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CONTROL OF VEHICLE SUSPENSION USING NONLINEAR ENERGY SINK CONTROLLER

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Abstract

The **N**onlinear **E**nergy **S**ink (**NES**) controller is designed and used to control actively or semi-actively the dynamics of vehicle suspension systems. A vertical vehicle dynamic model with seven degrees of freedom is developed and used to demonstrate the effectiveness of the NES controller. The proposed controller relies in its operation on specially designed nonlinear spring elements which are attached to the primary suspension system in order to induce optimal nonlinear vibration energy sinks. The obtained performance characteristics are compared with those of passive suspension and suspension using classical LQR controller. Simulation results show that the vibration energy transmitted by road disturbance has been successfully attenuated and the best possible ride comfort and handling performance can be potentially achieved by the proposed NES controller. It is also observed that the cost function of energy consumption using the NES controller is less than that using the LQR controller. The obtained results demonstrate the potential of the NES controller as an effective means for attenuating the vibration of vehicle suspension systems.

1. INTRODUCTION

The fundamental goal of any suspension system is to isolate the vehicle from external excitations through the use of appropriately selected set of springs and viscous dampers. The characteristics of these elements can be selected in order to enhance the ride comfort and road holding performance. An optimally designed suspension system should to be able to minimize the vehicle body accelerations, dynamic tire forces, and energy consumption while satisfying the constraints imposed on the design space.

Passive suspensions have inherent limitations as a consequence of the trade-off between the spring stiffness and damping characteristics in order to achieve acceptable behavior over a wide range of working frequencies. These limitations have motivated the development of controlled suspension systems.Controlled suspensions (both active and semi-active) have been investigated widely and a variety of designs and related control schemes have been proposed and presented over the past decades. The literature in this topic has been rich, a good review of the state-of-the-art of controlled suspensions can be found in references [1-2]. Various advanced control strategies have also been introduced such as LQR[3], nonlinear H_∞ [4]and so on.

Inducing energy sinks in vibrating systems as a way of localizing and eliminating unwanted disturbances has been considered in previous works, either through active means [5-6] or passive means [7]. Alternatively, linear passive sinks such as the classical dynamic absorber can operate only in the neighborhood of a single frequency, and are incapable of attracting robustly broadband transient disturbances. Vakakis [8] has proven that **N**onlinear **E**nergy **S**inks (NESs) attached to a linear extended system (the primary system) can be designed to absorb transient disturbances in a one-way irreversible manner.

Using NESs is a simple and viable means for achieving shock isolation of broadband unwanted disturbances. In this method, NESs are nonlinearly coupled to the main (primary) system by means of essential nonlinear stiffnesses. Such nonlinear coupling elements are necessary in order to achieve energy pumping of unwanted disturbances in a sufficiently fast timescale.

In this paper, the idea of NESs is utilized to isolate the vibration of vehicles from the road disturbance. Because the nonlinear energy sinks (NESs) can absorb transient disturbances in a sufficiently fast timescale and one-way irreversible manner, this motivates the present study. This study aims at: (1)using the idea of the NESs to control the vertical dynamics of vehicles with seven degree-of-freedom in order to achieve energy pumping of road disturbance in a one-way irreversible manner to nonlinear attachments; (2)using the nonlinear stiffness elements to achieve desired vehicle body and wheel performance.

2. VERTICAL VEHICLE DYNAMICS

Assume a vehicle driven on a straight road in a steady-state condition, i.e., with constant thrust and without brake action. In this case, the vertical dynamics of the vehicle include: the car body heave, roll, and pitch motions, and the four-wheel bounce motions. This is a typical seven degrees of freedom characterization of the vertical dynamics used for the computer controlled suspension as is shown in Fig.1.

The vertical vehicle dynamics model could be written using Newton-Euler method as follows:

$$M_b\ddot{q} + H^{'}K_s(Hq - z_w) + H^{'}C_s(H\dot{q} - \dot{z}_w) = H^{'}u \tag{1}$$

$$M_w\ddot{z}_w - K_s(Hq - z_w) - C_s(H\dot{q} - \dot{z}_w) + K_t(z_w - w) = -u \tag{2}$$

where $z_w = \begin{bmatrix} z_{w1} & z_{w2} & z_{w3} & z_{w4} \end{bmatrix}^T, q = \begin{bmatrix} h & r & p \end{bmatrix}^T,\ u = \begin{bmatrix} u_1 & u_2 & u_3 & u_4 \end{bmatrix}^T$

$$M_b = \begin{bmatrix} M_s & 0 & 0 \\ 0 & I_{xx} & 0 \\ 0 & 0 & I_{yy} \end{bmatrix},\ M_w = \begin{bmatrix} M_{w1} & 0 & 0 & 0 \\ 0 & M_{w2} & 0 & 0 \\ 0 & 0 & M_{w3} & 0 \\ 0 & 0 & 0 & M_{w4} \end{bmatrix}, K_s = \begin{bmatrix} K_{s1} & 0 & 0 & 0 \\ 0 & K_{s2} & 0 & 0 \\ 0 & 0 & K_{s3} & 0 \\ 0 & 0 & 0 & K_{s4} \end{bmatrix},$$

$$C_s = \begin{bmatrix} C_{s1} & 0 & 0 & 0 \\ 0 & C_{s2} & 0 & 0 \\ 0 & 0 & C_{s3} & 0 \\ 0 & 0 & 0 & C_{s4} \end{bmatrix}, \; K_t = \begin{bmatrix} K_{t1} & 0 & 0 & 0 \\ 0 & K_{t2} & 0 & 0 \\ 0 & 0 & K_{t3} & 0 \\ 0 & 0 & 0 & K_{t4} \end{bmatrix}, \quad H = \begin{bmatrix} 1 & l_{ylf} & -l_{xf} \\ 1 & -l_{ylf} & -l_{xf} \\ 1 & l_{ylr} & l_{xr} \\ 1 & -l_{ylr} & l_{xr} \end{bmatrix}$$

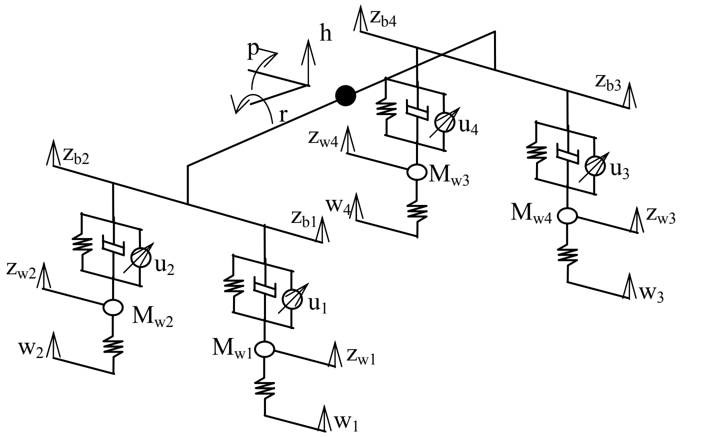

Figure1. A vertical vehicle dynamics model with seven degrees of freedom

where M_s is sprung mass, I_{xx} and I_{yy} are the roll and pitch moments of inertia of the car body, respectively. M_{wi} is the unsprung mass at the i^{th} corner, K_{ti} is the i^{th} tire stiffness. K_{si} as the passive suspension spring rate at the i^{th} corner, and C_{si} as the passive damping rate at the i^{th} corner. z_w is be the displacement vector whose i th element denotes the absolute displacement of the i th wheel of the vehicle. h is the heave displacement of the center of gravity of the car body (sprung mass), r is the car body's roll angle, and p is the car body's pitch angle. z_b is the vertical displacement vector of the car body, l_{xf} and l_{xr} are the distances from the front and rear axle to car body center of gravity, respectively, l_{ylf} and l_{ylr} are half of the front and rear wheel tracks.

Let $\hat{z} = \begin{bmatrix} q & z_w \end{bmatrix}^T$, in a more compact form, equations (1) and (2) can be rewritten as:

$$M\ddot{\hat{z}} + D\dot{\hat{z}} + K\hat{z} = E_1 w + E_2 u \tag{3}$$

where $M = \begin{bmatrix} M_b & 0 \\ 0 & M_w \end{bmatrix}, K = \begin{bmatrix} H' K_s H & -H' K_s \\ -K_s H & K_t + K_s \end{bmatrix}, D = \begin{bmatrix} H' C_s H & -H' C_s \\ -C_s H & C_s \end{bmatrix},$

$$E_1 = \begin{bmatrix} 0 \\ K_t \end{bmatrix} \; E_2 = \begin{bmatrix} H' \\ -I \end{bmatrix}$$

Define $\hat{x} = \begin{bmatrix} \hat{z} & \dot{\hat{z}} \end{bmatrix}^T$, then equation (3) can be expressed in the following state-space form:

$$\dot{\hat{x}} = A\hat{x} + B_1 w + B_2 u \tag{4}$$

where $A = \begin{bmatrix} 0 & I \\ -M^{-1}K & -M^{-1}D \end{bmatrix}$, $B_1 = \begin{bmatrix} 0 \\ M^{-1}E_1 \end{bmatrix}$ and $B_2 = \begin{bmatrix} 0 \\ M^{-1}E_2 \end{bmatrix}$

In this study, the performance variables for ride comfort are chosen as the acceleration of the heave, roll, and pitch and the performance variables for the handling performance are chosen as the tire defection at each corner due to the fact that the small tire deflection implies small tire dynamic load. A road excitation is used as input singals to measure the effectiveness of the NES controller and compare its performance with both the classical LQR controller and passive system. It is a single rectangular bump with a height of 0.01m and a width of 0.5m to reveal the transient response characteristic. The vehicle travels over the bump at a speed of 3 km/h. The input to rear wheel is assumed to be one time- delayed from the front wheel.

3. DESIGN AND ARCHITECTURE OF VEHICLE SUSPENSION CONTROLLERS

Four controllers are used to control the vertical dynamics of the vehicle, one controller for each corner as shown in figure1. In this case, the work of each controller is independent and uncoupled. This architecture is relative easy to accomplish and the hardware implementation becomes relatively simple.

3.1. LQR Controller

In order to design four independent LQR controllers, a quarter model is used as shown in figure2. The dynamic equations can be derived by Newton's method as follows:

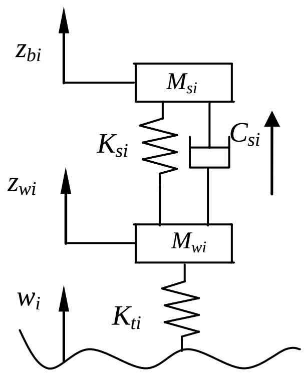

Figure2. A quarter vehicle dynamics model

$$M_{si}\ddot{z}_{bi} + f_{ki} + f_{ci} = u_i$$

$$\ddot{M}_{wi}\ddot{z}_{wi} - f_{ki} - f_{ci} + f_{kti} = -u_i \tag{5}$$

where M_{si} is sprung mass distributed by M_s at each corner. $f_{ki} = K_{si}(z_{bi} - z_{wi})$, $f_{ci} = C_{si}(\dot{z}_{bi} - \dot{z}_{wi})$, $f_{kti} = K_{ti}(z_{wi} - w_i)$.

Define state variables $x_i = [z_{bi} - z_{wi} \quad \dot{z}_{bi} \quad z_{wi} - w_i \quad \dot{z}_{wi}]^T$ and the output variables $y_i = [\ddot{z}_{bi} \quad z_{bi} - z_{wi} \quad z_{wi} - w_i]^T$, then the state-space and the output equations can be expressed as:

$$\dot{x}_i = A_i x_i + B_{1i}\dot{w}_i + B_{2i}u_i \quad (6)$$

$$y_i = C_i x_i + D_i u_i \quad (7)$$

In this case, the cost function for LQR control is expressed as follows:

$$J = \frac{1}{2}\int_0^{\infty} (y_i^T Q_i y_i + r u_i^2)dt \quad (8)$$

where Q_i is the positive definite matrix defined as $Q_i = diag(q_{1i}, q_{2i,} q_{3i})$, q_{1i}, q_{2i}, q_{3i} are the weights for y_{1i}, y_{2i} and y_{3i} respectively. Also, r denotes the weight for the control force u_i.In this way, the optimal gain matrix k_i can be obtained by following the design procedures of classical LQR controller.The optimal control force or controllable damping force in each corner can thus be derived as follows:

$$u_i = -k_i x_i \quad (9)$$

3.2. NES Controller

In this paper, a novel nonlinear controller is proposed based on the concept of the Nonlinear Energy Sinks (NES) mentioned above. These nonlinear energy sinks can be integrated in the vehicle vibration system through attaching nonlinear stiffness elements to the linear primary vibration system of vehicle under certain conditions. Once the nonlinear energy sinks are incorporated, the vibration energy excited by road disturbance can be pumped into the nonlinear attachments in one way, irreversible fashion.

In order to design these controllers, a new set of state variables are selected as follows:

$$g_i = \begin{bmatrix} (z_{bi} - z_{wi})^3 & \dot{z}_{bi} & (z_{wi} - w_i)^3 & \dot{z}_{wi} \end{bmatrix}^T$$

Compared with the LQR controller, it can be seen that nonlinear stiffness elements $(z_{bi} - z_{wi})^3$ and $(z_{bi} - z_{wi})^3$ have been introduced into subsystems of the primary vibration system of vehicle. We also assume that there exists a NES feedback gain matrix which can further induce nonlinear energy sinks in the subsystem. In this case, the active control force supplied by actuator can be described into:

$$u_i = -N_i g_i \quad (10)$$

where N_i is the feedback gain matrix of the i^{th} NES controller which is given by

$$N_i = [\alpha_{1i}, \alpha_{2i}, \alpha_{3i}, \alpha_{4i}]$$

The current question is how to select the feedback gain matrix to induce the nonlinear energy sinks so that the vibration energy excited by road disturbance can be pumped into the nonlinear attachments in one way, irreversible fashion.

Here the simplex search method [9] is used to determine the optimal feedback gain matrix N_i in order to minimize the cost function. At each step of the search, a new point in or near the current simplex is generated. The cost function value at the new point is compared with the cost function's values at the vertices of the simplex and, usually, one of the vertices is replaced by the new point, giving a new simplex. This step is repeated until the diameter of the simplex is less than the specified tolerance.

4. NUMERICAL EXAMPLES

Consider the vertical dynamics model with seven degrees of freedom depicted in figure1, parameters of the full car model are summarized in Table 1.

Table 1 -Vehicle parameters

Parameter	Unit	Value
M_s	*kg*	1583
I_{xx} / I_{yy}	*kg m*^{2}	531/2555
$M_{w1} / M_{w2} / M_{w3} / M_{w4}$	*kg*	48/48/74/74
$K_{s1} / K_{s2} / K_{s3} / K_{s4}$	*N/m*	35000/35000/34000/34000
$C_{s1} / C_{s2} / C_{s3} / C_{s4}$	*Ns/m*	400/400/200/200
$K_{t1} / K_{t2} / K_{t3} / K_{t4}$	*N/m*	220000/220000/220000/220000
I_{xf} / I_{xr}	*m*	1.116/1.438
$l_{ylf} / l_{yrf} / l_{ylr} / l_{yrr}$	*m*	0.77/0.77/0.765/0.765

For the LQR controllers, the weights are chosen as follows:

$$q_{1i} = 2.18\times10^4, q_{2i} = 9.9\times10^5, \text{ and } q_{3i} = 9.39\times10^6 \ (i=1,2,3,\text{and } 4)$$

$$k_1 = k_2 = [-1762.6, 846, 789.6, 3.3], \text{ and } k_3 = k_4 = [-2716.8, 1078.2, 1486.1, -58.9]$$

The optimal feedback gain matrix N_i for NES controllers is searched by the simplex search method are as follows:

$$N_1 = N_2 = [-110309.17, 1844.33, 399854.96, -18.10]$$

$$N_3 = N_4 = [57480.40, 1748.45, 364278.32, -87.20]$$

Fig.3 shows the transient responses when the vehicle travels over a bump. It can be seen that the control effectiveness using NES controllers is much better than that using LQR controllers. The body heave, roll and pitch accelerations, tire deflection are significantly reduced by the NES controllers. This means that the vibration energy from road excitation is absorbed rapidly.

The RMS values for the different control strategies are listed in Table 2. It can be clearly seen that the NES control strategy is more effective in attenuating the vibration of the vehicle body and improving the driving safety.

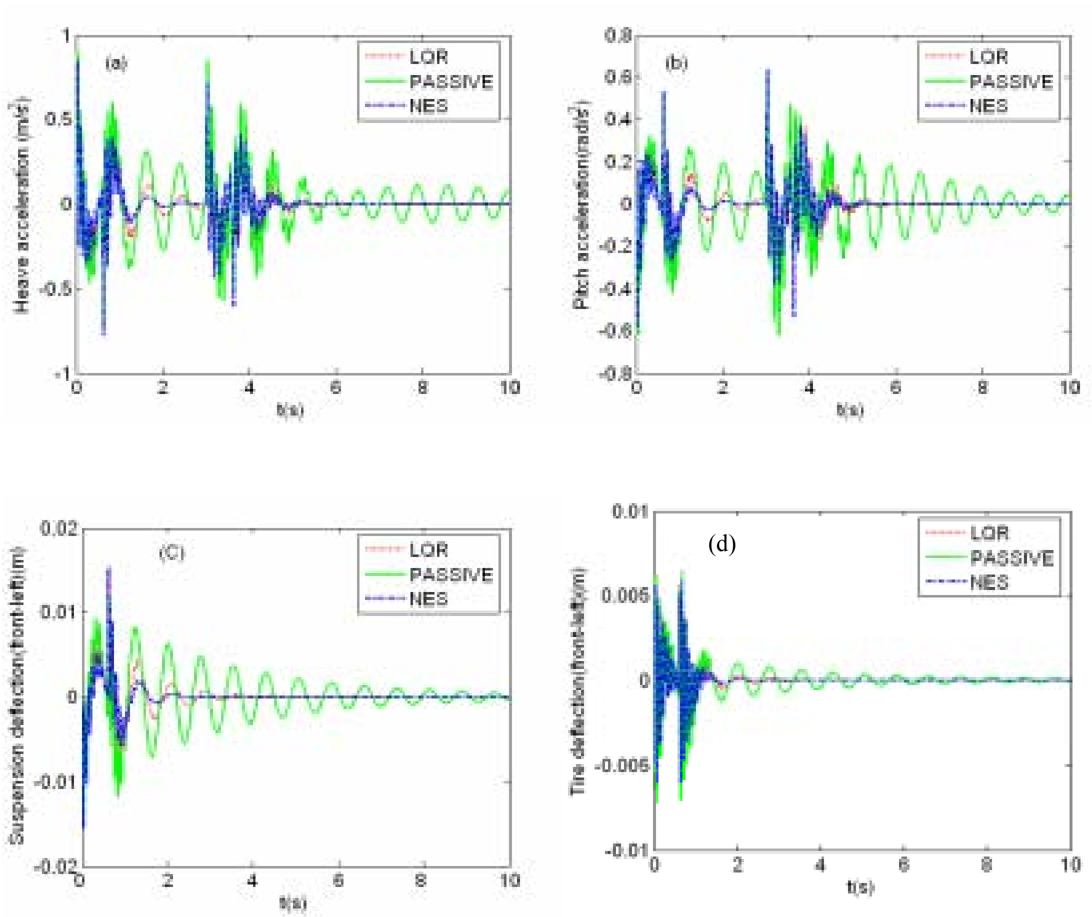

Figure3. Transient response when the vehicle passes over a bump
the body (a,b), suspension (c), and tire (d)

Table 2 - RMS for different control strategies in time domain

Parameter	RMS for passive	RMS for LQR	RMS for NES
heave acceleration	9.4858	7.0207	6.3112
roll acceleration	32.0980	18.8666	15.9603
pitch acceleration	9.4396	7.4042	7.2000
Suspension deflection(front-left)	0.1933	0.1370	0.1263
(front-right)	0.1840	0.1331	0.1276
(rear-left)	0.2728	0.1888	0.1643
(rear-right)	0.2616	0.1895	0.1669
Tire deflection (front-left)	0.0978	0.0948	0.0934
(front-right)	0.1007	0.0977	0.0962
(rear-left)	0.1861	0.1551	0.1363
(rear-right)	0.1939	0.1587	0.1378

Further comparisons between the different control strategies can be achieved by considering the cost function including vibration and control energy. Table 3 lists the response cost function to evaluate vibration energy and the total cost functions considering the control energy for both the LQR and NES controllers. It can be seen that vibration energy using the NES controllers is 27% smaller than that of the linear LQR controllers. At the same time, the total cost function with the NES controllers is about 13% less than that of the LQR controllers.

This dramatic result suggests the potential of the NES control strategy as an effective means for attenuating vehicle vibration .

Table 3 - The cost function comparison in time domain

Controller	The response cost function	The total cost function
LQR	3.4066×10^7	3.6140×10^7
NES	2.4645×10^7	3.1231×10^7

5. CONCLUSIONS

In this paper, the vertical dynamics model of vehicle with seven degrees of freedom is described. The model is used to measure the effectiveness of a nonlinear energy sink (NES) controller as compared to the classical LQR controller. The simulation results show that the NES controllers are more effective than the LQR controllers and passive systems for both the ride comfort or handling performance. This implies that the NES controller can induce effective nonlinear energy pumping into nonlinear attachments and thus an effective vibration isolation is reached. Experimental verification of the performance characteristics of the NES controller is a natural extension of the present study.

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