ICSV14
Cairns • Australia
9-12 July, 2007

14th International Congress on Sound & Vibration

LAMINAR BOUNDARY LAYER INSTABILITY NOISE PRODUCED BY AN AEROFOIL

Michael Kingan[1], John Pearse[1]

[1]Department of Mechanical Engineering, University of Canterbury
Christchurch, New Zealand
m.kingan@mech.canterbury.ac.nz

Abstract

When a laminar boundary layer exists on the surface of an aerofoil up to the trailing edge, a tone or a number of tones are sometimes produced. These tones have been the subject of a number of investigations which have proposed a variety of different mechanisms regarding their production. This paper describes the tone generation mechanism and then describes the development of a theoretical model to estimate the tone frequencies. The model is validated against experimental results for two cases.

1. INTRODUCTION

When a laminar boundary layer exists on one surface (usually the pressure surface) of an aerofoil up to the trailing edge, in certain instances a tone or a number of high amplitude tones are produced. This noise is referred to here as laminar boundary layer instability noise although it is sometimes referred to as laminar boundary layer vortex shedding noise (e.g. Brooks, Pope and Marcolini [3]). The first comprehensive study on the noise generated in the way was by including Paterson, Vogt, Fink and Munch [9] who observed (1) the 'peak frequency', f_s, of the noise produced by the interaction of airflow with the aerofoil scaled approximately according to $f_s \propto U_\infty{}^{1.5}$, (2) the presence of the tone was associated with a laminar boundary layer existing up to the trailing edge of the aerofoil (3) the frequency of the tones when plotted against free stream velocity U_∞ exhibited a ladder type variation for small variations in the free stream velocity and (4) that the frequency of the tones on each rung of the ladder was approximately proportional to $U_\infty{}^{0.85}$ (for the cases that they investigated).

Tam [11], Wright [13], Longhouse [6], Fink [4] and Arbey and Bataille [4] all proposed that the tones were produced by the amplification of boundary layer instabilities (known as Tollmein-Shlichting (T-S) waves) on the pressure surface of the aerofoil and an acoustic feedback loop which reinforced/excited the T-S waves. Arbey and Bataille's feedback mechanism is described in detail here and is used to develop a model to predict the frequency of tones produced by laminar boundary layer instability noise.

Arbey and Bataille conducted a comprehensive experimental and theoretical investigation into the noise generated by an aerofoil immersed in a laminar flow and found that the noise consisted of a broadband contribution which peaked at f_s and a discrete contribution at equidistant frequencies f_n. They concluded that the broadband contribution could be attributed to the diffraction of T-S waves in the developing boundary layer by the trailing edge, whereas the equi-spaced discrete frequencies were due to an aeroacoustic feedback loop, between the aerofoil trailing edge and the point of first instability. Arbey and Bataille's proposed feedback loop is summarized in figure 1 below.

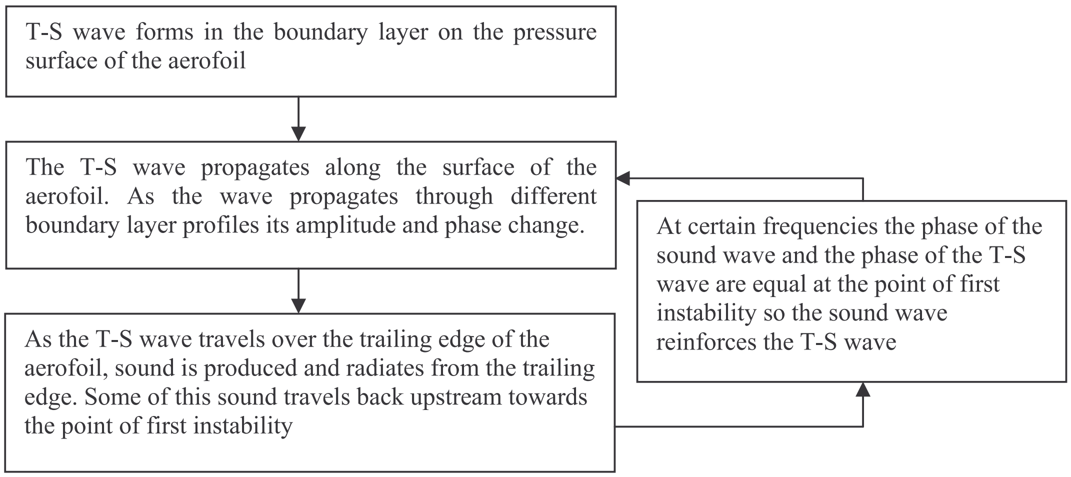

Figure 1. Flow diagram of the feedback loop proposed by Arbey and Bataille

McAlpine *et al.* [7] and Nash *et al.* [8] proposed a new mechanism explaining laminar boundary layer instability noise. Using linear stability analysis and experimental measurements of the boundary layer profiles over the pressure surface of an aerofoil they were able to calculate the total amplification of T-S waves over the surface. They proposed that the frequency of the tonal noise produced in these cases was close to the frequency of the T-S wave with maximum linear amplification over the aerofoil and that the majority of amplification occurred across a small separation bubble close to the trailing edge which was present in the cases they were investigating.

This paper describes an accurate method of calculating the frequency of laminar boundary layer instability tones produced by an aerofoil according to Arbey and Batille's feedback loop. In §2 a method based on that proposed by McAlpine *et al.* and Nash *et al.* to calculate the amplification of a T-S wave over the surface of an aerofoil using an easily automated method of solving the Orr-Sommerfeld equation is described. In §3 a model based on the feedback mechanism proposed by Arbey and Bataille is developed. While in §4 the model is validated against experimental results for two cases.

2. MODELLING T-S WAVE PROPAGATION

Following the method of McAlpine *et al.* and Nash *et al.* it is assumed that the two-dimensional T-S waves can be modeled by spatial modes of fixed frequency with slowly changing wavelengths. The stream function $\psi(\hat{\xi},\hat{\eta},t)$ of these T-S waves is described by the following expression

$$\psi\left(\hat{\xi},\hat{\eta},t\right)=\phi\left(\hat{\eta}\right)e^{i\left(\int\hat{\alpha}\left(\hat{\xi}\right)d\hat{\xi}-\hat{\omega}t\right)} \tag{1}$$

where $i=\sqrt{-1}$, t is time, ϕ is the perturbation amplitude and $\hat{\eta}$ is the stream-normal coordinate. The complex wavenumber $\hat{\alpha}$ is assumed to vary slowly with the streamwise coordinate $\hat{\xi}$ and the wavenumber of the least stable mode at a point along the surface of the aerofoil is calculated for a given velocity profile U, Reynolds number R and frequency ω by solving the Orr-Sommerfeld equation

$$(U\alpha-\omega)(\phi^{//}-\alpha^2\phi)-U^{//}\alpha\phi+\frac{i}{R}(\phi^{iv}-2\alpha^2\phi^{//}+\alpha^4\phi)=0 \quad (2)$$

The prime denotes differentiation with respect to the dimensionless coordinate $\eta = \hat{\eta}/L$, $R = U_\infty L/\nu$ is the Reynolds number and the variables in the Orr-Sommerfeld equation (3) are all non-dimensionalised using the free stream velocity U_∞, kinematic viscosity ν, and the Falkner-Skan viscous length L [10] (dimensional variables are indicated by a hat). For boundary layer flow equation (2) is subject to the following boundary conditions

$$\eta=0,\ \phi=0,\ \phi^{/}=0;\ \lim_{\eta\to\infty},\ \phi=0,\ \phi^{/}=0 \quad (3)$$

2.1 Numerical method

For the spatial stability case the eigenvalue appears to the fourth power in the Orr-Sommerfeld equation. To reduce the order of the eigenvalue problem from a fourth order problem to a second order problem a transformation is introduced (following Haj-Hariri [5])

$$\phi = \Phi e^{-\alpha\eta} \quad (4)$$

The reduced Orr-Sommerfeld equation is thus

$$(U\alpha-\omega)(\Phi^{//}-2\alpha\Phi^{/})-U^{//}\alpha\Phi+\frac{i}{R}(\Phi^{iv}-4\Phi^{///}+4\alpha^2\Phi^{///})=0 \quad (5)$$

with the same boundary conditions as equations (6) with ϕ replaced by Φ. The perturbation amplitude $\Phi(\eta)$ was approximated by a series of Chebyshev polynomials $T_n(x)$, defined on the interval $-1 \le x \le 1$ at the Gauss-Lobbotto collocation points x_i.

$$\Phi(\eta)=\sum_{n=0}^{N}a_nT_n(x),\ T_n(x)=\cos(n\cos^{-1}(x)) \text{ and } x_i=\cos\left(\frac{\pi i}{N}\right),\ i=0,\dots,N \quad (6)$$

The derivatives of Φ were found by differentiating the coefficients a_n through the use of derivative operators $\hat{\mathbf{D}}$.

$$\Phi^k(\eta_i)=\sum_{j=0}^{N}\hat{\mathbf{D}}_{ij}^k a_j T_j(x_i) \quad (7)$$

The derivative operator $\mathbf{D}$ was constructed as a matrix (see for example Trefethan [12]) and higher order derivative operators are simply defined $\mathbf{D}^k = (\mathbf{D}^1)^k$.

The derivative operators $\mathbf{D}^k$ are defined on the Chebyshev domain ($-1 \le x \le 1$) and are required to be mapped onto the semi-infinite domain ($0 \le \eta \le \infty$). This was done using an algebraic transformation which required the application of the chain rule to the derivative matrices $\mathbf{D}^k$. The derivative matrices for the semi-infinite domain are denoted $\hat{\mathbf{D}}^k$.

Substituting the series representation for Φ (equation (7)) into the reduced Orr-Sommerfeld eigenvalue problem (5) yields a second order eigenvalue problem of the form

$$\left(\mathbf{A}\alpha^2 + \mathbf{B}\alpha + \mathbf{C}\right)\Phi = 0 \tag{8}$$

where **A**, **B**, and **C** are matrices which need to be determined for each boundary layer profile, Reynolds number R and frequency ω. Boundary conditions (equation (3)) are applied to the first and last two rows of matrices **A**, **B** and **C**. Following Bridges and Morris [2], the eigenvalues of the companion matrix are the roots of the corresponding polynomial equation, a companion matrix for equation (8) can be written as

$$\left\{\begin{bmatrix} -\mathbf{B} & -\mathbf{C} \\ \mathbf{I} & 0 \end{bmatrix} - \alpha\begin{bmatrix} \mathbf{A} & 0 \\ 0 & \mathbf{I} \end{bmatrix}\right\}\begin{bmatrix} \alpha\Phi \\ \Phi \end{bmatrix} = 0 \tag{9}$$

This equation represents a complex generalized eigenvalue problem and was solved using the QZ algorithm.

2.2 Flow solver

The velocity profiles at a number of stations on the surface of the aerofoil were calculated using XFOIL. XFOIL solves the flow over the aerofoil using a coupled potential flow solver and boundary layer integral method. XFOIL was selected because of its ability to (very) quickly and accurately determine boundary layer profiles on an aerofoil surface even for mildly separated flows.

The boundary layer displacement thickness δ^*, the shape factor H were calculated using XFOIL at each station on the pressure surface of the aerofoil. The boundary layer velocity profile at each station was defined as the Falkner-Skan velocity profile with an identical shape factor to that calculated using XFOIL. Falkner-Skan boundary layer profiles were calculated (from the Falkner-Skan equations) using a parallel shooting technique employing a fourth order Runge-Kutta-Gill solver.

2.3 T-S wave amplification

The total amplification A of a T-S wave with slowly varying complex wavenumber between $\hat{\xi} = a$ and $\hat{\xi} = b$ (where b is downstream of a) over the aerofoil surface S is

$$A = \exp\left(-\int_a^b \hat{\alpha}_i\left(\hat{\xi}\right) dS\left(\hat{\xi}\right)\right) \tag{10}$$

For the calculations presented in this paper the integral in equation (10) was evaluated by the rectangle rule using stations located at 2% chord intervals from the leading edge to the trailing edge.

3. EXTENSION OF THE MODEL TO INCLUDE A TONE SELECTION MECHANISM

The model is based on the feedback loop proposed by Arbey and Bataille described in figure 1.

Assuming that sound is produced at the trailing edge and propagates upstream outside the boundary layer to the point of first instability, by calculating the phase change of both the sound and the T-S wave around the feedback loop, the frequency at which T-S waves will become excited\reinforced and produce tones may be calculated. The total phase change around the feedback loop may be deduced by considering each component of the loop separately.

Considering first the phase change of the T-S wave as it propagates downstream along the aerofoil surface. From inspection of the stream function of the T-S wave (equation (1)) the phase change over the aerofoil surface S between the point of first instability (at a) and the trailing edge (at b) is

$$\int_a^b \hat{\alpha}_r\left(\hat{\omega},\hat{\xi}\right)dS\left(\hat{\xi}\right) \tag{11}$$

The integral in equation (11) is a function of frequency and was evaluated using the rectangle rule using stations located at 2% chord intervals from the leading edge to the trailing edge.

According to Arbey and Bataille, Yu and Tam [14] showed that diffraction of the T-S wave at the trailing edge results in a phase shift of π radians which must be added into the phase change around the feedback loop.

The sound wave generated at the trailing edge then propagates back upstream at a speed of approximately c_0 - $U_{\infty,L}$ where c_0 is the speed of sound and $U_{\infty,L}$ is the average free-stream airflow speed over the surface of the aerofoil between the trailing edge and the point of first instability. The phase change of the sound wave is thus

$$\frac{2\pi fL}{c_0 - U_{\infty,L}} \tag{12}$$

Where L is the distance along the aerofoil surface between points a and b. At each frequency the point of first instability, a, was taken to be the first station where α_i became negative. When the total phase change around the feedback loop is equal to a multiple of 2π the acoustic wave will excite\reinforce the T-S wave at the point of first instability and will result in a tone being produced i.e. tones will occur at frequencies at which the following relationship is satisfied

$$\frac{1}{2\pi}\int_a^b \hat{\alpha}_r(x_1)dx_1 + \frac{1}{2} + \frac{fL}{c_0 - U_{\infty,L}} \equiv F = n\ ,\ n = 1,2,3... \tag{13}$$

In the current method the phase function F was calculated over the range of frequencies for which T-S wave amplification occurs. A least squares polynomial was fitted to F and frequencies which satisfy $F = n$ (which correspond to frequencies at which tones occur) were determined.

4. MODEL VALIDATION

For all results presented here, wind tunnel corrections were not applied when calculating the flow around the aerofoils. This may be a source of some small error, however it is assumed to be minimal. (Hopefully wind tunnel corrected data will be presented at the conference).

In this section the models described in §2 and §3 are used to predict the tone frequencies produced by an aerofoil for two cases. The calculations are compared with the results of two published experimental investigations.

4.1 300mm chord NACA0012 aerofoil inclined at 4° to a 29.7m/s airflow

As part of a comprehensive investigation into the laminar boundary layer instability noise produced by an aerofoil McAlpine *et al*. experimentally measured the tonal noise produced by a 300mm chord NACA0012 aerofoil inclined at 4° to a 29.7m/s airflow. For this case a single high amplitude tone was observed at 1048Hz. They measured the boundary layer velocity profiles over the pressure surface of the aerofoil using laser Doppler anemometry and calculated the total T-S wave amplification using a method similar to that described in §2 (they used a different method of solving the Orr-Sommerfeld equation). McAlpine *et al.* observed a small region of separated flow close to the trailing edge of the aerofoil and observed that the T-S waves underwent large amplification in these regions.

The method described in §2 predicted peak T-S wave amplification at 1000Hz (which is very close to the 1050Hz calculated by McAlpine *et al.* using experimentally measured boundary layer velocity profiles).

McAlpine *et al.* only observed one tone for this case (at 1048Hz). However, using the method described in §3 frequencies at which feedback would occur were calculated. The calculated tones were spaced approximately 60Hz apart and one was calculated to occur at 1042Hz.

Inspection of the local T-S wave amplification rates (see figure 2 below) showed that, as observed by McAlpine *et al.*, the regions of mildly separated flow close to the aerofoil trailing edge produced relatively high levels of T-S wave amplification.

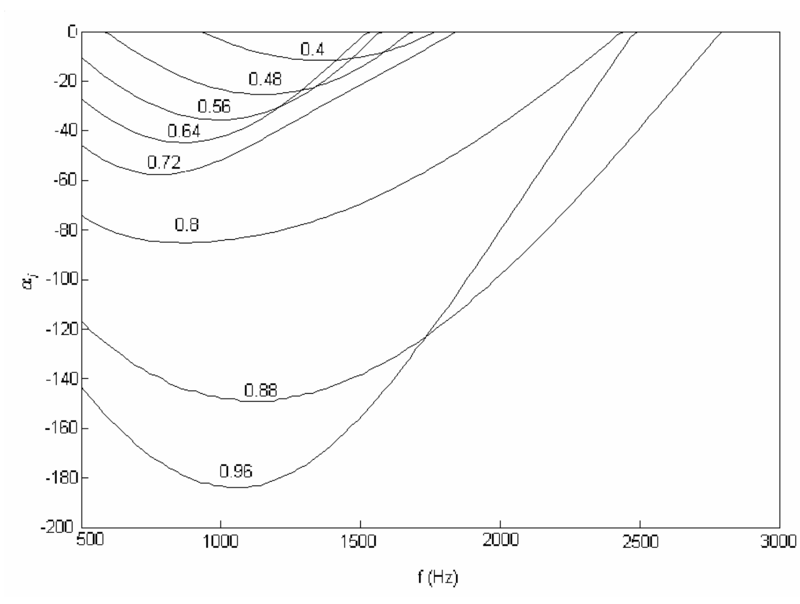

Figure 2. Local T-S wave amplification rates (α_i) at various chordwise positions along the aerofoil surface (the number adjacent to each curve corresponds to the station position as a fraction of the chord from the leading edge)

4.2 9" chord NACA0012 aerofoil inclined at 6° to a 30-60m/s airflow

Paterson, Vogt, Fink and Monk measured the aeroacoustic noise produced by a 9" (228mm) chord NACA0012 aerofoil inclined at 6° to a 30-60m/s airflow. They observed that when the frequency of the tones were plotted against airflow speed the tones arranged themselves into a 'ladder', which for large variations in airflow speed the tone frequency was proportional to $\sim U_\infty^{1.5}$ but for small variations in airflow speed the tone frequency was proportional to $\sim U_\infty^{0.85}$ (these are the 'rungs' of the ladder). In this section the models described in §2 and §3 are used to predict the frequency of the tones produced by the aerofoil at airflow speeds between 30m/s and 60m/s. The results of the calculation are shown in Table 8 below and are compared with the frequencies given by an empirical equation given by Paterson *et al.* which was derived from their experimental data.

Table 8. f_s calculated using a number of different methods

U (m/s)	30	33	36	39	42	45	48	51	54	57	60
f_s (Hz): this paper	1090	1260	1430	1580	1760	1950	2120	2330	2500	2720	2880
f_s (Hz): Paterson *et al.*	973	1123	1279	1443	1612	1788	1970	2157	2350	2549	2753

The frequency of maximum T-S wave amplification predicted using the method presented in §2 is in reasonable agreement with those predicted by the empirical relationship given by Paterson *et al*.

It was observed that the boundary layer displacement thickness at the trailing edge predicted by XFOIL did not scale with $U_\infty^{0.5}$ but yet the frequency of maximum T-S wave amplification f_s calculated using the method of §2 scales with $U_\infty^{1.4}$ which is very close to the observed $U^{1.5}$ scaling. This indicates that the f_s does not necessarily scale with boundary layer size at the trailing edge and thus caution should be exercised when using models which assume that it does.

The frequency of the tones calculated using the models described in §2 and §3 are plotted against free stream airflow speed in figure 3 below. The classic 'ladder' diagram is reproduced with the frequency of maximum T-S wave amplification scaling in proportion to $U_\infty^{1.4}$ and the tone frequencies scaling in proportion to $U^{0.79\text{-}0.85}$ (close to the $\sim U^{0.85}$ scaling observed by Paterson *et al.*).

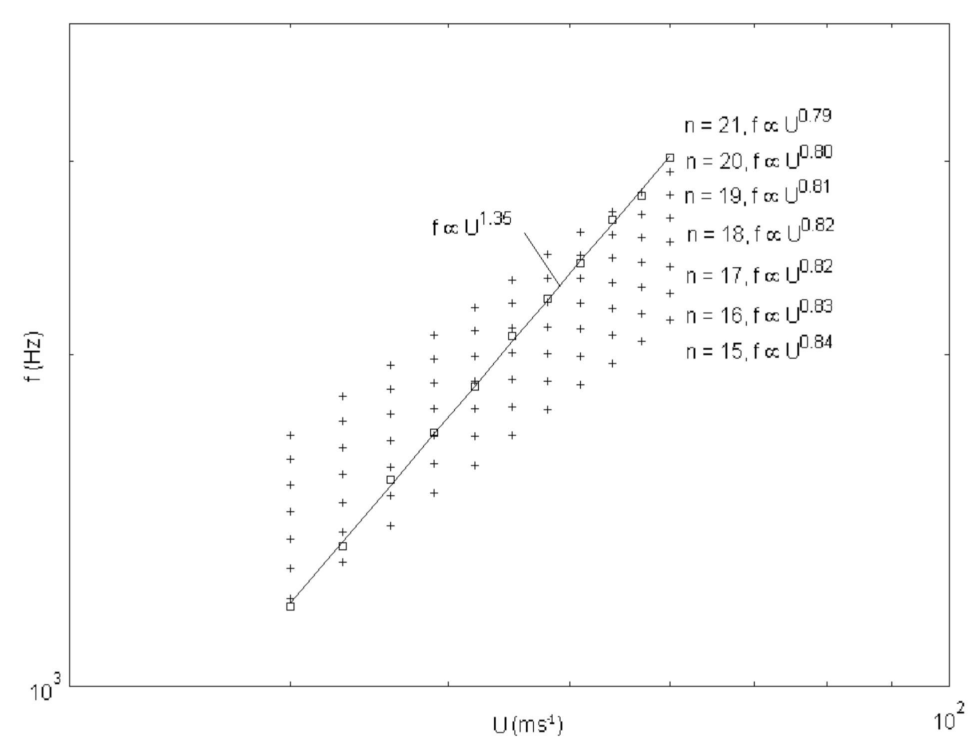

Figure 3. 'Ladder diagram' calculated using the method described in §2 and §3

As in the first case the XFOIL flow simulation predicted a region of mildly separated flow close to the trailing edge of the aerofoil on the pressure surface for all airflow speeds. For all cases the T-S waves were calculated to be highly amplified in these separated flow regions.

5. CONCLUSIONS

A theoretical model for laminar boundary layer instability noise proposed by McAlpine *et al.* and Nash *et al.* was extended to incorporate a tone selection mechanism based on the feedback mechanism proposed by Arbey and Bataille and was used to predict the frequencies of the tones for two cases. The model employed a global method of solving the Orr-Sommerfeld equation which led to easy automation. The models could be used as a predictive tool for laminar boundary layer instability noise on arbitrary aerofoil shapes. This is significant as no accurate method currently exists for making laminar boundary layer instability noise predictions, (although empirical models exist for the NACA0012 aerofoil e.g. [3]). The model requires further validation (against experiments) and in particular wind tunnel corrections need to be incorporated.

Confirming the hypothesis of McAlpine *et al.* and Nash *et al.* for all cases the XFOIL simulations predicted a region of mildly separated flow close to the trailing edge in which T-S waves underwent high amplification.

Due to the non-linear nature of the feedback mechanism, the level of tones is not related to their linear amplification over the pressure surface of the aerofoil. Thus the model can only be used for predicting frequencies at which tones will occur (and not the level of the tones).

REFERENCES

[1] H. Arbey, J. Bataille, “Noise generated by airfoil profiles placed in a uniform laminar flow”, *Journal of Fluid Mechanics* **134**, 1983, pp 33-47

[2] T. J. Bridges, P. J. Morris, “Differential Eigenvalue Problems in Which the Parameter Appears Nonlinearly” *Journal of Computational Physics* **55,** 437-460 (1984)

[3] T. F. Brooks, D. S. Pope, M. A. Marcolini, “Airfoil self-noise and prediction”, NASA RP-1218, 1-137, (1989)

[4] M. R. Fink, “Fine structure of airfoil tone frequency”, *Paper H3, presented at the 95th meeting of the Acoustical Society of America* (1978)

[5] H. Haj-Hariri, “Transformations reducing the order of the parameter in differential eigenvalue problems”, *Journal of Computational Physics* **77**, 472-484 (1998)

[6] R. E. Longhouse, “Vortex shedding noise of low Tip Speed Axial Flow Machines”, *Journal of Sound and Vibration* **53**, 25-46 (1977)

[7] A. McAlpine, E. C. Nash, M. V. Lowson, “On the generation of discrete frequency tones by the flow around an aerofoil”, *Journal of Sound and Vibration* **222**, 753-779 (1999)

[8] E. C. Nash, M. V. Lowson, A. McAlpine “Boundary layer instability noise on aerofoils”, *Journal of Fluid Mechanics* **382**, 27-61 (1999)

[9] R. W. Paterson, P. Vogt, M. Fink, C. Munch, “Vortex noise of isolated airfoils”, *Journal of Aircraft* **10**, 296-302 (1973)

[10] H. Schlichting, “Boundary-layer theory (7th ed.)”, McGraw-Hill, New York, (1979)

[11] C. K. W. Tam, “Discrete tones of isolated airfoils”, *Journal of the Acoustical Society of America* **55**, 1173-1177 (1974)

[12] L. Trefethan, “Spectral methods in Matlab”, SIAM Philadelphia, (1998)

[13] S. E. Wright, “The acoustic spectrum of axial flow machines”, *Journal of Sound and Vibration*, **45**, 165-223 (1976)

[14] J. C. Yu, C. K. W. Tam, “Experimental investigation of the trailing edge noise mechanism”, *Journal of the AIAA* **16**, 1046-1052 (1977)