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OPERATIONAL MODAL ANALYSIS IN PRESENCE OF UNKNOWN VARYING HARMONIC FORCES

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Abstract

In many applications of operational modal analysis one is confronted with unknown harmonic forces. These harmonic components are usually clearly present in the measured response data. The classical approach to deal with such data consists in eliminating these disturbing components from the data. Several techniques can be used to do so. However, when the frequency of these harmonic components varies during the measurements, this is not always obvious. In this contribution, an alternative approach will be proposed to deal with harmonic disturbances. This approach is based on parametrically identified multivariable transmissibilities.

1. INTRODUCTION

Recently, a new approach to identify modal parameters from scalar transmissibility measurements has been proposed. By combining (scalar) transmissibility measurements under different loading conditions, it has been shown in [1] that the model parameters can be identified from transmissibilities. Classical output-only techniques often require the operational forces to be white noise. This is not necessary for the proposed transmissibility-based approach. The unknown operational forces can be arbitrary (colored noise, swept sine, impact, ...) as long as they are persistently exciting in the frequency band of interest.

When several uncorrelated operational forces are acting on the structure, the scalar transmissibility approach is still applicable although a multivariable approach is expected to result in more accurate models. In the present paper the multivariable approach will be presented that can be used to derive modal parameters from arbitrary operational forces — including operational forces that contain important (time varying) harmonics components. These operational forces should be persistently exciting in the frequency band of interest.

2. EXPERIMENTAL AND OPERATIONAL MODAL ANALYSIS

During the last decade modal analysis has become a key technology in structural dynamics analysis [2, 3, 4]. Experimental modal analysis (EMA) identifies a modal model, $[H(\omega)]$, from the measured forces applied to the test structure, $\{F(\omega)\}$, and the measured vibration responses $\{X(\omega)\}$,

$$\{X(\omega)\} = [H(\omega)]\{F(\omega)\} \tag{1}$$

with

$$[H(\omega)] = \sum_{m=1}^{N_m} \frac{\{\phi_m\}\{L_m\}^T}{i\omega - \lambda_m} + \frac{\{\phi_m\}^*\{L_m\}^H}{i\omega - \lambda_m^*} \tag{2}$$

and

$$\lambda_m = -\sigma_m + i\omega_{dm} \tag{3}$$

The modal model (2) expresses the dynamical behavior of the structure as a linear combination of N_m resonant modes. Each mode is defined by a damped resonant frequency, $f_{dm} = \omega_{dm}/2\pi$, a damping ratio, $\zeta_m = \sigma_m/|\lambda_m|$, a mode shape vector, $\{\phi_m\}$, and a modal participation vector, $\{L_m\}$. These modal parameters depend on the geometry, material properties and boundary conditions of the structure.

More recently, system identification techniques were developed to identify the modal model from the structure under its operational conditions using output-only data [5, 6, 7, 8]. These techniques, referred to as operational modal analysis (OMA) or output-only modal analysis, take advantage of the ambient excitation due to e.g. traffic and wind. During an EMA, the structure is often removed from its operating environment and tested in laboratory conditions. The laboratory experimental situation can differ significantly from the real-life operating conditions. An important advantage of OMA is that the structure can remain in its normal operating condition. This allows the identification of more realistic modal models for in-operation structures. Frequency-domain output-only estimators start from power spectra. It can be shown that — assuming the operational forces to be white noise sequences — the power spectrum matrix or covarinace matrix, $[S_X(\omega)] = \text{cov}(\{X(\omega)\})$, safisfies

$$\begin{aligned}[S_X(\omega)] = \sum_{m=1}^{N_m} \frac{\{\phi_m\}\{K_m\}^T}{i\omega - \lambda_m} &+ \frac{\{\phi_m\}^*\{K_m\}^H}{i\omega - \lambda_m^*} \\ &- \frac{\{\phi_m\}\{K_m\}^T}{i\omega + \lambda_m} - \frac{\{\phi_m\}^*\{K_m\}^H}{i\omega + \lambda_m^*}\end{aligned} \tag{4}$$

with $\{K_m\}$ the operational participation vectors, which depend on the modal participation vector, $\{L_m\}$, and the power spectrum matrix of the unknown operational forces.

In the next section the scalar transmissibility approach will be briefly recapitulated. This approach does not require the operational forces to be white noise sequences.

3. RECAP OF SCALAR TRANSMISSIBILITY APPROACH

In general, it is not possible to identify modal parameters from transmissibility measurements. Scalar transmissibilities are obtained by taking the ratio of two response spectra, i.e. $T_{or}(\omega) = \frac{X_o(\omega)}{X_r(\omega)}$. By assuming a single force that is located in, say, the input degree of freedom (DOF) k, it is readily verified that the transmissibility reduces to

$$T_{or}(\omega) = \frac{X_o(\omega)}{X_r(\omega)} = \frac{H_{ok}(\omega)F_k(\omega)}{H_{rk}(\omega)F_k(\omega)} = \frac{N_{ok}(\omega)}{N_{rk}(\omega)} \triangleq T^k_{or}(\omega) \tag{5}$$

with $N_{ik}(\omega)$ and $N_{jk}(\omega)$ the numerator polynomials occurring in the transfer-function models $H_{ok} = \frac{N_{ok}(\omega)}{D(\omega)}$ and $H_{rk} = \frac{N_{rk}(\omega)}{D(\omega)}$. Note that the common-denominator polynomial, $D(\omega)$, which roots are the system's poles, λ_m, disappears by taking the ratio of the two response spectra. Consequently, the poles of the transmissibility function (5) equal the zeroes of transfer function $H_{rk}(\omega)$, i.e. the roots of the numerator polynomial $N_{rk}(\omega)$. So, in general, the peaks in the magnitude of a transmissibility function do not at all coincide with the resonances of the system.

Another important observation is that the transmissibility function is deterministic: the (stochastic) force disappears by taking the ratio in (5). The power spectra that are used in the traditional operational modal analysis approaches remain stochastic (the random contribution of the force is still present). To obtain smooth power spectra measurements averaging techniques are requires.

Making use of the modal model (6) between input DOF, k, and, say, output DOF, o,

$$H_{ok}(\omega) = \sum_{m=1}^{N_m} \frac{\phi_{om}L_{km}}{i\omega - \lambda_m} + \frac{\phi^*_{om}L^*_{km}}{i\omega - \lambda^*_m} \tag{6}$$

one concludes that the limit value of the transmissibility function (5) for the Laplace variable s (replace $i\omega$ by s in (6)) going to the system's poles, λ_m, converges to

$$\lim_{s\to\lambda_m} T^k_{or}(s) = \frac{\phi_{im}L_{km}}{\phi_{jm}L_{km}} = \frac{\phi_{im}}{\phi_{jm}} \tag{7}$$

and is independent of the (unknown) force at input DOF k. Consequently, the substraction of two transmissibility functions with the same output DOFs, (o, r), but with different input DOFs, (k, l) satisfies

$$\lim_{s\to\lambda_m} \left(T^k_{or}(s) - T^l_{or}(s)\right) = \frac{\phi_{om}}{\phi_{rm}} - \frac{\phi_{om}}{\phi_{rm}} = 0 \tag{8}$$

To sum up, the system's poles, λ_m, are zeroes of the rational function $\Delta T^{kl}_{or}(s) \triangleq T^k_{or}(s) - T^l_{or}(s)$, and, consequently, poles of its inverse, i.e.

$$\Delta^{-1}T^{kl}_{or}(s) \triangleq \frac{1}{\Delta T^{kl}_{or}(s)} = \frac{1}{T^k_{or}(s) - T^l_{or}(s)} \tag{9}$$

4. MULTIVARIABLE TRANSMISSIBILITY APPROACH

When several (uncorrelated) operational forces are exciting the structure, the scalar transmissibility is in general not deterministic anymore. Indeed, when e.g. 2 operational forces are present, (5) becomes

$$T_{or}(\omega) = \frac{X_o(\omega)}{X_r(\omega)} = \frac{H_{ok}(\omega)F_k(\omega) + H_{ol}(\omega)F_l(\omega)}{H_{rk}(\omega)F_k(\omega) + H_{rl}(\omega)F_l(\omega)} \tag{10}$$

and the forces $F_k(\omega)$ and $F_l(\omega)$ cannot be eliminated anymore. To do so, a multivariable transmissibility approach is required. Starting from the transfer function matrix, which related the reference output spectra, $\{X_R(\omega)\}$, and the remaining output spectra, $\{X_O(\omega)\}$, to the operational forces, $\{F^K(\omega)\}$, with $K = \{k, l, \ldots\}$ a set representing the operational force locations,

$$\{X_R(\omega)\} = [H_R^K(\omega)]\{F^K(\omega)\} \tag{11}$$

$$\{X_O(\omega)\} = [H_O^K(\omega)]\{F^K(\omega)\} \tag{12}$$

it is readily shown that

$$\{X_O(\omega)\} = [H_O^K(\omega)][H_R^K(\omega)]^{-1}\{X_R(\omega)\} \tag{13}$$

and

$$[T_{O,R}^K(\omega)] = [H_O^K(\omega)][H_R^K(\omega)]^{-1} = [N_O^K(\omega)][N_R^K(\omega)]^{-1} \tag{14}$$

with $[H_R^K(\omega)] = [N_R^K(\omega)][D(\omega)]^{-1}$, $[H_O^K(\omega)] = [N_O^K(\omega)][D(\omega)]^{-1}$ so-called Right Matrix-Fraction Descriptions (RMFD). One notice that — in analogy with scalar transmissibilities — also here, the common denominator matrix, $[D(\omega)]$, disappear in (14).

5. ESTIMATING MULTIVARIABLE TRANSMISSIBILITIES

Transmissibilities can be obtained using non-parametric estimators (e.g., the H_1 estimator in analogy with FRF measurements) or by means of parametric estimators. First, it is shown that the non-parametric estimation of multivariable transmissibilities is an ill-posed problem for lightly-damped structures: the multivariable transmissibilities can become singular at frequencies close to the resonances. Indeed, the H_1 estimate of $[T_{O,R}(\omega)]$ is given by

$$[\hat{T}_{O,R}(\omega)] = [G_{X_O X_R}(\omega)][G_{X_R X_R}(\omega)]^{-1} \tag{15}$$

Close to a resonance frequency, the response of the system is proportional to the corresponding mode shape. This imply that the auto-power matrix $[G_{X_R X_R}(\omega)]$ could become singular close to (lightly-damped) resonant frequencies. This is certainly true when a large number of reference outputs is required.

To circumvent this problem, a parametric frequency-domain identification approach starting directly from input and output spectra [3] can be used to obtain multivariable transmissibilites in a well-conditioned way. The reference outputs, $\{X_R(\omega)\}$, are used as input spectra, and $\{X_O(\omega)\}$ are the output spectra.

Another asset of the parametric approach is that only one measured sequence of the outputs is need to derive the multivariable transmissibility matrix; the non-parametric approach requires at less N_r (uncorrelated) sequences with N_r the number of reference signals (which should be equal to the number of uncorrelated operational forces) in order to be able to solve to non-parametric least-squares problem.

6. DERIVING SYSTEM POLES FROM MULTIVARIABLE TRANSMISSIBILITY FUNCTIONS

In [1] it has been shown that (scalar) transmissibility function under different loading conditions are identical in the system poles (see Section 3). For lightly damped systems, this means that the transmissibility functions cross each other at the resonant frequencies of the system. This is illustrated in Figure 1(a) for scalar transmissibilities. This property is verified for multivariable transmissibility functions in Figure 1(b). It turns out that this property is not valid anymore for multivariable transmissibilities: they do NOT cross each other at the resonant frequencies of the system! To understand why this is not the case, let us try to find what is the relationship

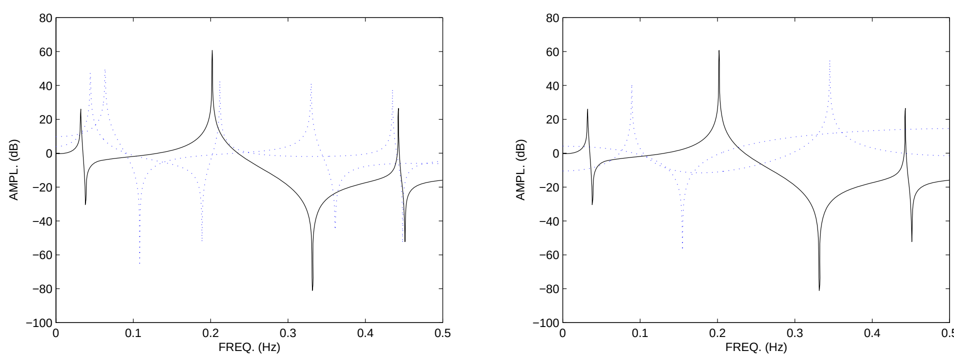

Figure 1. (a) Left - Scalar transmissibilities (dotted lines) for different loading conditions cross at the resonances frequencies. (b) Right - Multivariable transmissibilities (dotted lines) for different loading conditions do not cross at the resonances frequencies. The solid line is a frequency response function.

between scalar and multivariable transmissibility functions. For simplicity, 2 references will be considered, $R = \{1, 2\}$, and 1 output, $O = \{3\}$,

$$X_3(\omega) = T^K_{3,1}(\omega)X_1(\omega) + T^K_{3,2}(\omega)X_2(\omega) \tag{16}$$

Dividing by $X_1(\omega)$ yields,

$$\frac{X_3(\omega)}{X_1(\omega)} = T^K_{3,1}(\omega) + T^K_{3,2}(\omega)\frac{X_2(\omega)}{X_1(\omega)} \tag{17}$$

where $\frac{X_2(\omega)}{X_1(\omega)} \triangleq t^K_{2,1}(\omega)$ and $\frac{X_3(\omega)}{X_1(\omega)} \triangleq t^K_{3,1}(\omega)$ are scalar transmissibility functions corresponding with a load case $K = \{k, l\}$; $T^K_{2,1}(\omega)$ and $T^K_{3,1}(\omega)$ are the multivariable transmissibilities. This can be reformulated as

$$\begin{bmatrix} -T^K_{3,2}(\omega) & 1 \end{bmatrix} \begin{Bmatrix} t^K_{2,1}(\omega) \\ t^K_{3,1}(\omega) \end{Bmatrix} = T^K_{3,1}(\omega) \tag{18}$$

If for instance the amplitudes of the 2 operational forces in points $K = \{k, l\}$ are varied, the multivariable transmissibilities remain unchanged but the scalar ones will change. However, all possible scalar transmissibilities that can be generated by the 2 operational forces in points $K = \{k, l\}$ have to satisfy (18). A unique solution can be obtained by considering a second load case L, e.g. $L = \{l, m\}$. This unique solution can be obtained by solving

$$\begin{bmatrix} -T^K_{3,2}(\omega) & 1 \\ -T^L_{3,2}(\omega) & 1 \end{bmatrix} \begin{Bmatrix} t^{K,L}_{2,1}(\omega) \\ t^{K,L}_{3,1}(\omega) \end{Bmatrix} = \begin{Bmatrix} T^K_{3,1}(\omega) \\ T^L_{3,1}(\omega) \end{Bmatrix} \tag{19}$$

resulting in $\begin{Bmatrix} t^{K,L}_{2,1}(\omega) \\ t^{K,L}_{3,1}(\omega) \end{Bmatrix}$. By considering a third loading condition M, a second pair of "virtual" scalar transmissibilities, $\begin{Bmatrix} t^{K,M}_{2,1}(\omega) \\ t^{K,M}_{3,1}(\omega) \end{Bmatrix}$, can be derived, e.g.,

$$\begin{bmatrix} -T^K_{3,2}(\omega) & 1 \\ -T^M_{3,2}(\omega) & 1 \end{bmatrix} \begin{Bmatrix} t^{K,M}_{2,1}(\omega) \\ t^{K,M}_{3,1}(\omega) \end{Bmatrix} = \begin{Bmatrix} T^K_{3,1}(\omega) \\ T^M_{3,1}(\omega) \end{Bmatrix} \tag{20}$$

These "virtual" scalar transmissibilities (that are derived from multivariable transmissibilities) have to cross each other again in the system poles (because in the system poles the scalar transmissibilities are uniquely determined by the mode shapes [1], and so, also the "virtual" scalar transmissibilities have to be unique in the system poles). From these "virtual" scalar transmissibilities it is possible to derive "virtual" frequency response functions (in analogy with the scalar transmissibility approach [9])

$$[H]^{K,L|K,M}(\omega) = \begin{bmatrix} t^{K,L}_{2,1}(\omega) & t^{K,M}_{2,1}(\omega) \\ t^{K,L}_{3,1}(\omega) & t^{K,M}_{3,1}(\omega) \end{bmatrix}^{-1} \tag{21}$$

The poles estimated from these "virtual" frequency response functions, $[H]^{K,L|K,M}(\omega)$, correspond with the exact system poles. In Figure 3 the stabilization chart obtain from frequency response functions (classical EMA approach) is compared with the stabilization chart obtain from "virtual" frequency response functions (transmissibility-based OMA approach). One can observe that the proposed output-only technique yields stabilization charts that are comparable with the one derived from FRF measurements. The classical OMA approach could not be apply because the considered operational forces contain time-varying harmonic components, and thus, the white noise assumption is severely violated here.

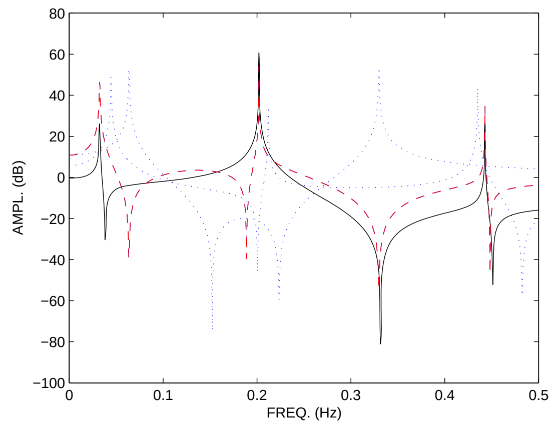

Figure 2. Solid line: a frequency response function. Dotted lines: two “virtual” scalar transmissibilities. Dashed line: a “virtual” frequency response function.

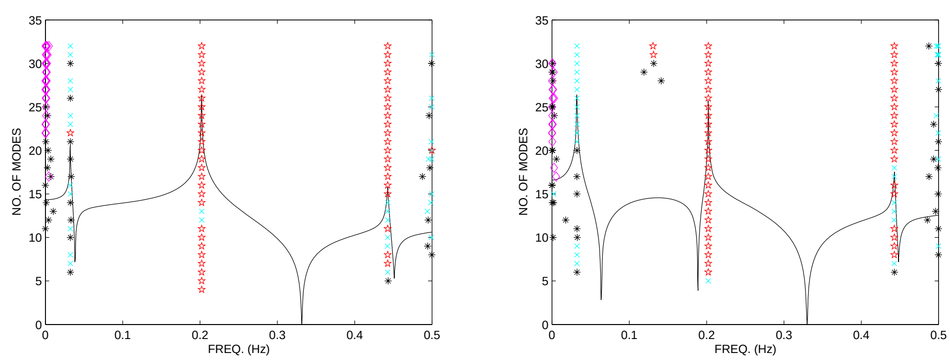

Figure 3. (a) Left - Stabilization chart obtain from a frequency response function. (b) Right - Stabilization chart obtain from a “virtual” frequency response function.

7. CONCLUSIONS

It has been shown in this paper that correct system’s poles can be identified starting from multivariable transmissibility measurements. The theoretical results are verified by means of simulation data.

Classical output-only techniques often require the operational forces to be white noise. This is not necessary for the proposed transmissibility-based approach. In this paper arbitrary unknown operational forces are considered that can contain (time-varying) harmonic components. The classical approach to deal with such data consists in eliminating these disturbing harmonic components from the data. Several techniques can be used to do so. However, when the frequency of these harmonic components varies during the measurements, this is not always obvious. In this contribution, an alternative approach has be proposed that can handle such time-varying harmonic disturbances.

A parametric approach is suggested in this paper because less measurements are required to identify the multivariable transmissibility functions compared to a non-parametric approach. This parametric approach is generally better numerically conditioned, less measurememts are required, and it is possible to compensate for leakage errors by including transient polynomials in the parametric model [3, 8].

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