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SIMPLIFICATIONS AND SOLUTIONS OF DIFFRACTION FUNCTIONS IN NONLINEAR ACOUSTIC PARAMETER MEASUREMENT

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Abstract

This paper presents an analytical formulation for correcting the diffraction associated to the second harmonic of an acoustic wave, more compact than that usually used. This new formulation, resulting from an approximation of the correction applied to fundamental, makes it possible to obtain simple solutions for the second harmonic of the average acoustic pressure, but sufficiently precise for measuring the parameter of nonlinearity B/A in a finite amplitude method. Comparison with other expressions requiring numerical integration, show the solutions are precise in the nearfield.

1. INTRODUCTION

In acoustic parameters measurements of a medium, it is necessary to take into account the diffraction effects of the ultrasonic source to improve the precision of measurements. The measurement cells usually used in transmission consist of two circular transducers (one used as source and the second as detector). In these situations the detector will translate into electric voltage the average acoustic pressure on its reception area. The analytical solutions describing this average pressure can be formulated as the sum of two terms, one corresponding to the propagation of a plane wave, and the other including the effects of diffraction generated by the geometry of the source-detector unit.

The attenuation α and velocity c can be obtained in the case of linear acoustics. Different authors [1- 4] gave exact and asymptotic expressions of the average pressure received by a circular transducer. These expressions permit to establish correction functions of diffraction in velocity and attenuation measurements [5, 6].

On the other hand B/A parameter is measured in the field of nonlinear acoustics. The first measurements of B/A parameter by finite amplitude methods rested on an analytical expression of the second harmonic by considering the propagation of a plane wave [7-9]. Various authors [10, 11] then improved the precision of these methods by including a function to correct diffraction effect resulting from the relation established by Ingenito and Williams [12] for the average pressure exerted by the second harmonic. However, the correction of diffraction obtained is not very practical because it can be evaluated only by numerical integration.

The objective of this paper is to show that one can obtain a simple and precise form by simplifying the correction function of diffraction for the fundamental. Then we will give simple expressions of the average pressure exerted by the second harmonic, including diffraction and attenuation effects. We will show that the results obtained are equivalent to those establish by Coob and validated in measurement systems [10]. But before establishing this result it is necessary to present the various corrections of diffraction applicable to fundamental from the acoustic pressure.

2. CORRECTION OF DIFFRACTION FOR THE FUNDAMENTAL

2.1. Function *$D_1(z)$* of diffraction correction for the fundamental

For the nondissipative case $(\alpha_1 = 0)$, Williams [1] gave the exact expression of the average velocity potential (figure 1) :

$$\langle \phi_1(r,z) \rangle = \frac{jU_0}{k} e^{jkz} - \frac{j\,4U_0}{k\pi} \int_0^{\pi/2} e^{jk[z^2+4a^2\cos(\theta)^2]^{1/2}} \sin(\theta)^2 \, d\theta \qquad (1)$$

The first term represents the velocity potential in the case of a plane wave, therefore the average velocity potential on the area of reception is $\phi_{10}(z) = \frac{jU_0}{k} e^{jkz} = \langle \phi_{10}(r,z) \rangle$. The second part of equation (1) corresponds to diffraction effect on the velocity potential, with U_0 the source amplitude velocity and k the wave number.

The average acoustic pressure applied on the receiver is expressed in the form: $\langle p_1(r,z) \rangle = -j\rho_o \omega \langle \phi_1(r,z) \rangle$

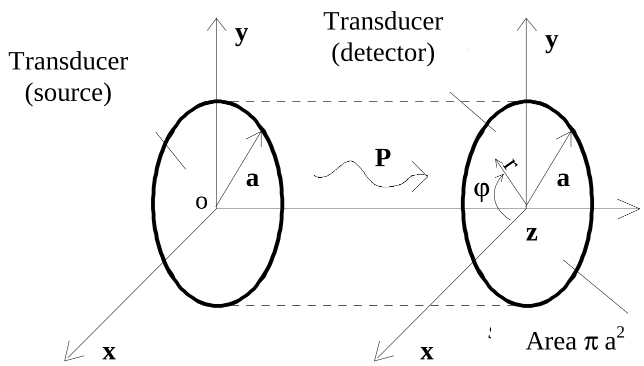


Figure 1. Geometrical configuration of the source-detector.

The correction diffraction function *$D_1(z)$* allows to adapt the theoretical plane wave to a real situation. Consequently :

$$D_1(z) = \frac{\langle \phi_1(r,z) \rangle}{\langle \phi_{10}(r,z) \rangle} = \frac{\langle p_1(r,z) \rangle}{\langle p_{10}(r,z) \rangle} \quad \text{with} \quad \langle p_{10}(r,z) \rangle = P_0 e^{jk'z} \quad (2)$$

$<p_{10}(r,z)>$ is the average pressure provided by the fundamental in the case of a plane wave, with $P_0 = \rho_0\, c_0\, U_0$ is the average acoustic pressure (the source). Thus the module of the average pressure is given in dissipative medium in the form:

$$\left|\langle p_1(r,z)\rangle\right| = P_0\, e^{-\alpha_1 z}\left|D_1(z)\right| \quad (3)$$

The exact expression of $D_1(z)$ is obtained with the Williams solution (1) :

$$D_1(z) = 1 - \frac{4}{\pi} e^{-jkz} \int_0^{\frac{\pi}{2}} e^{\,jk[z^2 + 4a^2 \cos(\theta)^2]^{\frac{1}{2}}} \sin(\theta)^2\, d\theta \quad (4)$$

2.2. Simplifications of the function of correction $D_1(z)$

For z > a Bass [2] gave a very good approximation of the solution (4) which can be simplified for $\xi(z) >> 1$ in the form:

$$D_1(z) \approx 1 - \left(1 - \frac{\xi(z)^2}{2(ka)^2}\right)\left(\frac{2}{\pi\xi(z)}\right)^{\frac{1}{2}} e^{-j\frac{\pi}{4}} \quad (5)$$

with $\xi(z) = \frac{k}{2}\left(\sqrt{z^2 + 4a^2} - z\right)$. This expression was used by Coob [10] to reduce $D_1(z)^2$ and to evaluate the average pressure of the second harmonic.

By limiting to the 1st order the development of $[\ ]^{1/2}$ in the solution (4), Rogers *et al.* [4] obtained a good approximation in the form:

$$D_1(z) \approx 1 - e^{-j\frac{ka^2}{z}}\left[J_0\left(\frac{ka^2}{z}\right) + jJ_1\left(\frac{ka^2}{z}\right)\right] \quad (6)$$

It is valid for all the values of z/a if $(ka)^{1/2} >> 1$, and the error take back by this simplification compared to the exact solution (4) is lower than 0.4 % for $ka = 100$ for $z/a<(ka)^{1/2}$, the preceding condition implies $ka^2/z > (ka)^{1/2} >> 1$, and one can reduce the expression (6) by using the asymptotic developments of the Bessel functions :

$$D_1(z) \approx 1 - \left(\frac{2z}{\pi k a^2}\right)^{1/2} e^{-j\pi/4} = 1 - g(z) \quad (7)$$

where we define $g(z)$ as the diffraction function, related to the parameters of source a and k, and having this property $\lim_{ka\to\infty}[g(z)] = 0$ (plane wave case).

2.3. Comparison of the various expressions of $D_1(z)$

Figures 2a and 2b represent the module $|D_1|$ of the different expressions. We use for y axis two variables z/a and $s = z\lambda/a^2 = 2\pi z/ka^2$. With the variable s, we can distinguish the near field $(s \le 1)$ and the far field $(s > 1)$. Simulations are obtained with $a = 1$ cm and $ka = 125$.

Simplification (6) is confused with the exact solution (4), and the asymptotic expressions (7) constitute a good approximations in the near field (fig. 2a). They diverge from the exact solution for $z/a > 60$ ($s > 3$) (fig. 2b). The relative error (fig. 2c) confirms the range of validity $z/a < (ka)^{1/2}$ ($s < 2\pi/(ka)^{1/2}$) for $(ka)^{1/2} >> 1$. Thus the relative error is lower than *0.7 %*. The lower limit being in any event limited in experiments to the appearance of standing waves in the measuring cell.

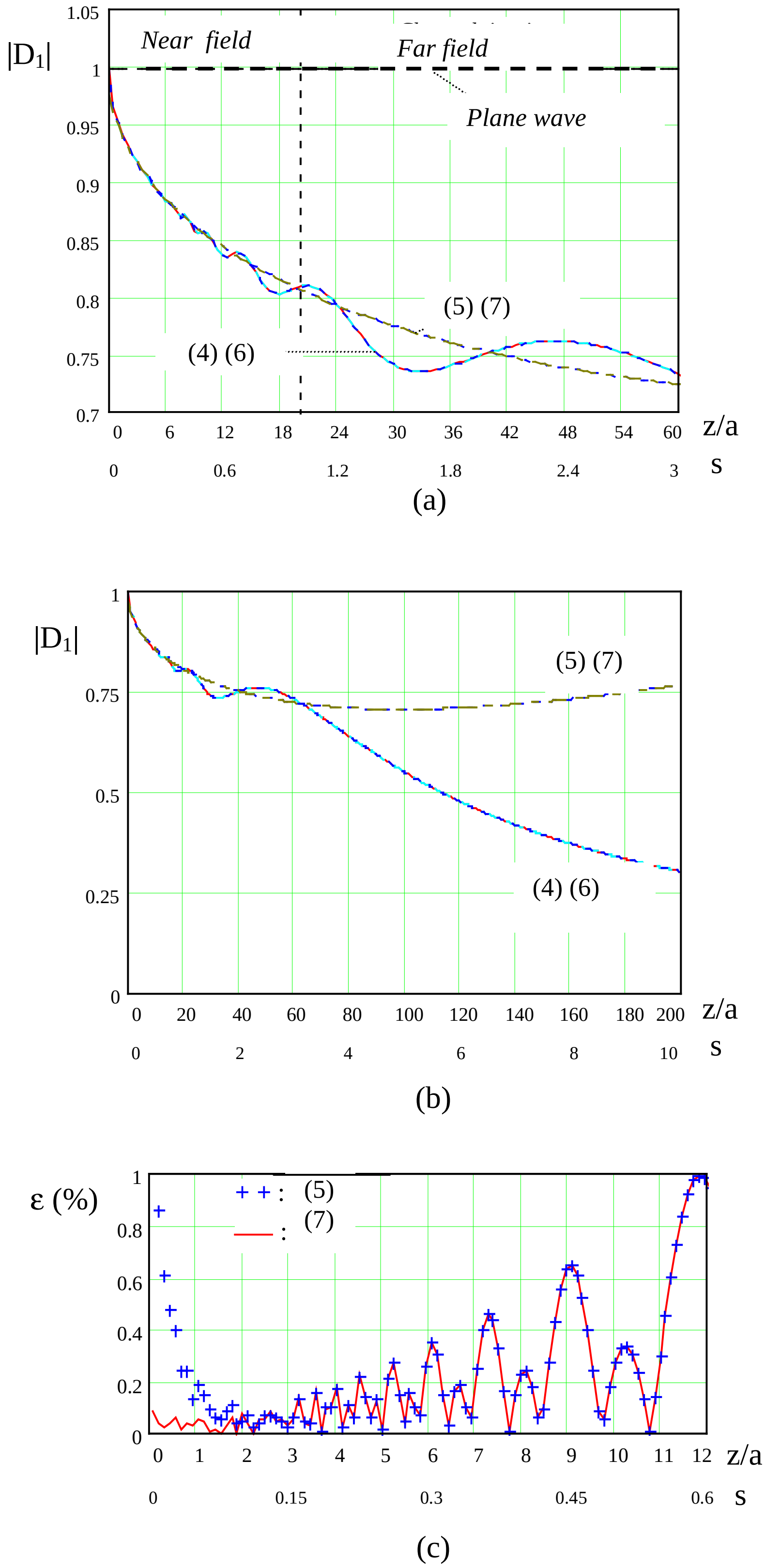


Figure 2. Functions of diffraction correction . Comparison with the exact solution of Williams (4). Figure (c) present the relative error between exact solution (4) and solutions (5) and (7)

3. CORRECTION OF DIFFRACTION OF THE 2ND HARMONIC

3.1. Function of diffraction correction for the second harmonic

Ingenito and Williams [12] obtain an equation for the second harmonic in the case of monochromatic wave in non dissipative medium. We can find in [10] a good approximation of this solution which can be used in the dissipative case.

Average potential ϕ_2 is given by:

$$\langle \phi_2(r,z)\rangle \approx -\frac{\beta k^2}{4c_0}\int_0^z e^{jk\psi} e^{-\alpha_2\psi}\left\langle \phi_1\left(r, z-\frac{\psi}{2}\right)^2\right\rangle e^{-2\alpha_1(z-\psi)}d\psi \quad (8)$$

with : $\beta = 1+\frac{1}{2}B/A$ and α_2 : the second harmonic attenuation,

$p_2 = -2j\rho_o\omega\phi_2$, and B/A is the parameter of non-linearity.

The relation (8) is the reference analytical solution for second harmonic average velocity potential in dissipative medium. Ingenito and Williams [12] showed that a good approximation consisted in replacing $<\phi_1^2>$ by $<\phi_1>^2$ in the expression of $<\phi_2>$. Thus, we can write:

$$\langle \phi_1(r,z)\rangle^2 = \langle \phi_{10}(r,z)\rangle^2\left[1-\left\{2g(z)-g(z)^2\right\}\right] = \langle \phi_{10}(r,z)\rangle^2\left[1-f(z)\right] \quad (9)$$

$$\text{with } f(z) = 2g(z)-g(z)^2 = 1-D_1(z)^2 \quad (10)$$

with these relations and some arrangements one obtain for the average pressure of the second harmonic according to $D_1(z)$:

$$\langle p_2(r,z)\rangle \approx \left(KP_0^2 e^{-2\alpha_1 z}\int_0^z e^{(2\alpha_1-\alpha_2)\psi} D_1\left(z-\frac{\psi}{2}\right)^2 d\psi\right) e^{j2kz} \quad (11)$$

with $K = \frac{(2+B/A)\omega}{4\rho_0 c_0^3}$.

The function of diffraction of the second harmonic can be given by $D_2(z) = \frac{\langle p_2(r,z)\rangle}{\langle p_{20}(r,z)\rangle}$.

Thus while considering $D_2(z)$ independent of the attenuation, which amount to separating the effects of the attenuation and diffraction, we obtain :

$$D_2(z) = 1-\frac{1}{z}\int_0^z f\left(z-\frac{\psi}{2}\right)d\psi = \frac{1}{z}\int_0^z D_1\left(z-\frac{\psi}{2}\right)^2 d\psi \quad (12)$$

3.2. Simplifications of $D_2(z)$ and $<p_2(r,z)>$

According to (12) the correction $D_2(z)$ is related to $D_1(z)^2$ wich can be simplified. Since $D_1(z)^2 = 1-2g(z)+g(z)^2$ and $\lim_{ka\to\infty}\left[g(z)\right]=0$, we can neglect the term $g(z)^2$ for large value of ka. Thus the solution (7) and (5) with the following conditions: $z/a < (ka)^{1/2}$ and $(ka)^{1/2} >> 1$, bring to this simplified expression :

$$D_1(z)^2 \approx 1-2\left(\frac{2.z}{\pi k\, a^2}\right)^{1/2} e^{-j\pi/4} \quad (13)$$

$$D_1(z)^2 \approx 1-2\left(1-\frac{\xi(z)^2}{2(ka)^2}\right)\left(\frac{2}{\pi\xi(z)}\right)^{\frac{1}{2}} e^{-j\frac{\pi}{4}} \quad (14)$$

We can thus take advantage of the simpler expression (13) to calculate a diffraction function $D_2(z)$. In this case the integral (12) can be evaluated, and it's give :

$$D_2(z) \cong 1-C\sqrt{\frac{z}{k\,a^2}}\, e^{-j\pi/4} \quad \text{with} \quad \mathrm{C}=\frac{4}{3\sqrt{\pi}}\left(2\sqrt{2}-1\right)\approx 1.375 \quad (15)$$

Finally with (11), we can established a simple expression sufficiently precise able to give the average pressure provided by the second harmonic on a receiver with the same dimensions of the source:

$$\left|\left\langle p_2(r,z)\right\rangle\right| \approx K\; P_0{}^2\left(\frac{e^{-\alpha_2 z}-e^{-2\alpha_1 z}}{2\alpha_1-\alpha_2}\right)\left|D_2(z)\right| \quad (16)$$

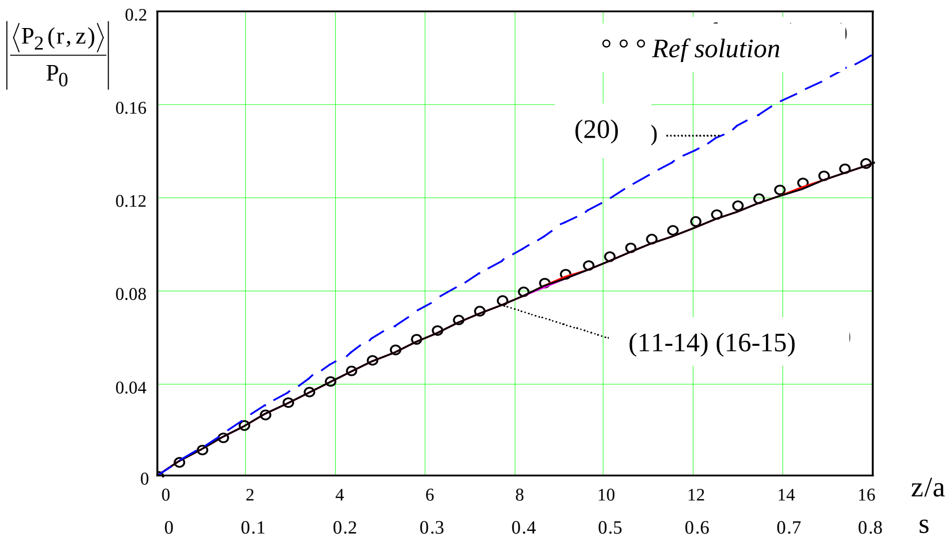
3.3. Comparison of solutions for the average presses $|\langle p_2(r,z)\rangle|$

We simulate the expressions of the relative average pressure $|\langle p_2(r,z)\rangle|/P_0$ in two extreme mediums in term of attenuation and nonlinear effects:
Water : $c_0=1483$ m/s, $\rho_0=1000$ kg/m^3, $\alpha_0=0.25.10^{-13}$ Npm^{-1}Hz^{-2}, $\alpha_2=4.\alpha_1$, $B/A=5.2$
Glycerol : $c_0=1909$ m/s, $\rho_0=1260$ kg/m^3, $\alpha_0=26.10^{-13}$ Npm^{-1}Hz^{-2}, $\alpha_2=4.\alpha_1$, $B/A=9.4$

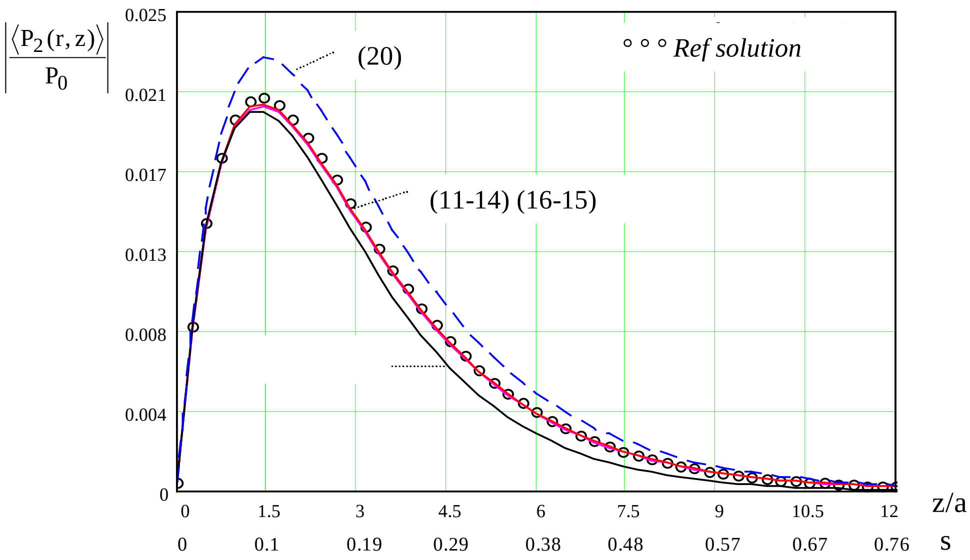
The conditions, close to the Coob experiments are: f = 3 MHz, a = 1 cm, I_0 = 0.5 W/cm² for water and I_0 =10 W/cm² for glycerol, with $I_0 = P_0^2/(2\rho_0 c_0)$.

The results are presented on figure 3 and one notes that our solution (15-14) is similar with that obtained by Coob [10]. Importance of the diffraction correction $D_2(z)$ is visualized by the representation of the simple case of a plane wave, i.e. for $D_2(z) = 1$.
We also simulated the relative average pressure obtained with the reference solution (8) and the King integral [13-14] for fundamental ϕ_1.

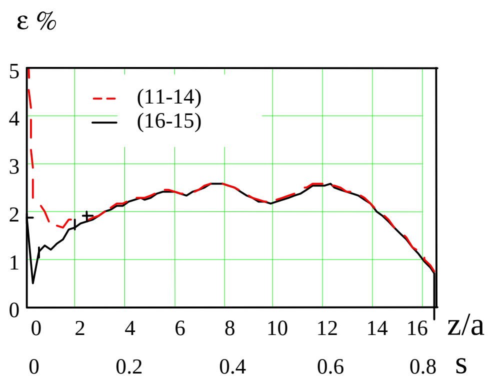
Relative errors between the reference solution and the solutions (15-14) and (11-14), are presented figure 3 c-d for water and glycerol, where simulations are carried out with a tolerance of 10^{-6} for the calculation of the integrals with the Mathcad software®. They show that the solution (16-15) is overall more precise than the solutions (11-14) and under the conditions adopted for simulation. Moreover, the computing time necessary to the solution (16-15) is much weaker than that of the reference solution which include a triple integral.

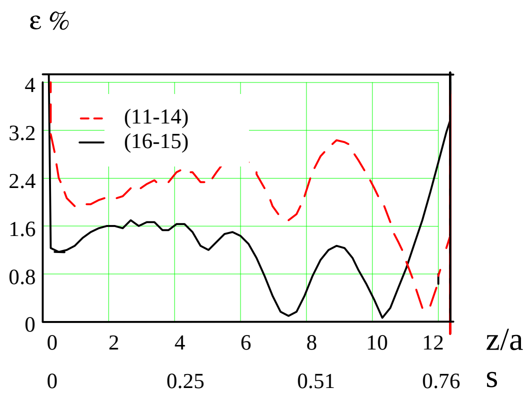


(a)



(b)





(c) (d)

Figure 3. Simulations of analytic solutions of the second harmonic average pressure for water (a) and glycerol (b) Relative variation between reference solution and solutions (16-15), (11-14) for water (c) and glycerol (d).

4. CONCLUSION

We showed in this article that we can obtain a function of diffraction correction for the second harmonic much simpler than those usually used. This new formulation is obtained from a simplification of the correction applied to the fundamental acoustic pressure.
We can use this new and simple expression to describe, with a very good precision, the second harmonic pressure detected by a transducer. It's can be exploited in measurements of non-linearity parameter B/A. Another interest of these simple analytical solutions is the significant reduction of the computing times when they are used in processes of simulation of systems working in the field of nonlinear acoustics.

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