ICSV14
Cairns • Australia
9-12 July, 2007
14th International Congress on Sound & Vibration

NONLINEAR CONTROL OF A ONE AXIS MAGNETIC SPRING

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Abstract

This paper presents the initial investigation of nonlinear control for a magnetic spring. The combination of a magnet pair in repulsion and a magnet pair in attraction create a spring with a quadratic force curve. At its nominal position, it is a marginally stable system that has interesting vibration isolation properties. Non-linear control is shown to be effective in stabilising the system.

1. INTRODUCTION

The performance of a vibration isolation table in attentuating disturbances from the ground is governed in the first order by the stiffness of the primary supports. Higher stiffnesses are more convenient to support large loads, but lower stiffnesses give better passive vibration suppression into lower frequency ranges. In high precision contexts, a resonance frequency of less than 1 Hz is often required, for which very soft pneumatic springs are typically used [1].

However, as the stiffness of a spring decreases its bulk increases, and there are practical limits on how large the springs can grow. The use of more complex support structures is a promising method of reducing the effects of low frequency disturbance noise when simply reducing the stiffness of the basic spring supports becomes infeasible.

One approach is to use a mechanical linkage [2]. In the limiting sense, the ideal support would have zero stiffness, in which displacement of the ground produces zero disturbance force on the load. Zero stiffness can be achieved with mechanical supports, but only for local regions of displacement. Since the zero stiffness property is not global, these devices are more accurately said to have 'quasi-zero stiffness' (although the 'quasi' term is often omitted).

All zero stiffness systems are a combination of positive and negative stiffness nonlinear springs. Others have shown clever arrangements of mechanical springs that achieve this [3–5], while permanent magnets have also been used [6, 7] and will be used here.

Generally, the zero stiffness property only occurs at a point of marginal stability, and some form of active control is required to keep the system stable at such an operating position. The focus of this paper is to investigate nonlinear control techniques for this purpose.

2. SYSTEM MODELLING

For the preliminary investigation, a single degree of freedom system is created with two pairs of magnets to generate the superposition of positive and negative spring stiffnesses. A schematic of the model is shown in Figure 1, in which vertical displacement, x, of the beam-tip constitutes vibration that is to be suppressed. In order to restrict the motion to a single degree of freedom, the magnet pairs are attached to a beam that is pinned at one end. The use of a large lever arm results in motion that is almost completely constrained to the vertical; horizontal and rotational displacements are negligible. A push-pull electromagnet pair acting on a central magnet is used to apply control force to the system via coil current I.

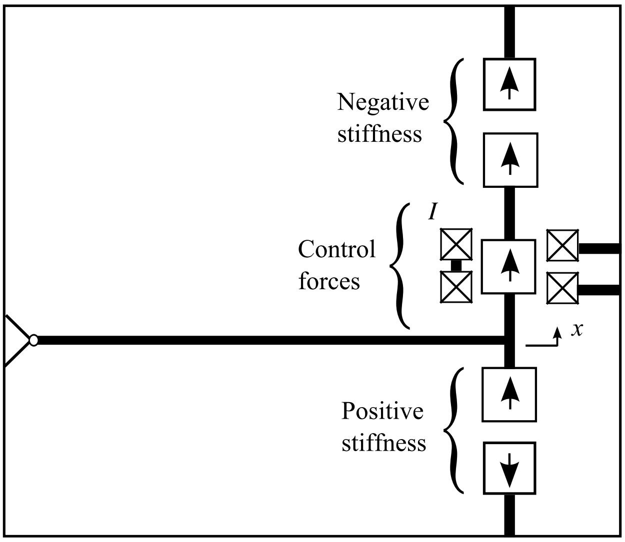


Figure 1. Schematic of the nonlinear magnetic system. Hollow blocks denote magnets with polarisation indicated by arrows. The centre magnet is encompassed within an electromagnet coil pair (blocks with crosses), wound in opposite directions, to apply control forces to the magnetic spring to stabilise the system. The frame around the system is rigid and attached to the ground.

2.1. Zero stiffness spring

The most simple form of the zero stiffness magnetic spring is the combination of a magnet pair in repulsion (lower pair in Figure 1) and a magnet pair in attraction (upper pair in Figure 1). The force versus displacement curve for this system can be described approximately by the quadratic [6]

$$F_m = K_m(x - x_0)^2 + F_{m0}, \tag{1}$$

where K_m is the spring constant, F_{m0} the load bearing capacity of the spring at zero stiffness, and x_0 the zero stiffness position due to

$$\left.\frac{\partial F_m}{\partial x}\right|_{x=x_0} = 0. \tag{2}$$

The spring constant K_m and the load bearing capacity F_{m0} are functions of the strength of the magnets and the gaps between them. For application purposes, the spring must be capable of bearing a variety of loads at zero stiffness ($F_g = -Mg$ for a range of M), which requires that the fixed magnet positions be adjustable. The mass being supported is assumed here to be constant.

Air resistance on the moving body and eddy currents induced by the permanent magnets will induce damping effects that dissipate energy from the vibrating system. The overall damping force, F_d, can be assumed to be viscous (and relatively small):

$$F_d = -C_d \dot{x}, \tag{3}$$

where C_d is the damping coefficient.

2.2. Actuator dynamics

The actuator is modelled as a simple dual-coil winding surrounding a permanent magnet. To be specific, the force this coil will produce, F_c, will be a function of both position and current, but a reasonable approximation for small displacements is

$$F_c = K_c I. \tag{4}$$

When driving the coil with a voltage amplifier, there will be electrical dynamics as well. In terms of coil impedance L, coil resistance R, voltage gain G, and input voltage u, the coil dynamics are given by

$$L\dot{I} + IR + K_e \dot{x} = Gu, \tag{5}$$

which also incorporates the back-emf term $K_e \dot{x}$ from the moving magnet inside the coil.

2.3. Dynamic model

From the preceeding subsections, the complete model of the system is given by

$$\begin{aligned} M\ddot{x} &= F_g + F_m + F_c + F_d, \\ &= -Mg + K_m(x - x_0)^2 + F_{m0} + K_c I - C_d \dot{x}, \\ L\dot{I} &= Gu - IR - K_c \dot{x}. \end{aligned} \tag{6}$$

This system must be expressed in the following form in order to implement a standard backstepping controller [8]:

$$\begin{aligned} \dot{x}_1 &= x_2 + \varphi_1(x_1)^{\mathrm{T}}\theta, \\ \dot{x}_2 &= b_2 x_3 + \varphi_2(x_1, x_2)^{\mathrm{T}}\theta, \\ \dot{x}_3 &= b_3 u + \varphi_3(x_1, x_2, x_3)^{\mathrm{T}}\theta, \end{aligned} \tag{7}$$

which is achieved by reformulating Equation (6) with $[x_1, x_2, x_3] = [x, \dot{x}, I]$ as

$$\begin{aligned} \dot{x}_1 &= x_2, \\ \dot{x}_2 &= \theta_1 + \theta_2 x_1 + \theta_3 x_1^2 + \theta_4 x_2 + b_2 x_3, \\ \dot{x}_3 &= \theta_5 x_2 + \theta_6 x_3 + b_3 u, \end{aligned} \tag{8}$$

where

$$\begin{aligned} \theta_1 &= K_m x_0^2/M - g + F_{m_0}/M, & \theta_3 &= K_m/M, & \theta_5 &= -K_c/L, & b_2 &= K_c/M, \\ \theta_2 &= -2x_0 K_m/M, & \theta_4 &= -C_d/M, & \theta_6 &= -R/L, & b_3 &= G/L, \end{aligned} \tag{9}$$

and for constant parameters θ and nonlinear functions φ_i defined as

$$\theta = [\theta_1, \theta_2, \theta_3, \theta_4, \theta_5, \theta_6]^{\mathrm{T}}, \tag{10}$$

$$\begin{aligned} \varphi_1(x_1)^{\mathrm{T}} &= [0, 0, 0, 0, 0, 0], \\ \varphi_2(x_1, x_2)^{\mathrm{T}} &= [1, x_1, x_1^2, x_2, 0, 0], \\ \varphi_3(x_1, x_2, x_3)^{\mathrm{T}} &= [0, 0, 0, 0, x_2, x_3]. \end{aligned} \tag{11}$$

3. NONLINEAR CONTROL

The authors have previously demonstrated a simple backstepping controller that is able to stabilise a system similar to that derived above, albeit without coil dynamics, for the purpose of vibration isolation [6]. That controller showed in simulation the advantages of using backstepping control with a zero stiffness system, but it required an explicit and exact model of the system to calculate the control law. In turn, a very precise system identification would be required for control to be possible; a more robust technique is therefore necessary for a practical implementation.

An approach without these limitations is the backstepping method using tuning functions [8, §4.5.1], which is a general method to generate tracking control systems for systems (for any number of states) of the form shown in Equation (7). The controllers thus created are adaptive and do not require knowledge of the coefficients b_2, b_3 or θ. The Lyapunov-based design ensures that the system states converge to their desired values and the unknown parameters converge to a bounded set. This latter property means that using this technique is not appropriate for system identification, since some parameter uncertainty is probable even after convergence.

When applying the tuning functions backstepping technique to Equation (7), the system of 'error variables' z_1, z_2, z_3 is defined with respect to the setpoint y and 'stabilising functions' α_1 and α_2 (with α_3 to appear):

$$z_1 = x_1 - y, \qquad z_2 = x_2 - \alpha_1, \qquad z_3 = x_3 - \alpha_2. \tag{12}$$

The voltage controller can now be shown to be defined as the following:

$$u = \varrho_3 \alpha_3, \tag{13}$$

with stabilising functions[2]

$$\begin{aligned} \alpha_3 = -\kappa_3 z_3 x_2^2 - \vartheta_5 x_2 + \partial_{x_1}\{\alpha_2\} x_2 - (x_1^4 + x_1^2 + x_2^2 + x_3^2 + 1)\kappa_3 z_3 \partial_{x_2}\{\alpha_2\}^2 \dots \\ + \partial_\vartheta\{\alpha_3\}\dot{\vartheta} - x_3\vartheta_6 - \beta_2 z_2 - \kappa_3 z_2^2 z_3 - c_3 z_3 - x_3^2 \kappa_3 z_3 \dots \\ + (\vartheta_1 + x_1(\vartheta_2 + x_1\vartheta_3) + x_2\vartheta_4 + x_3(\beta_2 + 2\kappa_3 z_2 z_3))\partial_{x_2}\{\alpha_2\} + \dot{\varrho}_2 \partial_{\varrho_2}\{\alpha_2\}, \end{aligned} \tag{14}$$

[2]The notation $\partial_x[y] \equiv \frac{\partial y}{\partial x}$ is used for clarity.

$$\alpha_2 = \varrho_2(-\vartheta_3 x_1^2 - \vartheta_2 x_1 - x_1 + y - \vartheta_1 - x_2\vartheta_4 - c_2 z_2 \dots \\ - (x_1^4 + x_1^2 + x_2^2 + 1)\kappa_2 z_2 + x_2 \partial_{x_1}\{\alpha_1\}), \quad (15)$$

$$\alpha_1 = -c_1 z_1. \quad (16)$$

In the controller above, c_1, c_2, and c_3 are the controller gains, and κ_2, κ_3 are the nonlinear damping gains. These gains can be adjusted to obtain a desirable controller response.

Parameter update laws for the estimates are shown in Equations (17) to (20), where ϑ are the estimates of parameters θ, β_2 and β_3 are the estimates of b_2 and b_3, and ϱ_2 and ϱ_3 are the estimates of $p_2 = 1/b_2$ and $p_3 = 1/b_3$. ϱ terms are used to avoid $1/\beta$ terms in the controller/update laws, which become problematic if $\beta \to 0$. Γ is the parameter update gain matrix for ϑ, and γ_2, γ_3 are update gains for ϱ_2 and ϱ_3.

$$\dot{\vartheta} = \Gamma[z_2 - z_3\partial_{x_2}\{\alpha_2\}, x_1(z_2 - z_3\partial_{x_2}\{\alpha_2\}), \dots \\ x_1^2(z_2 - z_3\partial_{x_2}\{\alpha_2\}), x_2(z_2 - z_3\partial_{x_2}\{\alpha_2\}), x_2 z_3, x_3 z_3]^{\mathrm{T}}, \quad (17)$$

and

$$\dot{\beta}_2 = -\gamma_2 z_3(-x_2 + \alpha_1 + x_3\partial_{x_2}\{\alpha_2\}), \quad (18)$$

$$\dot{\varrho}_2 = -\gamma_2 z_2 \alpha_2/\varrho_2, \quad (19)$$

$$\dot{\varrho}_3 = -\gamma_3 z_3 \alpha_3, \quad (20)$$

A characteristic of the tuning functions design is the tight coupling between the controller and the parameter update laws. Note that α_3 contains terms involving both $\dot{\vartheta}$ and $\dot{\varrho}_2$. No derivatives of the setpoint y appear, as it is assumed to be constant, but this restriction is not necessary in the general form of the controller.

4. RESULTS

To demonstrate the performance of the controller described above, a selection of simulation results are shown. When the system is initialised, the position of the fixed magnets needs to be set in order for the magnet force curve to provide a nominal supporting force to place the spring as close to the zero stiffness location as possible. For this to occur, $F_{m_0} = Mg$, where F_{m_0} is a function of the gaps between the fixed and floating magnets.

However, this calibration cannot be performed in open loop, because the desired location of the spring is in a position of marginal stability; therefore, the controller must be active while the fixed magnet positions are adjusted.

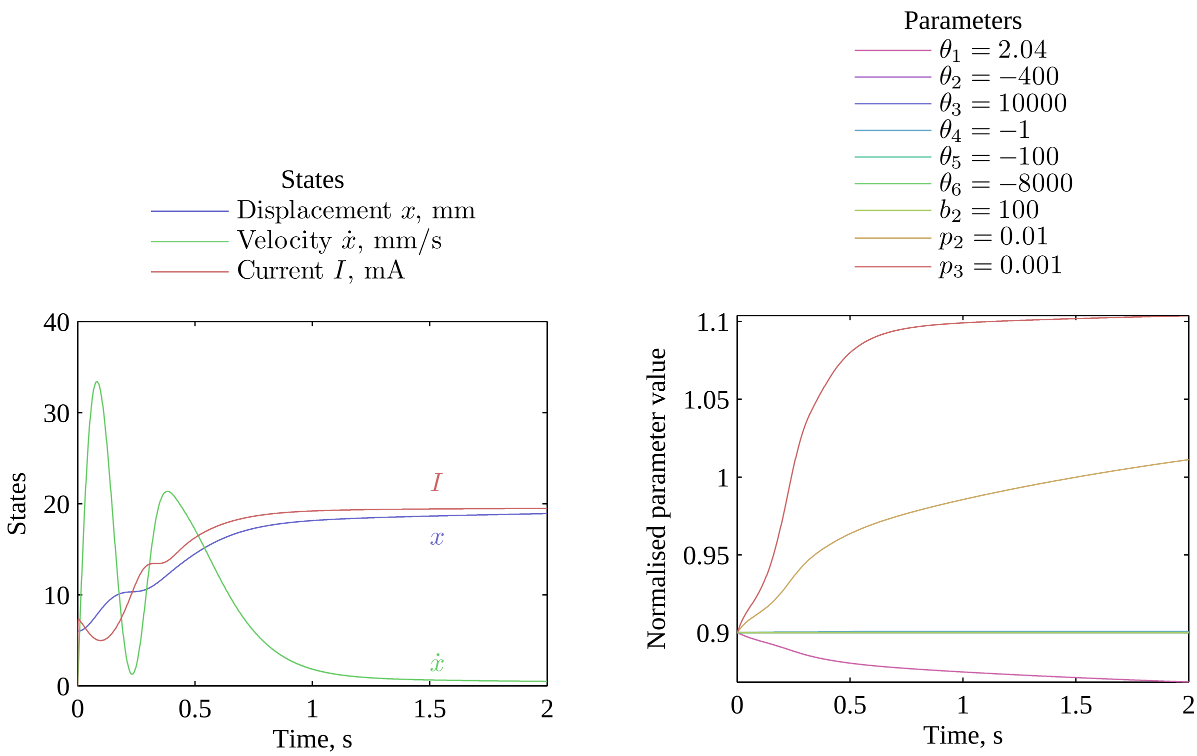
The first simulation results demonstrate the stability and convergence properties of the controller. The system parameters used in this simulation are shown in Table 1, with initial parameter estimates 10% below their actual value. Adaptation gains γ_2, γ_3, and Γ were chosen as the inverse of the orders of magnitude of the parameters they affect.

Figure 2a shows the states converging with setpoint following ($x = x_0$). During parameter convergence, the system approaches its desired position as the parameters converge to their steady state values. Figure 2b demonstrates that the parameters do not converge to their exact values, but to some nearby constant values instead.

Table 1. System parameters for simulation results.

Mechanical		Electrical		Controller	
Parameter	Value	Parameter	Value	Parameter	Value
M	$0.01\,\mathrm{kg}$	K_c	$1\,\mathrm{N/V}$	c_1, c_2, c_3	10
K_m	$100\,\mathrm{N/m^2}$	K_e	$0.1\,\mathrm{V/(m/s)}$	κ_2, κ_3	1
F_{m_0}	$0.8Mg$	R	$8\,\mathrm{Ohm}$	γ_2	0.01
C_d	$0.01\,\mathrm{kg/s}$	L	$0.001\,\mathrm{H}$	γ_3	0.001
x_0	$0.02\,\mathrm{m}$	G	1	Γ	See below

$$\Gamma = \mathrm{diag}(1, 0.001, 0.0001, 1, 0.01, 0.0001)$$



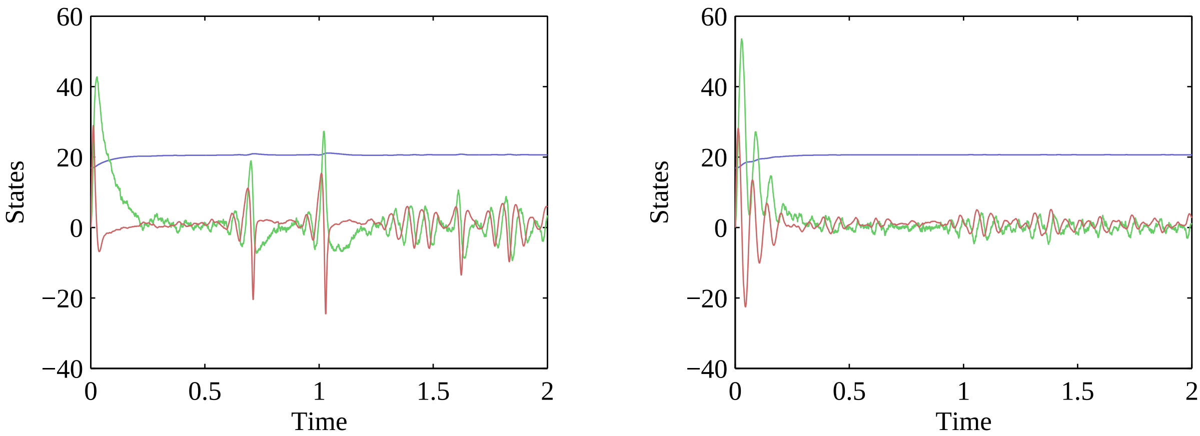
(a) States of the system

(b) Parameter estimates, normalised with their exact values (that is, ϑ_i/θ_i, β_2/b_2, ρ_i/p_i are plotted).

Figure 2. Example control with system parameters as shown in Table 1.

Once the parameters have converged and the magnet positions have been adjusted to achieve the desired nominal supporting force, the system is simulated again with perturbation on $M\ddot{x}$ introduced to demonstrate the behaviour under direct disturbance situations. For this simulation, the nominal force $F_{m_0} = 0.99Mg$; the 1% difference imposes a small initial displacement offset. The perturbation force on the isolated mass is Gaussian-distributed white noise with a variance of 0.001 N.

The simulation results are shown in Figure 3, for two cases: with and without parameter adaptation. Figure 3a can be seen to have a greater succeptibility to the noise disturbance compared to Figure 3b. These results show that once the system has converged, it is desirable to disable the adaptation ($\gamma_2, \gamma_3 = 0$, $\Gamma = \mathbf{0}$) to avoid perturbation in the system affecting the control performance.



(a) With parameter adaptation.

(b) Without parameter adaptation.

Figure 3. Control at steady state position with direct disturbance, with and without parameter adaptation. It is clear that noise upsets the performance of the parameter adaptation after convergence. See Figure 2a for legend.

5. SUMMARY AND FUTURE WORK

This article has investigated the feasibility nonlinear control for a magnetic spring system. We have shown that standard backstepping methods can be used to stabilise the system in the presence of parameter uncertainties. However, we have not yet extended this work to look at vibration isolation, which is the *raison d'etre* for looking at this quasi-zero stiffness system.

To investigate vibration isolation, additional states need to be added to the system to represent the vibration of the ground. The tuning functions backstepping method accommodates this additional complexity without trouble.

The current design uses full state feedback, which is not infeasible in practice with the use of laser displacement and velocity sensors. Such sensors are expensive, however, and a useful modification to the controller will be to add state observers (either velocity or displacement) in order to reduce the number of sensors required.

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