inc., New York, 1996.

[18] S.M. Kuo and D. Vijayan, "A Secondary Path Modeling Technique for Active Noise Control Systems"*, IEEE Transactions on Speech and Audio Processing*, **5**, 374-377 (1997).

[19] L. Ljung, *System Identification - Theory for the User,* Prentice Hall PTR, 1999.

[20] P.A. Nelson, S.J. Eliott, *Active Control of Sound,* Academic Press, London, 1992.

[21] R. Pintelon, J. Schoukens, *System Identification. A Frequency Domain Approach.* IEEE Press, New York, 2001.

[22] T. Söderström, P. Stoica, *System identification.* Prentice-Hall International, 1989.

ACKNOWLEDGEMENTS

The partial financial support of this research by The Polish Ministry of Higher Education and Science under grant 3 T11A 012 29 is gratefully acknowledged.