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DESIGN ANALYSIS OF A RUBBER MOUNT SYSTEM FOR A PUSH-TYPE CENTRIFUGE

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Abstract

The lack of balance, which occurs in all rotating machines, could cause troublesome vibration if the attachments to the foundation of the machine are unsuitable. This is particularly true for centrifuges, where uneven loading of the basket, for example, due to temporary irregularities in the feed or layers of salt sediment on the basket, may lead to particularly bad balancing. The aim of this investigation was to evaluate the use of rubber mounts and to predict feasible mount stiffness coefficients to be used. These mounts have to produce good vibration isolation to protect the supporting building from dynamic forces transmitted, but must also limit the static and dynamic mount displacements to allow acceptable motion of the centrifuge assembly. It was therefore necessary to use a six-degree of freedom mathematical model, implemented in computer programs to compute these static and dynamic mount displacements and mount forces, for 2 possible designs. An objective function as a measure of vibration transmitted, was also used. In addition, the mount system natural frequencies were also computed, to prevent resonance with the normal basket angular speed and also the forced frequency of the axial pusher mechanism.

1. INTRODUCTION

This investigation regarding vibration isolation was done for centrifuges located at the first level of supporting reinforced concrete building. A six-degree of freedom model was used. A direct approach in terms of forces transmitted to the support structure is considered.

2. MATHEMATICAL MODEL

A mount system model similar to that employed by Nel and Heyns is considered [3], [5], [6]. Hence the centrifuge assembly (centrifuge and base plate) is idealised as a rigid body of mass m attached to a rigid support structure by means of an arbitrary number n of elastic mounts with arbitrary positions and orientations with respect to the centrifuge assembly global co-ordinate system. The origin of the fixed global co-ordinate system *xyz* is located at *g*, the center of gravity (c.g.) of the centrifuge assembly as shown in figure 1.

It is assumed that the stiffness coefficient k_{xi}, k_{yi}, k_{zi} in the three co-ordinate directions are independent of each other if the rotational stiffness of the mount is neglected. For elastomeric materials, as are usually used at rubber mounts, a hysteretic damping model with a complex stiffness matrix is assumed here [2], [4], [5], see also figure 1, so that

$$\begin{bmatrix} k_x(1+j\eta_x) & 0 & 0 \\ 0 & k_y(1+j\eta_y) & 0 \\ 0 & 0 & k_z(1+j\eta_z) \end{bmatrix}_i \tag{1}$$

where $j = \sqrt{-1}$, and η_x, η_y and η_z are the mount loss factors in each of the three co-ordinate directions. A transformation is required to relate the translational displacements of each mount to translational and rotational displacements of the centrifuge assembly c.g. Assuming small centrifuge displacements, G_i is defined by

$$u_i = \begin{bmatrix} \Delta x \\ \Delta y \\ \Delta z \end{bmatrix}_i = \begin{bmatrix} 1 & 0 & 0 & 0 & z & -y \\ 0 & 1 & 0 & -z & 0 & x \\ 0 & 0 & 1 & y & -x & 0 \end{bmatrix}_i = \begin{bmatrix} \Delta x \\ \Delta y \\ \Delta z \\ \Delta\theta_x \\ \Delta\theta_y \\ \Delta\theta_z \end{bmatrix}_g = G_i U_g \tag{2}$$

where u_i is the translational displacement matrix at mounting point *i* and U_g are the centrifuge assembly c.g. displacements. $\Delta x, \Delta y, \Delta z$ are translational and $\Delta\theta_x$, $\Delta\theta_y$, $\Delta\theta_z$ rotational displacements. With *Fi* vector comprising the three forces and the three moments due to mount *i* acting on the centrifuge assembly along *x, y* and *z* and about *x, y* and *z* respectively, it now follows that

$$F_i = -\left[G_i^T \quad K_i \quad G_i\right] U_g \tag{3}$$

Assuming again small displacements, the rigid body equations of motion [1] is

$$M_g \ddot{U}_g = \mathrm{F} \tag{4}$$

with F the matrix of resultant total forces and moments on the centrifuge assembly. The system mass matrix is

$$M_g = \begin{bmatrix} m & 0 & 0 & 0 & 0 & 0 \\ 0 & m & 0 & 0 & 0 & 0 \\ 0 & 0 & m & 0 & 0 & 0 \\ 0 & 0 & 0 & I_{xx} & 0 & 0 \\ 0 & 0 & 0 & 0 & I_{yy} & 0 \\ 0 & 0 & 0 & 0 & 0 & I_{zz} \end{bmatrix} \tag{5}$$

where m is the total centrifuge assembly mass and I_{xx}, I_{yy} and I_{zz} are the elements of centrifuge assembly inertia expressed in terms of the global co-ordinate system.

The centrifuge assembly c.g. acceleration matrix is

$$\ddot{U}_g = \begin{bmatrix} \Delta\ddot{x} \\ \Delta\ddot{y} \\ \Delta\ddot{z} \\ \Delta\ddot{\theta}_x \\ \Delta\ddot{\theta}_y \\ \Delta\ddot{\theta}_z \end{bmatrix} \tag{6}$$

where $\Delta\ddot{x}$, $\Delta\ddot{y}$, $\Delta\ddot{z}$ are translational accelerations and $\Delta\ddot{\theta}_x$, $\Delta\ddot{\theta}_y$, $\Delta\ddot{\theta}_z$ angular accelerations. Adding together the effects of all the mounts of the centrifuge assembly, it follows that

$$\begin{aligned} \sum_{i=1}^{n} F_i &= - \left[\sum_{i=1}^{n} G_i^T K_i G_i \right] U_g \\ &= - K_g U_g \end{aligned} \tag{7}$$

where K_g is is a complex dynamic matrix which includes hysteretic loss effects at the centrifuge c.g. From this and equation (4) it now follows that

$$M_g \ddot{U}_g + K_g U_g = F_{ge} \tag{8}$$

where F_{ge} represents all forces and moments on the centrifuge assembly other than the mount reaction forces. In order to take advantage of the frequency domain approach, F_{ge} is assumed to comprise of v sinusoidal forces and moments included by $\overline{F}_{dh}$ with corresponding frequencies ω_h and phase angles $a_{h,h=1,2,\ldots,v}$. It may then be shown that

$$\overline{U}_g = \sum_{h=1}^{v} \left[K_g - \omega_h^2 M_g \right]^{-1} \left[\overline{F}_d \right]_h \tag{9}$$

where $\overline{U}_g$ and $\overline{F}_{dh}$ represent displacement and force phasors respectively [2].

The centrifuges assembly shaking forces and moments are

$$F_{ge} = \begin{bmatrix} F_{xe} \\ F_{ye} \\ F_{ze} \\ M_{xe} \\ M_{ye} \\ M_{ze} \end{bmatrix} = \begin{bmatrix} 0 \\ F_{ye} \\ F_{ze} \\ M_{xe} \\ M_{ye} \\ M_{ze} \end{bmatrix} \tag{10}$$

The model may be employed to determine an objective function. The dynamic displacements at each mount in the x, y and z directions are computed in the time domain. It is thus possible to compute for each force component at each mount the average value of the sum of the dynamic forces at each time increment *dt* over the period *T*. As criterion of vibration transmitted, an objective function is defined by

$$\varphi = \sqrt{\left| \frac{1}{T}\int_{O}^{T}\left\{\sum_{i=1}^{n}\left(f_{xi}^{2}(X)+f_{yi}^{2}(X)+f_{zi}^{2}(X)\right)\right\}dt \right|} \tag{11}$$

in which X is a vector of design variables chosen here to comprise stiffness coefficients. It is now required to minimize with respect to variables X, subject to inequality constraints

$$w = \left| \overline{U}_s + \max(\overline{U}_d) \right| - \overline{U}_c \leq 0 \tag{12}$$

where $\overline{U}_c$ is a vector of specified maximum acceptable mount translational displacement amplitudes. The maximum dynamic translational displacement elements in $\overline{U}_d$ are in the same direction as the static displacement elements in $\overline{U}_s$. These constraints also give an indication of acceptable centrifuge assembly motion and acceptable mount displacements.

3. CHARACTERISATION OF PARAMETERS

The manufacturer supplied the centrifuge and base plate mass and centre of mass properties. These properties were used in the computation of the moments of inertia for the total centrifuge and base plate assembly for each global axis. The total principal moments of inertia are 5272.7, 6395.1 and 7468.5 kgm^2 corresponding to the centrifuge global x, y and z axes as shown in figure 1. The total centrifuge assembly mass is 9800 kg. The global co-ordinates for each of the mounts and those for the basket centre were determined. The centrifuge assembly is supported by 4 mounts positioned at the following global co-ordinates:

Table 4. Mount global co-ordinates [m].

Mount	x	y	z
1	0.55	0.8	-0.795
2	-0.496	0.8	-0.795
3	-0.496	-0.8	-0.795
4	0.55	-0.8	-0.795

The global co-ordinates of the basket centre are:

Table 5. Basket centre global co-ordinates [m].

x_c	y_c	z_c
-0.925	0.000	0.420

The basket radius is 400 mm and the greatest possible excess mass acting on this radius is 2.3 kg (provided by the manufacturer). The normal running speed of the basket is 1200 rev/min.

The stiffness coefficients of the 2 existing very soft pipe compensators used were neglected, because these magnitudes were regarded as very small compared to the stiffness coefficients of typical rubber mounts. The dynamic mount stiffness coefficients of standard mounts available were chosen for 2 different possible designs A and B as shown in Table 6.

Table 6. Mount stiffness coefficients [kN/m].

Mount stiffness coefficients [kN/m]	Design A (Resonance)	Design B (Feasible)
$k_{x1}, k_{x2}, k_{x3}, k_{x4}$	12000	1000
$k_{y1}, k_{y2}, k_{y3}, k_{y4}$	12000	1000
$k_{z1}, k_{z2}, k_{z3}, k_{z4}$	15330	4560

Since mount loss factors in the range of 0.05 to 0.15 do not appreciably affect the dynamic behaviour, and rubber mounts normally have mount loss factors in this range, mount loss factor values of 0.1 were assumed for all the mounts in all 3 orthogonal directions.

4. COMPUTER IMPLEMENTATION

The mathematical model was implemented in computer programs in a MATLAB environment. The magnitudes of the characteristics of the parameters described above were used as input data. The centrifuge assembly shaking forces and moments were computed first, and then used in the computation of dynamic and static mount displacements, dynamic and static mount forces transmitted to the support structure through the mounts and also objective function values.

5. CENTRIFUGE ASSEMBLY SHAKING FORCES AND MOMENTS

The magnitudes of the parameters required for computation of the centrifuge assembly shaking forces and moments and mathematical equations implemented in computer programs were also used, to compute these forces and moments.

Figure 2 shows this force and moment time histories for the centrifuge angular speed at 1200 rev/min. The frequency observed from these wave forms, corresponds to the centrifuge basket angular speed. These time domain forces and moments were Fourier analysed by using the MATLAB fft.m function, and the Fourier coefficients at corresponding frequencies and phase angles were then used in the computation of the responses in the x, y and z directions at each mount by using the frequency domain computer program (see for example figure 3). Table 7 shows the force and moment amplitudes at phase angles. The force magnitude of the axial pusher mechanism was considered relatively small compared to centrifugal forces, and therefore neglected. The magnitude of this very low forced frequency force is important, and was taken into account when the mount system natural frequencies were evaluated (see Table 9), to avoid resonance.

Table 7. Shaking force [N] and moment [Nm] amplitudes at phase angles [rad] and angular speed of 125.46 rad/s.

	F_{ye}	F_{ze}	M_{xe}	M_{ye}	M_{ze}
Amplitudes [N] and [Nm]	1452.8	1452.8	6101.8	1343.8	1343.8
Phase angle [rad]	0	-1.5708	0	1.5708	-3.1416

6. RESULTS AND EVALUATION

As criterion or measure of vibration transmitted to the support structure, the objective function (equation 11) was used. This single objective function value was then computed for each of the 2 different designs A and B and also a rigid mounted system design, as shown in Table 8 for a basket speed at 1200 rev/min. Large objective function values indicate bad vibration isolation, and lower objective function values improved vibration isolation [5], [6].

Table 8. Objective function [kN] at 1200 rev/min.

	Design A (Resonance)	Design B (Feasible)	Rigid design
Objective function ψ [kN]	1320.8	47.01	570.03

By using the MATLAB eig.m function, the eigenvalues and eigenvectors were also computed for the 2 different designs A and B. Table 9 shows these natural frequencies and mode shapes. The stiffness coefficients used for designs A and B are shown in Table 6. These natural frequencies are important in order to avoid resonance with the basket normal angular speed (20 Hz), but also with the axial pusher mechanism force frequency (1 to 1.25 Hz).

Table 9. Natural frequencies [Hz] and corresponding mode shapes.

	Natural frequencies [Hz] and mode shapes					
Design A	5.41 x	7.65 y	12.20 θ_z	12.59 z	16.78 θ_y	20.00 θ_x
Design B	2.45 x	2.87 θ_y	3.52 θ_z	5.83 y	6.88 z	8.39 θ_x

7. CONCLUSIONS

The use of the very low stiffness coefficients renders the best vibration isolation, but has the largest static mount displacements. The use of very low stiffness coefficients also results in low system natural frequencies, which could easily correspond to the pusher mechanism force frequency (1 to 1.25 Hz), thus resonance. The use of very low stiffness coefficients also results in unacceptable centrifuge assembly motion, thus large mount dynamic displacements.

Design A with the moderately high stiffness coefficients renders the poorest vibration isolation (see Table 8 for comparison of objective function magnitudes), but also has smaller static mount displacements compared to design B, but results in the largest dynamic mount displacements. This vibration isolation is unacceptable. The use of higher stiffness coefficients in general tends to create natural frequencies which could easily correspond, or be near to the centrifuge basket operation speed, see also Table 9, which is the case for design A. When one of the mount system natural frequencies corresponds to the forced basket frequency, then resonance takes place, which is undesirable and dangerous and could lead to failures and mount durability problems. The objective function used has the largest values when resonance takes place (see Tables 8 and 9), which indicates that its use is effective for the measure of vibration transmitted.

Design B renders the best compromise between vibration isolation and acceptable mount displacements. The natural frequencies of design B are also far away enough from the basket's normal operational speed, and also the forced frequency of the pusher mechanism (see Table 9). The vibration isolation obtained for design B is also significantly better compared to a design where mount stiffness coefficients were chosen to represent a centrifuge which is rigidly attached (see Table 8 for comparison of objective function magnitudes).

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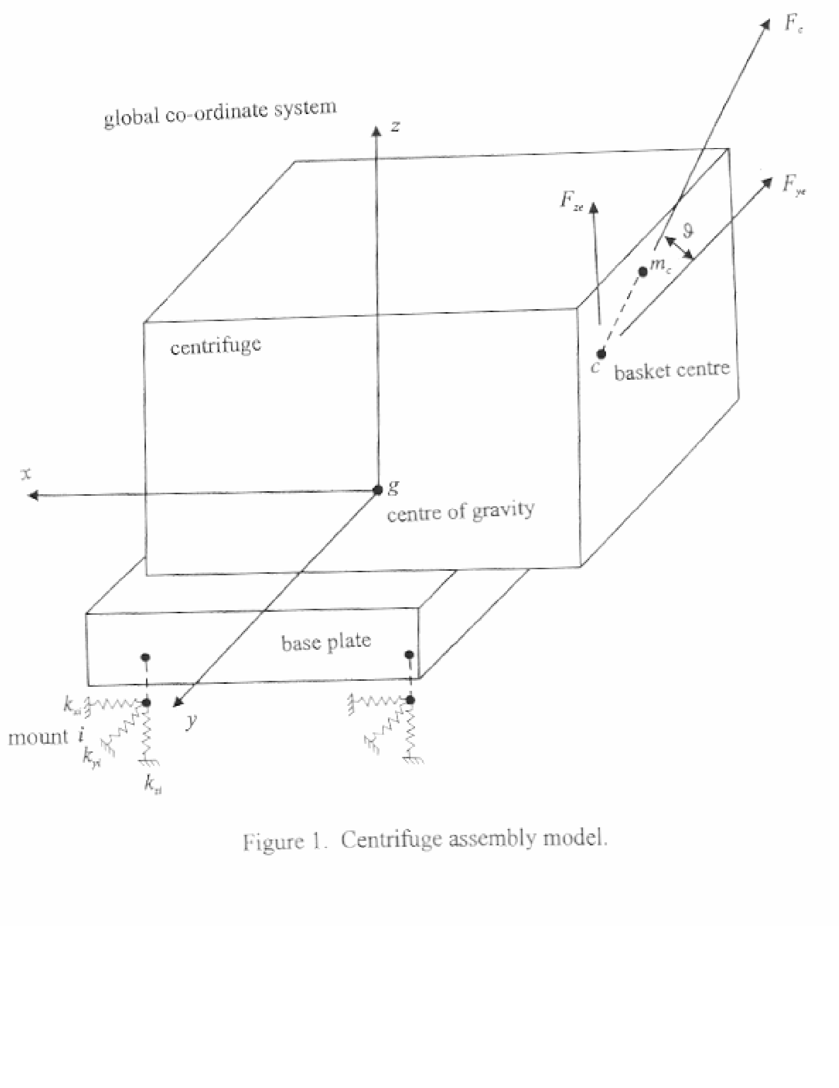

Figure 1. Centrifuge assembly model.

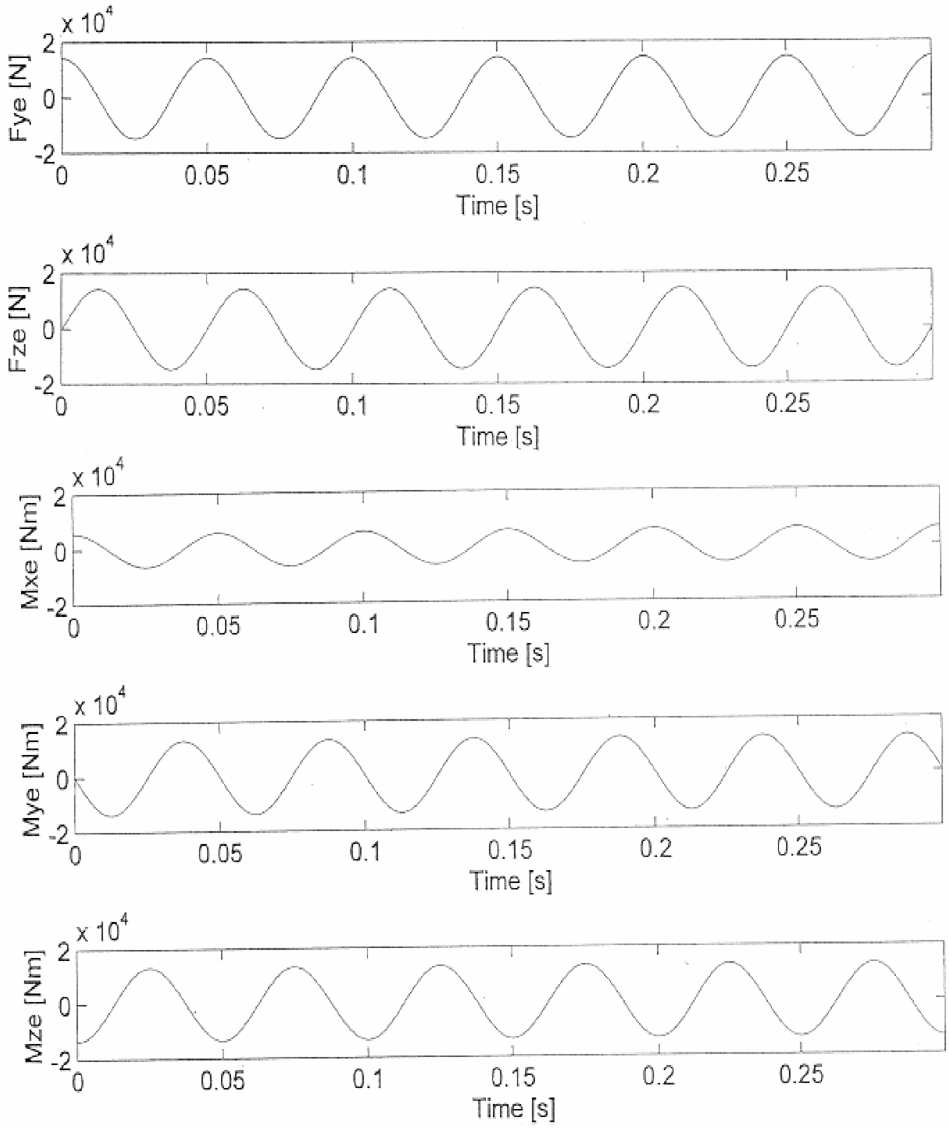

Figure 2. Shaking forces F_{xe} and F_{ye} moments M_{xe}, M_{ye} and M_{ze} in the time domain.

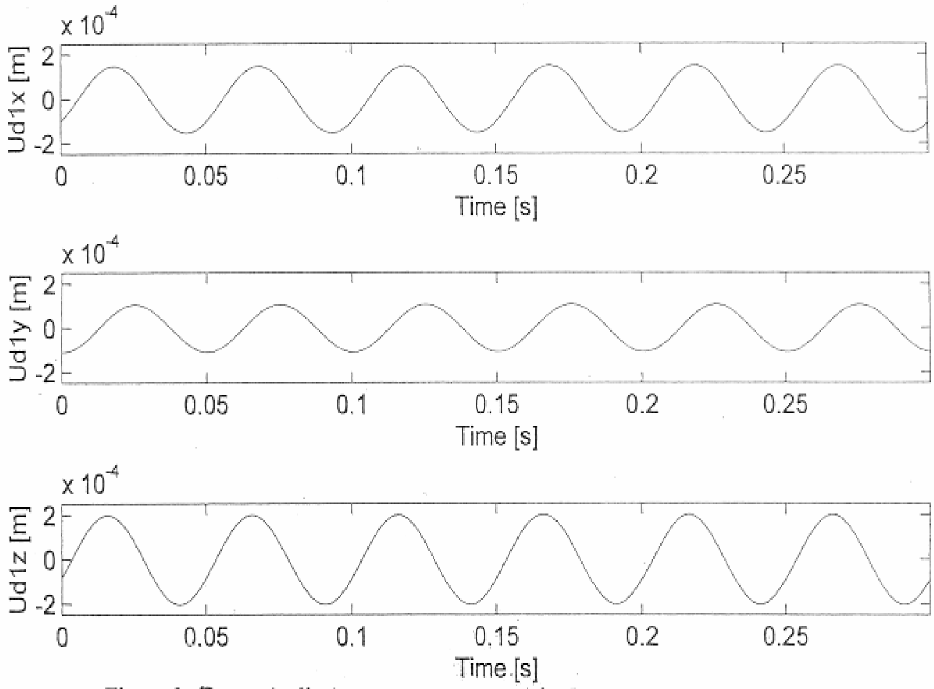

Figure 3. Dynamic displacements at mount 1 in the x, y and z directions.