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THE DIRECTIVITY OF THE FORCED RADIATION OF SOUND FROM PANELS AND OPENINGS

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Abstract

This paper presents a method for calculating the directivity of the radiation of sound from a two dimensional panel or opening, whose vibration is forced by the incidence of sound from the other side. The directivity of the radiation depends on the angular distribution of the incident sound energy. For panels or openings in the wall of a room, the angular distribution of the incident sound energy is predicted using a physical model which depends on the sound absorption coefficient of the room surfaces. For an opening at the end of a duct, the sound absorption coefficient model is used in conjunction with a two dimensional model for the directivity of the sound source in the duct. The finite size of the duct or panel is taken into account by using a two dimensional model for the real part of the radiation efficiency of the finite size panel or opening. For angles of radiation close to 90° to the normal to the panel or opening, the effect of the diffraction by the panel or opening, or by the finite baffle in which the panel or opening is mounted, needs to be included. The directivity of the radiation depends strongly on the length of the radiating object in the direction of the observer and only slightly on the width of the object at right angles to the direction of the observer. For panels, the plate wave impedance of the panel is used. Above its critical frequency, a single panel radiates strongly at the angle at which coincidence occurs. The method is compared with published experimental results.

1. INTRODUCTION

This paper describes a theoretical method for predicting the directivity of the sound radiated from a panel or opening excited by sound incident on the other side. This directivity needs to be known when predicting the sound level at a particular position, such as that due to the sound radiation from a factory roof, wall, ventilating duct or chimney flue. There is surprisingly little information on how to predict this directivity in the scientific literature. Most of this information is based on limited experimental data or its basis cannot be determined.

2. THEORY

The effective impedance $Z_e(\phi)$ of a finite panel in an infinite baffle to a plane sound wave incident at an angle of ϕ to the normal to the panel is [1]

$$Z_e(\phi) = Z_{wfi}(\phi) + Z_{wft}(\phi) + Z_{wp}(\phi) \quad (1)$$

where

$Z_{wfi}(\phi)$ is the wave impedance of the fluid as experienced by the finite panel in an infinite baffle, whose vibration is due to a plane sound wave incident at an angle of ϕ to the normal to the panel, on the side from which the plane sound wave is incident (this is the fluid loading on the incident side),

$Z_{wft}(\phi)$ is the wave impedance of the fluid as experienced by the finite panel in an infinite baffle, whose vibration is due to a plane sound wave incident at an angle of ϕ to the normal to the panel, on the side opposite to which the sound is incident (this is the fluid loading on the non-incident or transmitted side) and

$Z_{wp}(\phi)$ is the wave impedance of the finite panel in an infinite baffle to a plane sound wave incident at an angle of ϕ to the normal to the panel, ignoring fluid loading.

It will be assumed that the fluid wave impedances on both sides are the same and the imaginary part of the fluid wave impedance will be ignored [1]. That is

$$Z_{wfi}(\phi) = Z_{wft}(\phi) = \rho c \sigma(\phi) \quad (2)$$

where ρ is the density of the fluid, c is the speed of sound in the fluid and $\sigma(\phi)$ is the radiation efficiency into the fluid of one side of the finite panel in an infinite baffle, whose vibration is due to a plane sound wave incident at an angle of ϕ to the normal to the panel.

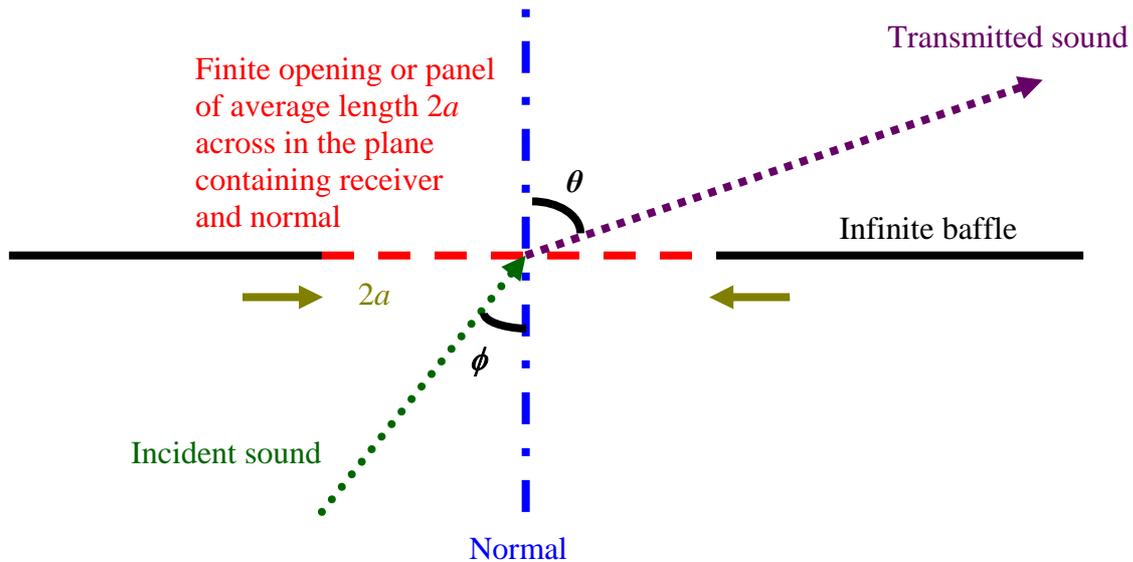


Figure 1. Sound incident at an angle of ϕ to the normal to a panel or opening and radiated at an angle of θ to the normal.

Reflections at the panel edges are ignored [1]. The rms normal velocity $v_{rms}(\phi)$ of the panel due to a plane sound wave incident at an angle of ϕ to the normal to the panel which

exerts an rms pressure $p_{irms}(\phi)$ is

$$v_{rms}(\phi) = \frac{p_{irms}(\phi)}{2\rho c\sigma(\phi) + Z_{wp}(\phi)}. \quad (3)$$

The transmitted rms sound pressure $p_{irms}(\theta, \phi)$ which is radiated by the panel on the non-incident side to a receiving point which is at an angle of θ to the normal to the centre of the panel and a large distance from the panel (see Figure 1) is [2]

$$p_{irms}(\theta, \phi) \propto v_{rms}(\phi) \frac{\sin[ka(\sin\theta - \sin\phi)]}{ka(\sin\theta - \sin\phi)} \quad (4)$$

where k is the wavenumber of the sound and $2a$ is the average length across the panel or opening in the plane containing the receiver and the normal to the panel or opening. Thus

$$p_{irms}(\theta, \phi) \propto \frac{p_{irms}(\phi)}{2\rho c\sigma(\phi) + Z_{wp}(\phi)} \frac{\sin[ka(\sin\theta - \sin\phi)]}{ka(\sin\theta - \sin\phi)}. \quad (5)$$

The case where the incident sound is generated by a sound source in a room or duct is now considered. We assume that the sound pressure waves are incident at different angles ϕ with random phases and mean squared sound pressures which are proportional to a weighting function $w(\phi)$.

$$|p_{irms}(\phi)|^2 \propto w(\phi). \quad (6)$$

The weighting function is to account for the fact that sound waves at grazing angles of incidence will have had to suffer more wall collisions and therefore be more attenuated before reaching the panel. The total mean square sound pressure $|p_{Trms}(\theta)|^2$ at the receiving point is

$$|p_{Trms}(\theta)|^2 \propto \int_{-\pi/2}^{\pi/2} \frac{w(\phi)}{|2\rho c\sigma(\phi) + Z_{wp}(\phi)|^2} \left\{ \frac{\sin[ka(\sin\theta - \sin\phi)]}{ka(\sin\theta - \sin\phi)} \right\}^2 d\phi. \quad (7)$$

The case when sound is incident from a source in a free field at an angle θ to the normal to the panel and the panel radiates at all angles ϕ into a room or duct is also of interest. In this case the weighting function $w(\phi)$ is to account for the fact that sound waves radiated at grazing angles will have had more wall collisions and therefore be more attenuated before reaching the receiving position which is assumed to be a reasonable distance from the panel or opening which is radiating the sound. In this second case, we have to integrate over all angles of radiation ϕ because of the reverberant nature of the sound. For this case, the impedance terms in the integral are functions of θ rather than ϕ and can be taken outside the integral. However in this study both cases are calculated using the formula for the first case which is shown above. This is because both cases should be the same by the principle of reciprocity and it is not clear which form of the formula is more appropriate.

For large values of ka , the two cases of the formula will be similar. If ka is much greater than 1, the function

$$\left\{ \frac{\sin [ka(\sin \theta - \sin \phi)]}{ka(\sin \theta - \sin \phi)} \right\}^2 \quad (8)$$

has a sharp maximum at $\phi = \theta$ and is symmetrical in both θ and ϕ about the point $\phi = \theta$. We can exploit these facts by evaluating the impedance terms for the first case at $\phi = \theta$ and taking them out side the integral. This gives the formula for the second case.

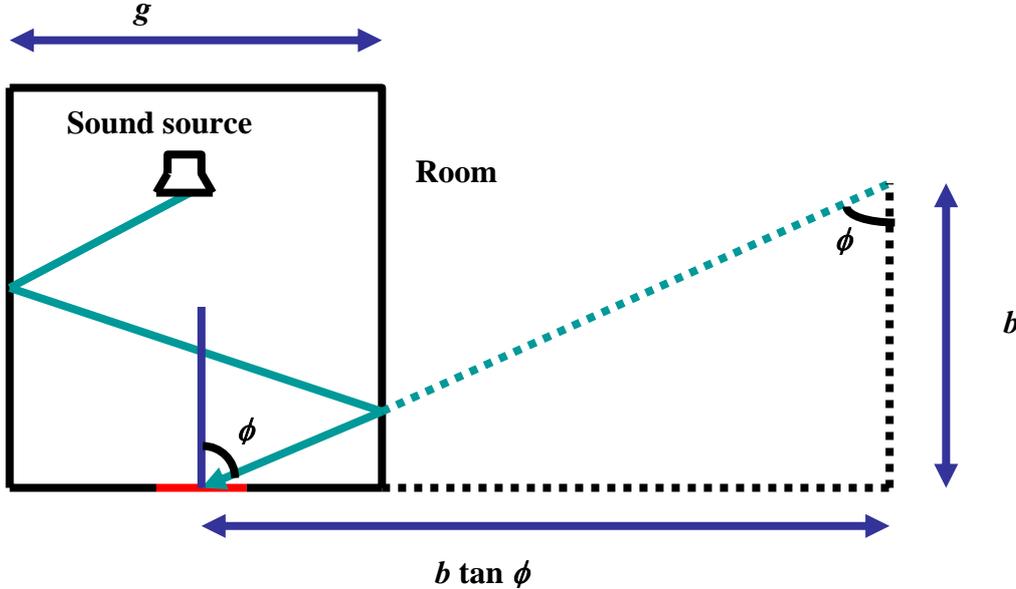


Figure 2. Calculating the number of wall reflections before sound hits the panel or opening at an angle of ϕ to the normal.

To derive the angular weighting function, we assume that the sound source is distance b from the surface of the room containing the panel or opening and that the room width is g in the plane containing the incident sound ray (see Figure 2). If the sound ray is incident at an angle of ϕ to the normal to the panel or opening, it travels a minimum distance of $b \tan \phi$ parallel to wall containing the panel or opening before hitting the wall. The sound which travels this minimum distance hits the wall approximately

$$n = \frac{b}{g} \tan \phi \quad (9)$$

times before reaching the panel or opening, where n is allowed to be a real number rather than an integer in order to give a smooth weighting function. If the sound absorption coefficient of the walls of the room is α , the sound intensity incident at an angle of ϕ to the normal is proportional to

$$w(\phi) = (1 - \alpha)^n = (1 - \alpha)^{\frac{b}{g} \tan |\phi|}. \quad (10)$$

Equation (10) gives us the weighting function $w(\phi)$. If α is zero, a uniform diffuse field will be obtained.

In this study we use the radiation efficiency of a panel of length $2a$, which we approximate with the following equation [2].

$$\sigma(\phi) = \begin{cases} \frac{1}{\frac{\pi}{2k^2a^2} + \cos \phi} & \text{if } |\phi| \leq \phi_l \\ \frac{1}{\frac{\pi}{2k^2a^2} + \frac{3\cos \phi_l - \cos \phi}{2}} & \text{if } \phi_l < |\phi| \leq \frac{\pi}{2} \end{cases} \quad (11)$$

where

$$\phi_l = \arccos \left(\sqrt{\frac{\pi}{2ka}} \right) \quad (12)$$

and k is the wavenumber of the sound and $2a$ is the length of the panel in the direction of the receiver.

For an opening with no panel in an infinite baffle we put $Z_{wp}(\phi) = 0$. For a finite panel in an infinite baffle we use the infinite panel result for $Z_{wp}(\phi)$. This result is expected to be the correct result when averaged over frequency, because this approach gives the correct result for point impedances when averaged over frequency and position on a finite panel [3].

$$Z_{wp}(\phi) = m\omega \left\{ j \left[1 - \left(\frac{\omega}{\omega_c} \right)^2 \sin^4(\phi) \right] + \eta \left(\frac{\omega}{\omega_c} \right)^2 \sin^4(\phi) \right\} \quad (13)$$

where m is the surface density (mass per unit area) of the panel, η is the damping loss factor of the panel, ω_c is the angular critical frequency of the panel and ω is the angular frequency of the sound.

In a duct, the directivity of the sound source is also included. The sound source is modelled as a line source of length $2r$ where r is the radius of the sound source. The directivity of the sound source is proportional to

$$\left[\frac{\sin(kr \sin \phi)}{kr \sin \phi} \right]^2 \quad (14)$$

where k is the wavenumber.

For angles of radiation close to 90° to the normal to the panel or opening, the effect of the diffraction by the panel or opening or by the finite baffle in which the panel or opening is mounted needs to be included [4]. $p(\theta)$ is the ratio of the increased sound pressure to the sound pressure without the baffle for an angle of incidence or radiation of θ . The increase in sound pressure due to radiation (or incidence) of sound pressure of wavenumber k normally from (or onto) a panel or opening in a baffle of average length $2L$ in the plane containing the receiver (or source) and the normal to the baffle is

$$p(0) = \begin{cases} 1 + \sin^2(kL) & \text{if } kL \leq \frac{\pi}{2} \\ 2 & \text{if } kL > \frac{\pi}{2} \end{cases}. \quad (15)$$

The limiting angle below which the sound pressure does not vary with angle of radiation (or incidence) is θ_m . Notice that if $L = a$, $\theta_m = \phi_l$.

$$\theta_m = \begin{cases} 0 & \text{if } kL \leq \frac{\pi}{2} \\ \arccos\left(\sqrt{\frac{\pi}{2kL}}\right) & \text{if } kL > \frac{\pi}{2} \end{cases}. \quad (16)$$

There is no increase of sound pressure at grazing angles of transmission (or incidence).

$$p\left(\frac{\pi}{2}\right) = 1. \quad (17)$$

$p(\theta)$ is obtained by linear interpolation.

$$p(\theta) = \begin{cases} p(0) & \text{if } |\theta| \leq \theta_m \\ \frac{p(0)\left(\frac{\pi}{2} - |\theta|\right) + p\left(\frac{\pi}{2}\right)(|\theta| - \theta_m)}{\frac{\pi}{2} - \theta_m} & \text{if } \theta_m < |\theta| \leq \frac{\pi}{2} \end{cases}. \quad (18)$$

The relative sound pressure level $L(\theta)$ in the direction θ is

$$L(\theta) = 20 \log_{10} \left(\left| p_{Trms}(\theta) \right| p(\theta) \right) - 20 \log_{10} \left(\left| p_{Trms}(0) \right| p(0) \right). \quad (19)$$

3. COMPARISON WITH PUBLISHED RESULTS

In this section, the prediction method described in the previous section is compared with experimental results and prediction methods for finite size panels and finite size openings from the literature. Results are presented on a logarithmic scale of Strouhal number. The Strouhal number is defined as the ratio of the average distance across the finite flat panel or finite opening, in the plane containing the receiver and the normal to panel or opening, to the wavelength of the sound in the air.

Stead [5] measured the sound insulation directivity of a window installed in one wall of a room. The sound was incident at an angle to the normal to the window from outside the room. This is the opposite direction to the calculation method used in this paper, but is expected to give similar results because of the principle of reciprocity. The window was 1.45 m wide by 2.12 m high. The glass was 6 mm thick. The wall of the room containing the window was part of the external wall of a larger building which served as a baffle. The

internal dimensions of the room were 2.88 m wide by 3 m high by 5.12 m deep. The loudspeaker was 20 m from the middle of the window. The edge of the building in the direction of the measurements was 11 m from the centre of the window. Thus the baffle length was set to twice this distance, namely 22 m. To show the result of the diffraction correction, Stead's results at an angle of 90° to the external normal to the window, in a horizontal plane through the centre of the window, are compared with the theory presented in this paper in Figure 3.

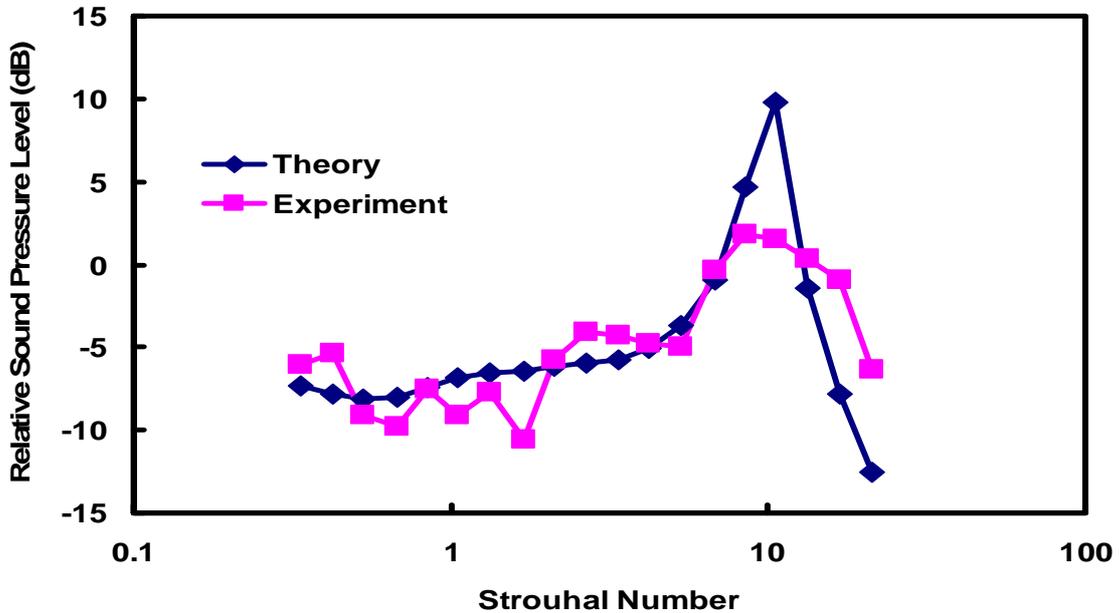


Figure 3. The sound pressure level at 90° relative to that at 0° as a function of Strouhal number for 6 mm thick glass installed in the wall of a room.

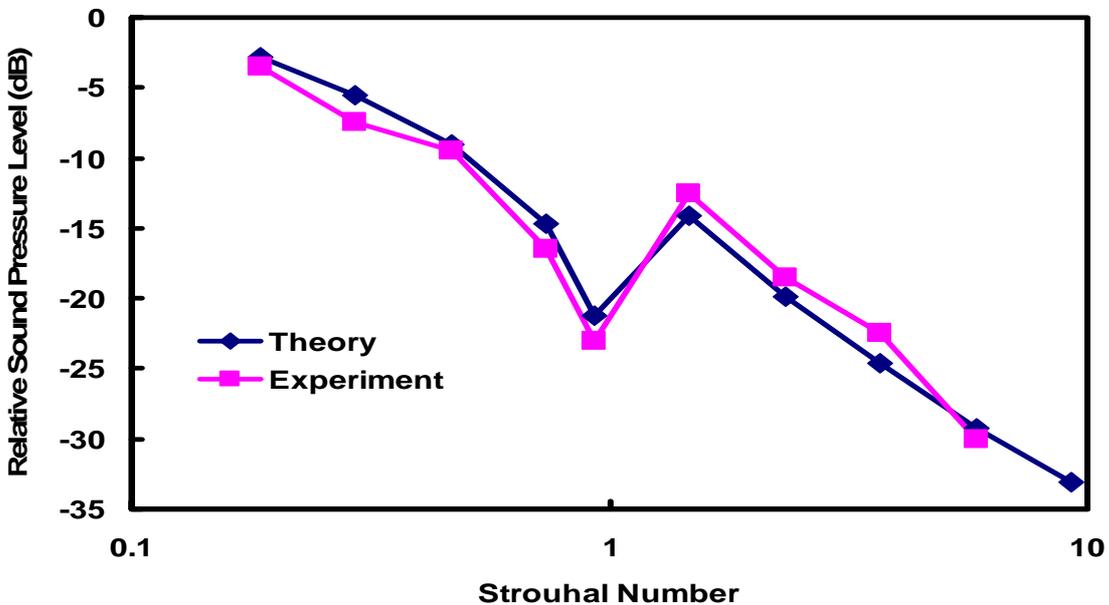


Figure 4. The sound pressure level at 90° relative to that at 0° as a function of Strouhal number for an un baffled duct end opening.

Stead's measured reverberation times were used to calculate the average wall absorption coefficients of the room for use in the calculation of the weighting function. Apart from some ripple in the experimental results, the agreement between theory and experiment is good below the frequency at which coincidence occurs. The agreement between the experimental

and theoretical results at coincidence is better at lower angles of incidence.

Sutton [6] measured the directivity of eight un baffled duct end openings in an anechoic room. Figure 4 shows a comparison of his measurements at 90° for an 80 mm square cross section duct of length 750 mm. Sutton used two sound sources. The sound source (loudspeaker) directivity was modelled by assuming a source diameter of 300 mm for Strouhal numbers less than one and a source diameter of 30 mm for higher Strouhal numbers. Note that the lower frequency sound source would not have actually been 300 mm in diameter. The 300 mm diameter is used to model the directivity of the sound radiation into the duct. If only plane waves had been excited in the duct, a sound source diameter of infinity would have been used. A practical absorption coefficient value of 0.05 was assumed for the internal walls of the duct. The agreement between theory and experiment is very good, although this agreement does depend on the ad hoc values adopted for the notional diameter of the sound sources.

4. CONCLUSIONS

The theoretical model presented in this paper can be used to successfully predict the sound pressure level radiated at a particular angle to the normal of a panel or opening, relative to the sound pressure level radiated in the direction of the normal. The theory depends on the average length across the radiating object, in the plane containing the receiver and the normal to the radiating object, divided by the wavelength of the sound in air, and is independent of the width of the object at right angles to the direction of the observer. The relative sound level radiated from a panel is relatively independent of the Strouhal number apart from a strong peak at coincidence. The relative sound level radiated from an opening decreases as the Strouhal number increases.

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