ICSV14
Cairns • Australia
9-12 July, 2007
14th International Congress on Sound & Vibration

IMPLEMENTATION OF A HYBRID MODEL FOR PREDICTION OF SOUND INSULATION OF FINITE-SIZE MULTILAYERED BUILDING ELEMENTS

Selma Kurra

Department of Architecture, Acoustics Unit, Bahcesehir University
Cıragan Cad. Osmanpasa Mektebi Sokak No: 4 – 6
Besiktas, Istanbul 34100 Turkey
skurra@bahcesehir.edu.tr

Abstract

In building acoustics, it is necessary to have a reliable prediction model to obtain the sound transmission loss of layered structures especially for the building facades exposed to noise. Traditional models are not very much applicable in most of the cases, since the building elements are multilayered structures comprised of different layer combinations with various physical characteristics, such as solid homogenous layers, porous or foamed material, elastic damping layers, etc. The previous model that has been developed by integrating the impedance approach for infinite-size layered systems and the windowing technique for finite-size elements, was computerized as a time efficient program. The predicted results were confirmed by the laboratory experiments as satisfactory for use in insulation practice for acquiring the 1/3 octave band sound transmission losses as well as the rating values; Rw(C; Ctr). This paper describes the model briefly and refers to some implementations in search for the effects of particular layer parameters.

1. INTRODUCTION

Structures composed of several layers made of different materials are used for noise control in industry and for the building elements comprising various protective layers and lightweight panels. The multilayered elements revealing a great diversity of physical and constructional characteristics in buildings with their nature like isotropic or orthotropic, viscous-damping, fibrous and poro-elastic porous layer, rigid (stiff) (with high stiffness-to-weight ratio like fibre-reinforced plastics) and non-rigid (flexible), plane or corrugated, with rigid or non-rigid layer connections etc., enhance the complexity regarding their acoustical and structural behaviours. Airborne sound transmission through these systems has been dealt with in the literature however some of the theoretical models are difficult to generalize for practical applications in building elements.

The physical parameters, like mass, density, size, thickness, stiffness, elasticity, flow resistivity, porosity, loss factor, Poisson ratio, etc. highly influence the total sound transmission coefficient of these composite elements, types and numbers of connections between layers and size (dimensions) of the panels. Since values of these parameters remain within the definable ranges for the building materials it might be possible to suggest their optimum values to lead to a design criteria. However this study should be based on an accurate and reliable prediction model that would be applicable to various types of constructions within a desired layer composition.

Considering the above target, this paper describes a computer model in connection with the former study of the author et al [1], its validation through the former experimental study and implementations in practice.

2. DESCRIPTION OF THE HYBRID MODEL FOR COMPUTATION OF SOUND TRANSMISSION LOSSES OF MULTILAYERED STRUCTURES

The prediction model is based on two approaches: A. The impedance approach for infinite size elements, B. The finite-size effect through the spatial windowing technique.

2.1 Impedance model for infinite size elements

The impedance approach was implemented by Au and Byrne to calculate the insertion losses of the multilayered elements and the results were experimentally validated by using the intensity technique [2]. This approach is capable of applications for the building elements both in the direct field and the diffuse field. The multilayered (lagging) structures consisting of various materials, such as hard, hard-damped with an elastic layer, porous layer and air gap can be taken into account in calculation of sound transmission losses. The boundary conditions of the model of which the two dimensional plane wave model (on x-y plane) is used, are: Wave number component parallel to the panel surface is the same in all of the layers. Acoustical pressure and the particle velocity at the interfaces of the layers are continuous. Layers are uniform having arbitrary thickness. Element is infinite on x-z plane. Number of layers (n) is infinite. Both sides of the element is in contact with air. However the model does not take into account the size of the elements. The major parameters in calculations are; complex characteristic impedance of the layers, complex bending stiffness and complex wave impedances at interfaces. The method is based on the complex wave impedance ratio of both sides of each layer according to the direction of sound intrusion and the ratio of the complex sound pressures. The impedances at input and terminating sides are matched at the interfaces between the layers (Figure 1). The relationship between the transmitted and the input pressures is given below:

$$p_I = p_T \frac{Z_I}{Z_T} \quad N/m^2 \tag{1}$$

p_I, p_T represent the sound pressures at source and receiver sides of the wall. The input impedance Z_I for each layer is equal to the summation of the terminating impedance Z_T and the separating impedance Z_s:

$$Z_I = Z_T + Z_s \quad N.s/m^3 \tag{2}$$

Z_I and Z_T values are the complex impedances of the input and transmitted sides of the element. Z_s depends on the complex bending stiffness of the layer, ω and k which are the wave speed ($2\pi f$) and wave number ($2\pi f/c$), ρ_s; the layer surface mass, kg/m^2. As known, B is related to the thickness, Young's modulus, Poisson ratio and the loss factor of the layer. For the isotropic plates; Z_s (separation impedance) and Z_I are given below [2] :

$$Z_s = j(\omega\rho_s - Bk_x^4/\omega) \tag{3}$$

$$Z_I = Z_T + \left[j(\omega\rho_s - \frac{1}{\omega}(B_x k_x^4 + 2B_{xy} k_x^2 k_y^2 + B_y k_y^4)\right] \quad (4)$$

k_y, k_x are the complex wave numbers, at x and y direction (Fig.1) and B_{xy} is the complex bending stiffness.

For impervious solid plates attached with a secondary layer, such as a damping material, the composite bending stiffness and the composite loss factor are calculated by specifying the physical parameters of this additional layer. For composite structures which include porous material in the cavity, the complex characteristic acoustical impedance of the porous layer as a function of the flow resistivity, is necessary to use and Z_I is calculated by inserting the complex wave number (k_a), the complex characteristic acoustical impedance (Z_A) of the bulk material and the thickness of the porous layer into the equations. The equations for the porous layer computations are presented in [1-2].

Complex acoustical impedances at the receiver side, Z_I and sound pressure at the transmitted side, p_T are computed by inserting the thickness of the porous layer (h_p). The resultant sound transmission coefficient is calculated as:

$$\tau_{(\varpi,\theta)} = \frac{P_T(1)^2}{(P_{source})^2} \quad (5)$$

In diffuse field conditions, the incident sound field is assumed that the plane waves are incident on the element surface from all directions with equal probability (0-90^o). Thus, the total sound transmission coefficient is calculated by integrating the results over the range of incident angles of sound waves as given below [3]:

$$\tau_{\text{diffuse}}(\omega) = \frac{\int_0^{\theta_{\lim}} \tau(\omega,\theta)\sin\theta\cos\theta\, d\theta}{\int_0^{\theta_{\lim}} \sin\theta\cos\theta\, d\theta} \quad (6)$$

$\boldsymbol{\tau(\omega,\theta)}$:Sound transmission index (coefficient) for each angle of incidence (θ). The maximum range of the incident angle (θ_{lim}) depends on the diffuse field conditions. Sound transmission

$$R_{\text{diffuse}}(\omega) = 10 \log \frac{1}{\tau_{\text{diffuse}}(\omega)} \quad dB \quad (7)$$

loss and its spectral profile for the diffuse field is calculated by the well known formula:

2.2 Improved model for finite-size structures (Spatial Windowing Technique)

The above model was improved for the finite-size elements based on the Spatial Windowing Technique for the finite-size elements that was developed by Villot et al. [4]. The model involves with calculating the vibration velocity field of the infinite structure and then spatially windowing this field before calculating the radiated field (Figure 2). Then the radiated sound power is obtained from the wavenumber spectrum of the velocity field by using the spatial Fourier transform. The radiation efficiency associated with spatial windowing is given below [4] by using the dimensions of the structure, L_x and L_y:

$$\sigma(k_p,\psi) = \frac{S}{\pi^2}\int_0^{k_a}\int_0^{2\pi} \frac{1-\cos((k_r\cos\phi - k_p\cos\psi)L_x)}{[(k_r\cos\phi - k_p\cos\psi)L_x]^2} \cdot \frac{1-\cos((k_r\sin\phi - k_p\sin\psi)L_y)}{[(k_r\sin\phi - k_p\sin\psi)L_y]^2} \frac{k_a k_r}{\sqrt{k_a^2 - k_r^2}}\, d\phi \cdot dk_r \quad (8)$$

k_p, k_a, k_r and ϕ: the wave number components in the polar coordinate system (Figure 3).

The resultant airborne sound transmission coefficient for the finite element is calculated as:

$$\tau_{\text{finite}}(\theta,\psi) = \tau_{inf}(\theta,\psi)[\sigma(k_a \sin\theta,\psi)\cos\theta]^2 \tag{9}$$

$\tau_{\text{inf}}(\theta,\psi)$; Sound transmission coefficient for the infinite structure, ψ; direction angle (from 0 to 2π) and θ; incidence angle of plane wave on the surface of the element. The transmission coefficient in a diffuse field is expressed as:

$$\tau_{\text{diffuse/finite}} = \frac{\int_0^{\theta_{\lim}} \int_0^{2\pi} \tau_{\text{finite}}(\theta,\psi) \sin\theta \cos\theta \, d\psi \, d\theta}{\int_0^{\theta_{\lim}} \int_0^{2\pi} \sin\theta \cdot \cos\theta \, d\psi \, d\theta} \tag{10}$$

The above spatial windowing technique has easily been constructed over the existing impedance model to determine sound transmission loss values of finite-size multilayered structures. Thus transmission index for the infinite structures, τ_{inf} (θ) is acquired from the impedance model and the resultant diffuse field transmission index is calculated by using the above equations for the structures whose dimensions are given.

3. DEVELOPMENT OF F MULAY DESCRIPTION OF THE COMPUTERIZED MODEL : F-MULAY

Since the model including a complex calculation procedure for both infinite and finite elements involve rather time-consuming processes by using the Matlab [37] due to the complex numbers and iterations, a computer program F-MULAY was developed to increase the computation efficiency and to facilitate the parametric studies. The algorithm on which the model was based consists of discrete functions for input impedances, output pressures and radiation efficiency. It gives the normal, oblique and diffuse sound transmission losses at octave, third octave and narrow bands within the range of 63 Hz - 6300 Hz as well as the single-number ratings (i.e., weighted sound reduction index: R_w ($C;C_{tr}$). The outputs are presented both in the form of tables and graphs. It is possible to obtain both the infinite and finite size TL values. A database for the physical characteristics of various building materials has been organized within the model. The program code (programming language) is C++ and MS Visual Basic which is used to build the user interface on the Windows operating system.

4. VALIDITATION STUDY

The predicted results obtained by employing this model were checked to search the compatibility with the original data presented by Au and Byrne and a complete match was found when similar material characteristics and the limiting angle of incidence (90^0) were used in the calculations [1]. The model has been verified by the basic double wall theories as of the modal shape in terms of mass-air-mass resonance frequencies (f_r), critical frequencies of layers (f_c), resonance modes as nodes and antinodes (f_{dm} and f_{dn}) and cross-cavity resonance (f_d) and the critical frequencies. Figure 3 gives the narrow band *TL*s for a sample structure revealing the complete agreement between the calculated modal behaviour of the infinite panels.

The numerical results from the infinite size-version of the model were also compared with the experimental data obtained through a former study of the author et al [2]. The acoustical tests that were performed at the IITRI Riverbank Acoustical Laboratories have provided sufficient amount of experimental data through various multilayered structures. The testing procedure, equipment and laboratory facilities were described in detail [1]. The test outputs are compared with the predicted data for both infinite and finite-size elements by considering the aspects to be taken into account in such comparisons with the laboratory data.

Total of 28 wall samples were used in the experimental study and the two specimen size 11 m^2 and 2.9 m^2. The wall samples are: isotrophic panels as single and in multilayered constructions: Steel plate with vinyl layer, double panels with identical layers with varying cavity thicknesses and with and without a porous material in the cavity and; various multilayered combinations (gypsum board, steel plate, vinyl layer, airgap and glasswool). The physical properties of the materials used in the calculations corresponding to the test materials are given in Table 1. The comparisons were made both in terms of the third octave bands and R_w rating units to provide sufficient evidence regarding the accuracy of the model especially between 125-3150 Hz. Some examples regarding the comparisons are given below.

1. The predictions at the third octave bands for the multilayered structures regardless of size revealed divergences from the lab data -as shown in Figure 4- especially at higher frequencies. This is a typical case due to the radiation efficiency of finite-size elements.
2. By taking into account the size of the element and employing FMULAY, the calculated results for double gypsum walls with air gap, could yield a perfect agreement with the measured results almost all through the frequency range (Figure 5). When the porous material is inserted into the cavity, the compatibility is again satisfactory except at the critical frequency where the dips on the calculated *TL* profile are more pronounced than which could be expected (Fig 6). The two-layered steel panel with an airgap in two widths, 5 and 10 cm, reveals a satisfactory compatibility to the measured data except at very low (<100 Hz) frequency (Figure 7). A multilayered wall sample in smaller size is given in Figure 8. The layer composition contains a steel plate damped with a vinyl layer, a gypsum board and glasswool of 10cm filled into the cavity. The result of the comparison displays a similar tendency as described above except at the frequencies above 2500 Hz.

As a result, for very low frequencies (i.e. at 100 Hz) the range of validity is limited due to the unreliability upon the measured results in the laboratory. However, the weighted insulation rating for the building elements is not influenced unless special concern exists regarding the lower frequencies.

5. IMPLEMENTATION OF THE MODEL: EFFECT OF POROUS LAYERS

As an implementation of the computerized hybrid model for calculation of sound transmission loss (*TL*) of multilayered elements, the effect of flow resistivity (R_1) of porous layer was investigated for various materials which are commonly used in buildings such as light and dense concrete, brick, plywood, aluminium and steel panels. Flow resistivity of rockwool was changed between 500-20000 Ns/m^4. By taking the limiting angle of sound incident as 89^0 , the model was employed in the narrow frequency bands with 10 Hz resolution to examine the resonance modes, as well as in 1/3 octave bands.

Figures 9 and 10 give two samples of the predicted narrow band *TL*’s. However it should be noted that the results indicate steeper *TL*’s at high frequencies since the element is accepted as infinite in line with the objective of this study. As a result, the effect of flow resistivity of the porous material seems to be inversely related to sound transmission losses at very low frequencies, i.e. the *TL* decreases while R_1 increases up to 2000 N.s/m^4, while at mid and higher frequencies, the identical and non-identical layered walls reveal different behaviours against the variation of the flow resistivity. In general, the modes occurred for the air cavity walls are suppressed when a porous layer is inserted in the cavity. The effect of flow resistivity is more significant for the double walls consisting of identical layers made of materials with lower density or thickness, whereas it is less significant when the layers are not identical and when at least one layer is relatively thicker and/or denser.

6. CONCLUSION

Below conclusions can be drawn from the validation study for the hybrid model determining sound transmission losses of finite-size multilayered structures:

- Based on the narrow band analyses performed for the structures consisting of up to 4 layers, the impedance model has been proved to have sufficient accuracy in terms of the modal behaviour of the entire structure as well as of the independent layers.
- The impedance model for infinite-size elements gives satisfactory agreement with the experimental data for the single layered elements in standard size of 10 m^2, however for the infinite-size multilayered elements a great discrepancy exists in compliance with the experimental results. When the windowing technique was combined with the impedance model, the experimental verification was proved to be more successful according to the results of various comparisons both in the frequency domain *TL*s and the insulation rating values.
- It was revealed that the physical parameters i.e. limiting angle of incidence, density, elasticity modulus, loss factor and flow resistivity, play an important role in the predicted sound transmission losses which are rather sensitive even to minor change in their values.
- F-MULAY is rather time-efficient program and facilitates the parametric studies because of the Windows interface. Therefore the computerized model can be useful for the problems in building acoustics, as well as in the optimization studies.

REFERENCES

[1] S. Kurra and D., Arditi, Determination of sound transmission loss of multilayered elements, Part 1: Predicted and measured results, International Journal on Acoustics: Acta Acustica, S.H.Verlag, 87 n.5 (2001) 582-592

[2] A.C.K. Au and. K.P. Byrne, On the insertion losses produced by acoustic lagging structures which incorporate flexurally orthotropic impervious barriers, Acustica 70, (1990) 284-291.

[3] L.Beranek, I.L. Ver, Noise and Vibration Control Engineering, Principles and Applications, John Wiley and Sons Inc. 1992

[4] M. Villot, C.Guigou and L. Gagliardini, Predicting the acoustical radiation of finite size multilayered structures by applying spatial windowing on infinite structures, Journal of Sound and Vibration, 245 (3), (2001) 433-455.

[5] Z. Maekawa and P. Lord, Environmental and Architectural Acoustics, Chapter 5, E&FN Spon, 1994

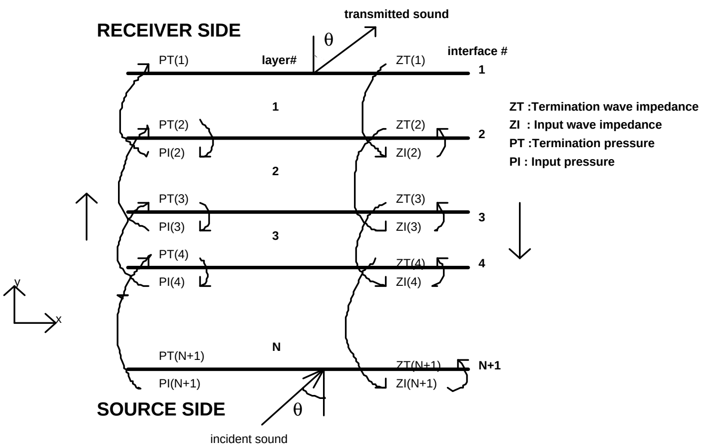


Figure 1. Calculation process of the impedance model for multi-layered elements (Arrows indicate the order of computations)

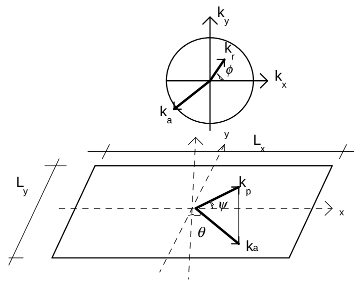


Figure 2. Wave parameters in windowing technique [4]. a) Wave number components contributing to radiated power in solid angle. b) Parameters relative to incident wave on a finite element.

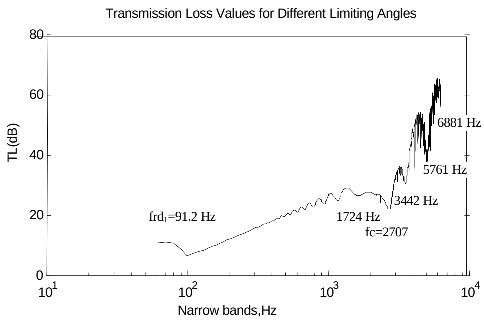


Figure 3. Predicted narrow band TL for a multilayered structure by using the infinite model MULAY and the calculated modes according to the basic theories. (Double gypsum board with 10 cm airgap)

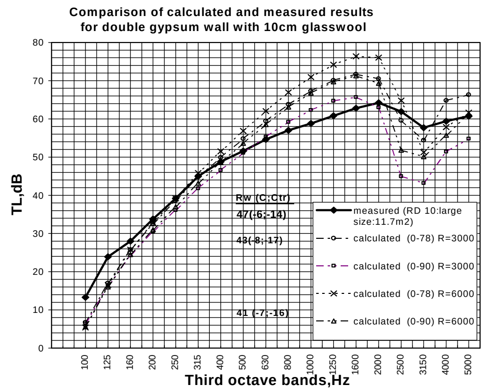


Figure 4. Comparison of calculated (as infinite) and the measured results for double gypsum board with 10cm glasswool (R: flow resistivity, angles of incidence are shown in the legend) [1]

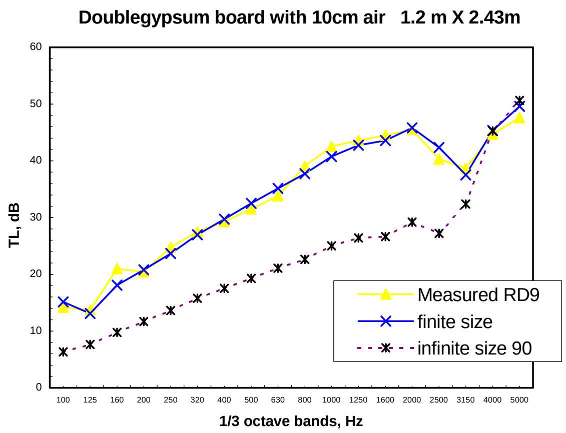


Figure 5. Comparison between the measured and calculated data for double gypsum board with 10cm air in small size: 1.2 mX2.43m

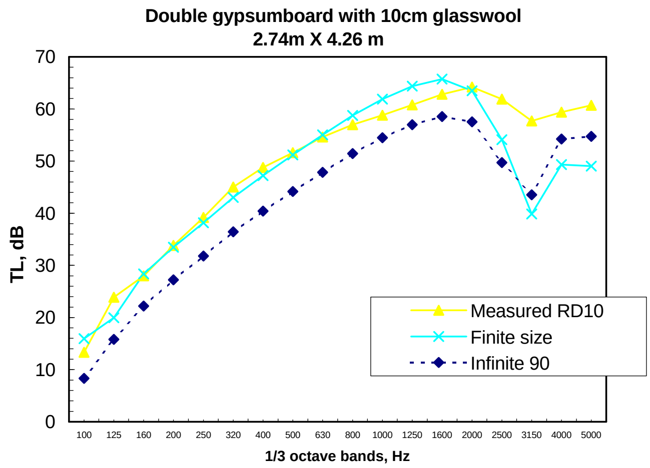


Figure 6. Comparison between the measured and calculated data for double gypsum board with 10 cm glasswool in large size: 2.74 m X4.26m

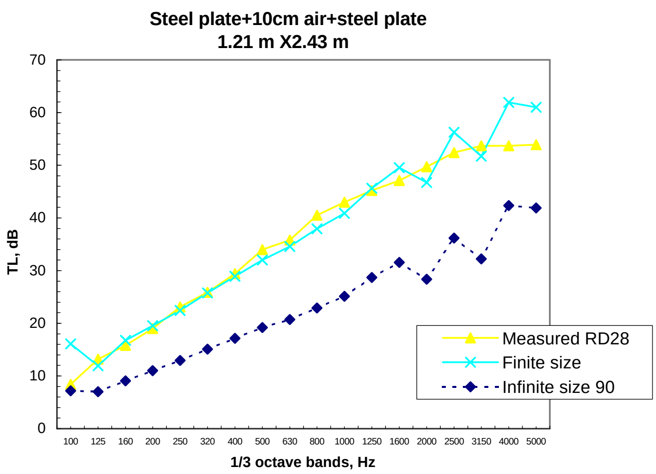


Figure 7. Comparison between the measured and calculated data for double steel plates with 10 cm air in small size: 1.21 m X 2.43 m

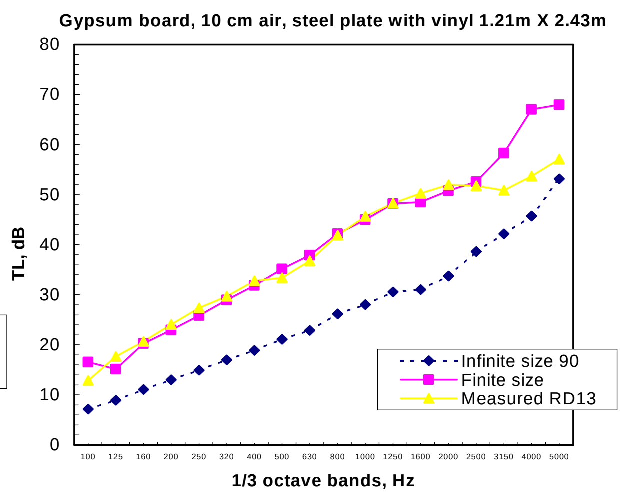
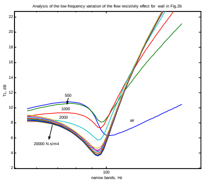


Figure 8. Comparison between the measured and calculated data for a multilayered structure in small size: 1.21 m X 2.43 m.



Low frequency detail of the plot

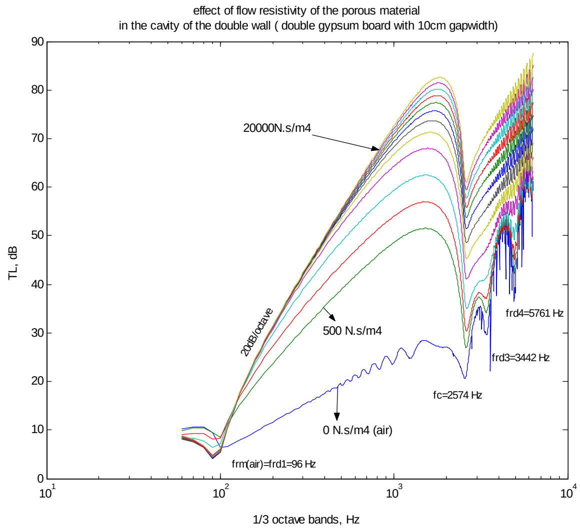


Figure 9. Variation of the narrow band sound transmission losses of double gypsum wall with 0.10m cavity (infinite size).

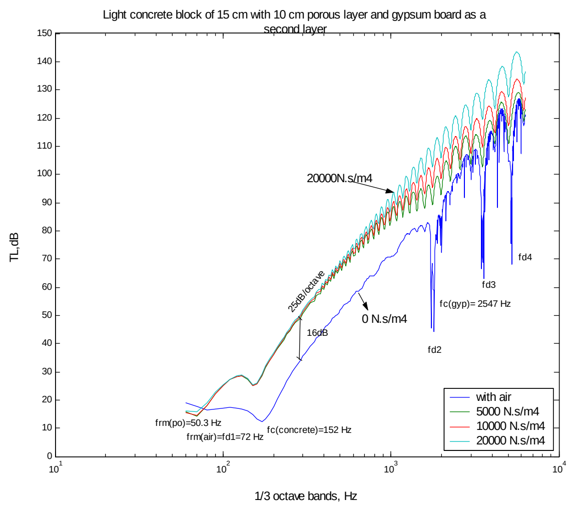


Figure 10. Variation of narrow band sound transmission losses of the wall type in Fig.4a with 0.10m gapwidth (infinite size).

Table1. Physical characteristics of the materials used in the sample walls during the experiment

Material	**Density (av.),kg/m^3**	**Thickness m**	**Elasticity modulus,N/m^2**	**Loss factor**	**Poisson ratio**	**Flow resistivity of porous layer, N.s/m^4**
Gypsum board	625-725	0.0127	1.3 e9-3 e9	0.01-0.03	0.3	_
Steel plate	7178 -7800	0.001	250 e9-310 e9	0.01 (10^{-4}-10^{-2})	0.31	_
Vinyl plate	2403 2000	0.00196	6 e9 - 9e9	0.5 Max:0.6	0.6	_
Glass-wool	8.01	0.05 0.10	_	_	_	6000-500
Air	1.24	0.05-0.10	_	_	_	0
Light concrete	400	0.15	3e9	0.015	0.3	-