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ESTIMATION OF THE ACTIVE AND REACTIVE SOUND POWER USING HYPER-MATRICES OF IMPEDANCE

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Abstract

A numerical approach to estimate the active and reactive sound power radiated by a baffled plane piston is presented. The active part of the sound power is related to the generation of sound and the reactive part is associated with the reaction of the medium. Usually, the sound power is estimated by a direct integration of the Rayleigh integral. However, this technique is time-consuming because of the numerical integration of the sound field. Here the vibrating surface is replaced by a number of small equivalent pistons, and the sound power is estimated by means of the radiation impedance matrix of the structure. Since the impedance matrix depends on the two-dimensional geometry and frequency, the matrix can be handled as an hyper-matrix which reduces significantly the computation time. This approach is applied to various piston shapes and the results are compared with traditional methods.

1. INTRODUCTION

The analysis of the sound radiation from a vibrating body is a persistent problem of significant practical importance which has generated a large body of papers in the technical literature. It is identified that sound waves are primarily created by the vibration of physical bodies in contact with a fluid medium, such as the air. The motion of the body causes the air in the immediate vicinity of the body to also move thus generating sound radiation. However, all of the energy produced near the vibrating surface does not flow. In particular, at low frequencies where the wavelength of the sound is much larger than the dimension of the vibrating surface, most of the vibration energy just remains in the vicinity of the vibrating surface (reactive sound power) and never flows. Thus, it simply adds to the mass of the vibrating surface. The efficiency with which the sound flows (active sound power) can be regarded as the fraction of the vibrating energy which actually makes it away from the surface and travels into the air as sound waves. Evidently, this efficiency will depend on issues such as the exact geometrical shape of the vibrating surface and the surroundings of the vibrating surface, among others.

It is well-known that the sound radiation field of a structure vibrating with a specified distribution of normal velocity on its surface may be computed from the Helmholtz integral if the pressure distribution on the surface is known. In special cases of low frequency, high frequency, or particular surface shapes, it is possible to estimate the surface pressures with sufficient accuracy and thus reduce the calculation of the sound pressure field to a simple quadrature over the surface. In more-general cases, the surface pressure may be calculated by numerical solution of a Fredholm integral equation. In principle, Fredholm integral equations can be solved numerically by several methods. In all methods, however, both the effort of the solution and the storage requirements for the computer increase as the square of the number of stations at which the functions are evaluated. In an old paper, Chertock [1] considered surface shapes and velocity distributions with a particular symmetry, in order to minimize the number of stations required and to increase the speed and precision of the calculation.

The problem of sound radiation from a circular plate mounted flush in an infinite or finite baffle was explained by Ginsberg et al. [2]. Integral formulations for the sound radiation efficiency from rectangular plates have been presented by Berry et al. [3]. Rdzanek has presented several theoretical papers [4-8] related to obtaining the complex self power of flat vibrating plates. In general, he uses closed path integral techniques and the Hankel representation of the sound power to derive the radiation efficiency of plates having different geometries and boundary conditions. The technique provides some very precise asymptotic estimates for the sound power of individual modes and mode pair valid for the high frequencies.

On the other hand, Fahnline and Koopmann [9] presented a lumped parameter model for the acoustic radiation from a vibrating structure which was defined by dividing the surface of the structure into elements, expanding the acoustic field from each of the elements in a multipole expansion, and truncating all but the lowest-order terms in the expansion. Later [10], they implemented the model numerically by requiring the boundary condition for the normal surface velocity to be satisfied in a lumped parameter sense. This approach eases the difficulties usually encountered in enforcing the boundary condition, leading to a relatively simple numerical solution with well-defined convergence properties. The basis functions for the numerical analysis were taken as the sound fields of discrete simple, dipole, and tripole sources located at the geometrical centres of the surface elements. In this way, the different source types were used to represent the radiation from different kinds of surface elements: simple sources for elements in the plane of an infinite baffle, dipole sources for very thin structures which deform only in bending, and tripole sources for elements associated with parts of a structure enclosing a finite volume.

In engineering applications the use of numerical procedures assist in solving very complex problems. However, not all numerical methods can be efficient and accurate enough to provide the best solution to the problem. When the velocity distribution on a plane surface is known, the sound radiation characteristics can be found by using an integral expression known as the Rayleigh Integral (which is derived from the Kirchhoff-Helmholtz integral relation). Exact solutions for the sound radiation field are possible only when the surface represents a level value of a coordinate in one of the few coordinate systems in which the wave equation can be separated. Therefore, most of the time a numerical scheme solution should be found. Since the calculation of the total sound power radiated by the vibrating structure requires a second surface integral to be computed, this approach can be very inefficient for certain applications. This fact can be more evident when results are required for a large number of discrete frequencies. At each frequency, a couple of surface integrals must be solved to obtain the final result. In addition, the size of the meshes used to discretize the numerical problem is usually related to the wavelength of the sound so, in the high frequency range, more restrictions are found. In this paper, an alternative to the direct numerical integration is presented. This alternative is based on a matrix approach for the impedance matrix and the discrete volume velocity distribution on the surface.

2. THEORY

The acoustic radiation impedance matrix Z_{ik} corresponds to a transfer function which relates the volume velocities **u** of a vibrating structure to the sound pressures **p** on its surface. It can be defined as

$$\mathbf{p} = \mathbf{Z}\mathbf{u}. \tag{1}$$

The complex matrix $\mathbf{Z} = \mathbf{Z}(\omega)$ is independent of the vibration and depends only on the geometry of the structure. Here ω is the circular frequency. The dimension of the matrix is given by the number of virtual sub-divisions of the structure.

Assuming that the characteristic length of the surface elements is small compared to a typical acoustic wavelength, then the pressure and velocity can be considered constant over each element and can be represented by an average value. Therefore, each element of the matrix is

$$Z_{ik} = \frac{p_i}{u_k} = R_{ik} + jX_{ik}, \tag{2}$$

for all $i, k = 1, 2, ..., N$, where R_{ik} is the resistance matrix, X_{ik} is the reactance matrix, and $j = \sqrt{-1}$.

2.1. Matrix approach for the sound power radiated

The sound power radiated by a vibrating structure is a single global quantity commonly used to characterize its source strength. The time-averaged total complex sound power radiated $\bar{\Pi}_{rad}$ by a baffled plate's surface vibrating at a frequency ω is

$$\bar{\Pi}_{rad} = \frac{1}{2} \iint_S \{p(\vec{r}) V_n(\vec{r})^*\} dS, \tag{3}$$

where S is the total surface area of the plate, $\vec{r}$ are the coordinates of the point on the surface, $p(\vec{r})$ is the complex sound pressure in the near field, $V_n(\vec{r})$ is the normal complex velocity on the plate, and $*$ denotes the complex conjugate. If the sound pressure is estimated from an integral representation, such as the Rayleigh integral, the evaluation of $\bar{\Pi}_{rad}$ requires to solve a quadruple integral.

A matrix approach can be obtained considering that the vibrating structure is divided into N small elements. Then, after the application of the principle of reciprocity to acoustic fields $Z_{ik} = Z_{ki}$, it can be shown that

$$\bar{\Pi}_{rad} = \bar{\Pi}_{act} + j\bar{\Pi}_{rea} = \frac{1}{2} \sum_{i=1}^{N} \sum_{k=1}^{N} Z_{ik} u_i u_k^* = \frac{1}{2} \mathbf{u}^H \mathbf{Z}\mathbf{u}, \tag{4}$$

where $\bar{\Pi}_{act}$ is the active sound power, $\bar{\Pi}_{rea}$ is the reactive sound power, $\mathbf{u}$ is the $N \times 1$ complex volume velocity vector, H denotes the Hermitian (conjugate transpose), and Z_{ik} are the elements of the complex $N \times N$ *acoustic radiation impedance matrix* $\mathbf{Z}$.

Therefore, if the elements of $\mathbf{Z}$ are known, the sound power can be calculated as

$$\bar{\Pi}_{rad} = \bar{\Pi}_{act} + j\bar{\Pi}_{rea} = \frac{1}{2}\mathbf{u}^H \Re\{\mathbf{Z}\}\mathbf{u} + j\frac{1}{2}\mathbf{u}^H \Im\{\mathbf{Z}\}\mathbf{u}. \tag{5}$$

For each element of the matrix Z_{ik}, if $i = k$ it corresponds to the self-impedance and when $i \neq k$ the element is a cross-impedance, which gives a measure of the total acoustic coupling between the ith and kth surface elements. Thus, Eq. (5) provides a convenient method to estimate the sound radiation for any assumed set of structural modes for a sound radiator of arbitrary shape.

2.2. Numerical evaluation of the impedance matrix

As discussed before, the sound power radiated by a baffled vibrating plate can be expressed in terms of the volume velocities of a number of elemental radiators. However, most of the mesh used to discretize the plates are straight-sided geometrical forms and the cross-impedances are given in terms of complex integral forms. However, if the vibrating plane structure (plate) is divided into N small virtual elements, each surface element can be treated as a circular piston having an area equal to that of the corresponding element [11]. Using this approach, the values for the self-impedance and cross-impedance are given approximately by

$$Z_{ii} = \rho_0 c S_i \left[1 - \frac{J_1(2k_0 a_i)}{k_0 a_i} + j\frac{S_1(2k_0 a_i)}{k_0 a_i}\right], \tag{6}$$

and

$$Z_{ik} = \frac{2\rho_0 c k_0^2 S_i S_k}{\pi}\left[\frac{J_1(k_0 a_i)}{k_0 a_i}\frac{J_1(k_0 a_k)}{k_0 a_k}\right]\left[\frac{\sin(k_0 r_{ik})}{k_0 r_{ik}} + j\frac{\cos(k_0 r_{ik})}{k_0 r_{ik}}\right], \tag{7}$$

respectively, for all $i, k = 1, 2, ..., N$, where c is the speed of sound, k_0 is the acoustic wavenumber, r_{ik} is the distance between ith and kth surface elements, S_i and S_k are the surfaces of the equivalent pistons, $a_i = \sqrt{S_i/\pi}$ and $a_k = \sqrt{S_k/\pi}$ are the radii of the pistons, J_1 is the first-order Bessel function and S_1 is the Rayleigh-Struve function. If two pistons of equal area are assumed, it can be seen that an increase in the distance between the pistons results in a decrease in the cross-resistance which adopts more negative values at low frequencies.

3. APPLICATIONS

In order to test the the performance of the matrix method presented above, numerical simulations were carried out for two acoustical applications: 1) the calculation of the mechanical radiation impedance of rectangular and elliptical plane baffled vibrating pistons, and 2) the calculation of active and reactive sound power from the structural axisymmetric modes of circular plates, having simply-supported and clamped boundary conditions. It is worth noticing that, since the matrix $\mathbf{Z}$ depends on the frequency ω, the matrix method can be implemented into a computer code by using hyper-matrices, i.e. multidimensional arrays Z_{ikm} where the subindex m is related to the frequency. This approach reduces significantly the computation time of the whole numerical process.

3.1. Determination of mechanical radiation impedances

For plane pistons, which are often encountered in engineering, the phase angles and the normal component of velocity are constant over the entire vibrating surface, so the vibrating surface is a rigid body. In terms of our matrix approach that means we can consider the volume velocity vector as a constant vector $\mathbf{u} = \alpha[1, 1, 1, ..., 1]^T$, where α is some constant real number.Here, we will assume that the piston is located on a plane which contains a rigid screen of infinite extent so there is no interaction of the acoustic fields on the two sides of the piston. It suffices then to study the field in the semi-space bounded by the screen. The mechanical radiation impedance of a circular piston of radius a has been extensively studied in the literature, and the result is in fact given by Eq. (6). Rectangular and elliptical pistons are more complex problems, so their mechanical radiation impedances were evaluated as an application example of the method presented above. To discretize the pistons $N = 400$ elements were used in the computation. Details on the discretization scheme can be found in ref. [12]. The parameter β is a measure of the aspect ratio and eccentricity for a rectangular and elliptical geometry, respectively. Thus, for a rectangular piston of sides a and b ($a > b$), $\beta = b/a$. For an elliptical piston of semiaxes a and b ($a > b$), $\beta = b/a$. Figure 1 shows the numerical results for rectangular pistons and Fig. 2 shows the same results for an elliptical piston.

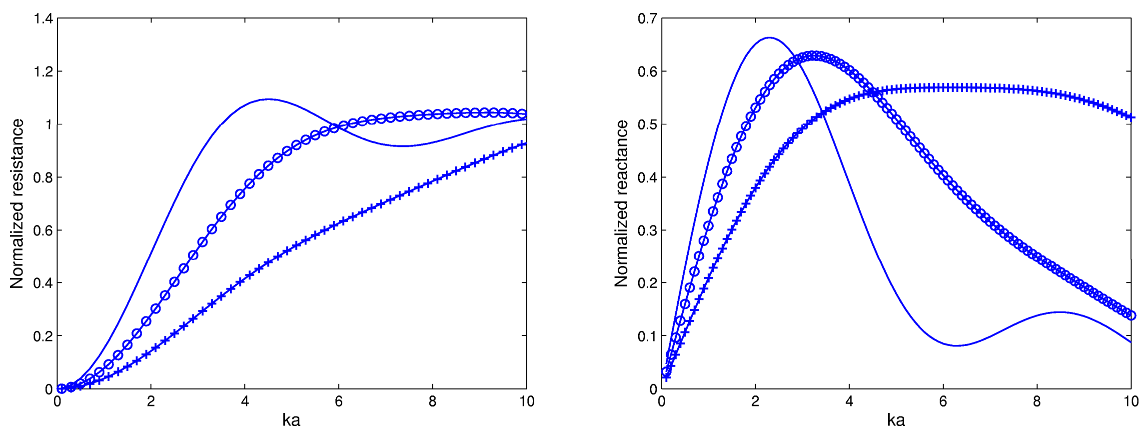

Figure 1. Normalized radiation resistance (left) and reactance (right) for a rectangular piston: $\beta = 1$ (—); $\beta = 0.5$ (–○–○–); $\beta = 0.25$ (+ + +).

Use of the matrix approach gives results quite close to the known radiation impedances for $ka < 4$ [13]. In this region, the results for the real part agree almost perfectly with those derived by Rayleigh radiation integral. However, the results for the imaginary part seem to underestimate the numerical values when compared to the integral approach. This fact is more evident for higher frequencies and it can be due to the degree of accuracy given by Eq. (7). However, from numerical experimentation it has been observed that the error increases linearly as a function of ka, which can be a good starting point for deriving a further correction to the method.

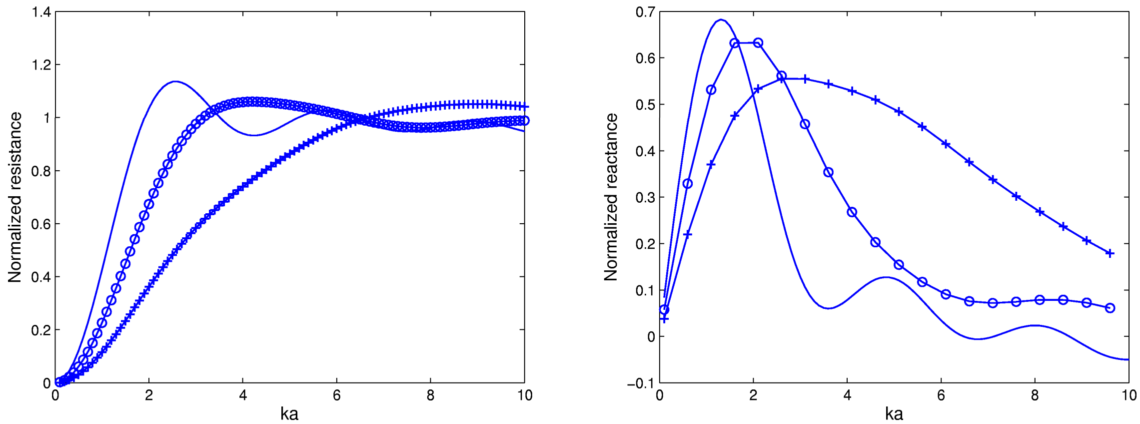

Figure 2. Normalized radiation resistance (left) and reactance (right) for an elliptical piston: $\beta = 1$ (—); $\beta = 0.5$ (–○–○–); $\beta = 0.25$ (+ + +).

3.2. Active and reactive sound power radiated

The active power can be calculated from the real part of the impedance matrix, i.e. from the resistance matrix. This real power is radiated by the vibrating source into the surrounding medium and does not return to the source, if it is not reflected in the space outside the source. On the other hand, a source, apart from the real power, produces the imaginary power, i.e. the reactive power which can be calculated from the imaginary part of the impedance matrix. This reactive power is related to the reaction of the medium as an overall dynamic behaviour of the source. The sound radiation from a plate which is vibrating in a mode of vibration is more complicated than for a piston. Thus, the sound radiation will depend on the manner of fixing the plate, i.e. the boundary conditions, its elastic properties and the frequency of vibrations. Details on the calculation of the surface velocity distribution can be found in ref. [12]. Figures 3 and 4 shows the numerical results of normalized sound power for a circular plate vibrating in an axisymmetric mode having simply-supported and clamped boundary conditions on its perimeter, respectively.

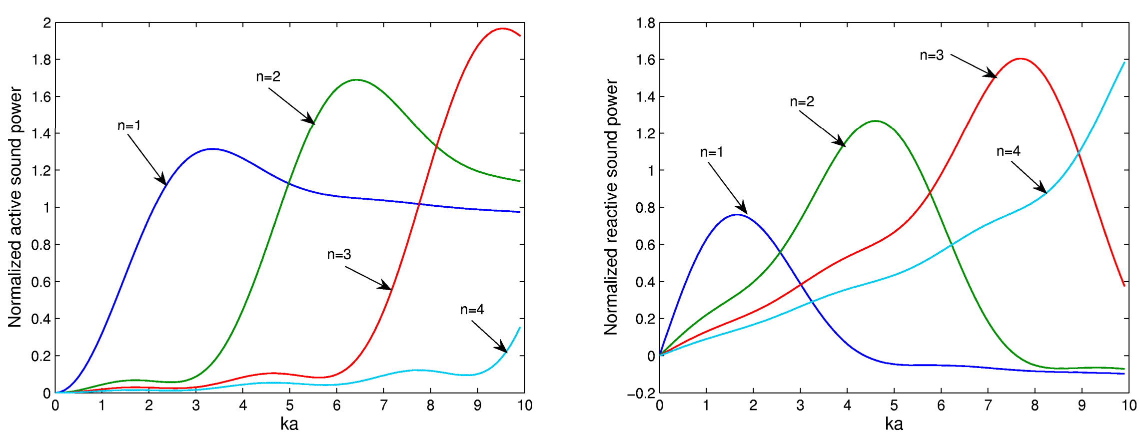

Figure 3. Results of normalized active (left) and reactive (right) sound power for a simply-supported circular plate of radius a for four axisymmetric modes.

The results in Figs. 3 and 4 are in good agreement with those presented by other authors which they determined from integral methods and asymptotic approximations [4]. It can be

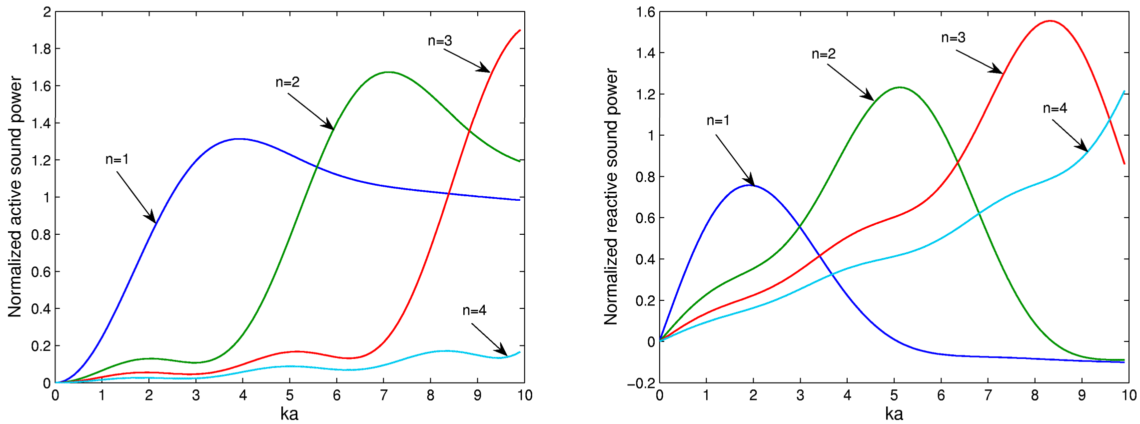

Figure 4. Results of normalized active (left) and reactive (right) sound power for a clamped circular plate of radius a for four axisymmetric modes.

observed that each normalized active sound power curve exhibits a peak, the height increasing with the axisymmetric mode numbers and the efficiency reaches values greater than unity. However, we observe that well above the critical frequency the curves for all the axisymmetric modes do not approach unity as theoretically expected. Clearly, this is because the method is discrete in nature. In order to improve the estimation, a larger number of elementary radiators should be considered, making the process very expensive in computation time. Therefore, for frequencies above the critical frequency, the use of well-known asymptotic approximations of the Rayleigh integral can be more accurate.

4. CONCLUSIONS

The characteristics of the complex sound radiation from some vibrating baffled plane structures have been numerically determined using a method based on hyper-matrices of acoustic impedance. The results show good agreement with previous theoretical and experimental results. The method presented here can be very useful in the modeling of vibro-acoustics problems. When compared with the numerical integration methods, a large reduction of computation time can be achieved. Computations that can take several hours were reduced to some few minutes of CPU time. The method is limited to the frequencies below the critical frequency. For higher frequencies a large number of virtual elements would be needed to obtain accurate prediction. Results obtained for the imaginary part of the radiation impedance and reactive sound power appear to be underestimated by the matrix approach. Further work is needed to make some linear correction to the results so their results can be brought into a better agreement with those obtained by the radiation integrals.

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