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SOUND ABSORPTION OF A VIBRATING PLATE WITH PIEZOELECTTIC MATERIALS AND PASSIVE ELECTRICAL NETWORKS

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Abstract

The possibility of sound absorbing by a vibrating plate with piezoelectric material shunted with passive electrical circuit is investigated. Using Lagrange's approach, a governing equation for the flexural vibration of the plate attached with piezoelectric wafer shunted with RL circuits is derived. As a consequence, the effective mechanical impedance for the corresponding plate is obtained. A measurement based on an impedance tube is also conducted to gauge the influence of the shunting RL circuits on the sound absorption. Good agreements achieved when the measured sound absorption coefficients are compared with the calculations from the mathematics model established in this paper.

1. INTRODUCTION

Piezoelectric materials shunted with electric circuits add additional damping to the structure, and subsequently reduce structure vibrations and noise radiations. This method is called passive shunt damping technique. Since Hagood and Flotow[1] presented a quantitative analysis of the piezoelectric materials shunted with a series RL circuit in 1991, much attention has been paid to the theoretical modelling and experimental validation of this technique. Among them, Wu[2] successfully introduced parallel RL circuits for the structure vibration control, while Hollkamp[3] and Wu[4] investigated multi-mode control by using only one piezoelectric wafer. Moheimani *et al.*[5] considered the problem of vibration control using shunted piezoelectric material as a feedback control technique. His observation paved a way for the potential application to modern and robust control design.

Although much work has been contributed to the beam or plate vibration control using shunted with piezoelectric materials. The literature seems to record few work on sound absorption with this technique. Kim[6] experimentally investigated the sound transmission noise reduction of a plate shunted with a RL circuit. Zhang[7] discussed the acoustic echo cancellations by a single piezoelectric layer shunted with various circuits. But no experimental results were presented.

In this paper, a theoretical model is established to predict the sound absorbing characteristics of a thin plate on which a piezoelectric patch shunted with RL circuits is mounted. A test based on an impedance tube and transfer function method is made to measure the sound absorption coefficient for a plate shunted with piezoelectric materials. Numerical results agree well with the measurements.

2. A SIMPLIFIED ANALYTICAL MODEL

Piezoelectric patches behave electrically like a capacitor and mechanically like a stiff spring. The equivalent electrical model of a piezoelectric patch shunted with a series RL circuit is shown in Fig.1. The equivalent compliance of piezoelectric patch shunted with RL circuits can be written as [1]

$$s_{ii}^{su} = s_{ii}^{E}\left(1 - k_{31}^{2}\overline{Z}^{EL}\right) \tag{1}$$

where s_{ii}^{E} is the shorted circuit compliance of the piezoelectric patch, k_{31} is the electromechanical coupling coefficient, $\overline{Z}^{EL}$ is the non-dimensional electrical impedance as defined in reference[1]. Equation (1) indicates that the compliance of the piezoelectric patch can be modified by the shunting RL circuit. In other words, the influence of the shunting circuit is implied in the changeable equivalent compliance s_{ii}^{su}.

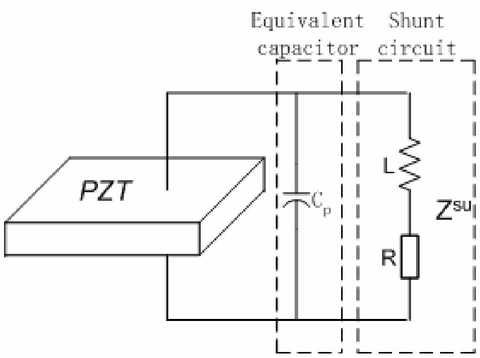

Fig. 1. Electric model of a piezoelectric patch

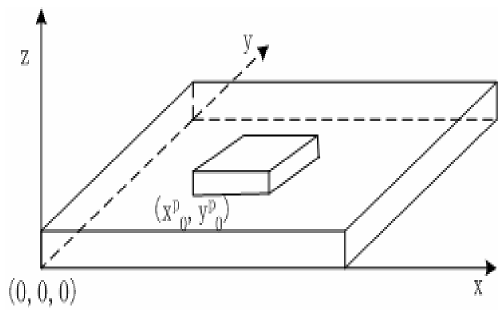

Fig.2. A thin plate attached with a shunted PZT

As shown in Fig.2, a piezoelectric patch shunted with a RL circuit is attached to a rectangular thin plate. The plate has a length L_a ,width L_b and thickness t_b . The piezoelectric patch has a length L_a^p , a width L_b^p and a thickness t_p . The thin plate makes flexural vibration based on the Kirchhoff's hypothesis, and the patch vibrates with the same displacement. In order to derive the equation of motion, the Lagrange’s equation is applied.

$$\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{q}_r(t)}\right) - \frac{\partial T}{\partial q_r(t)} + \frac{\partial V}{\partial q_r(t)} = Q_r(t) \tag{2}$$

where $T = E_p + E_b$, and E_p , E_b represent the kinetic energy of the piezoelectric patch and the thin plate, respectively. $V = U_p + U_b$, and U_p, U_b are the mechanical strain energy of the piezoelectric patch and the pate, respectively. The electrical energy is omitted in this system because the equivalent compliance of the patch, which is used in calculation of the mechanical strain energy, allows for the influence of RL circuits. The governing equation can be derived as

$$(j\omega \mathrm{M} + \frac{1}{j\omega}\mathrm{K}^b + \frac{1}{j\omega}\mathrm{K}^p)\overline{\dot{W}}(t) = \mathrm{N}(t) \tag{3}$$

where M, K^p, and K^b are the system mass, piezoelectric stiffness, and plate stiffness matrices respectively. $\mathrm{N}(t)$ is the generalized force column vector. $\overline{\dot{W}}(t)$ is the average mode velocity column vector, $\overline{\dot{W}}_{kl}(t) = \dot{q}_{kl}(t)\overline{\phi}_{kl}$, here $\overline{\phi}_{kl} = \frac{1}{L_a L_b}\int_0^{L_a}\int_0^{L_b}\phi_{kl}(x,y)dxdy$, and $\phi_{kl}(x,y)$ is the mode function. The sign '•' means the time differential. And

$$\begin{aligned}\mathrm{K}^p_{mn,kl} = {} & \frac{1}{3\overline{\phi}_{kl}}\left[\left(\frac{t_b}{2}+t_p\right)^3 - \left(\frac{t_b}{2}\right)^3\right]\frac{s_{11}^{su}}{(s_{11}^{su})^2-(s_{12}^{su})^2}\int_{x_0^p}^{x_0^p+L_a^p}\int_{y_0^p}^{y_0^p+L_b^p}\left[\frac{\partial^2\phi_{mn}}{\partial x^2}\frac{\partial^2\phi_{kl}}{\partial x^2}+\right.\\ & \left.\frac{\partial^2\phi_{mn}}{\partial y^2}\frac{\partial^2\phi_{kl}}{\partial y^2} - \frac{2s_{12}^{su}}{s_{11}^{su}}\frac{\partial^2\phi_{mn}}{\partial x^2}\frac{\partial^2\phi_{kl}}{\partial y^2} + \frac{2(s_{11}^{su}+s_{12}^{su})}{s_{11}^{su}}\frac{\partial^2\phi_{mn}}{\partial x\partial y}\frac{\partial^2\phi_{kl}}{\partial x\partial y}\right]dxdy\end{aligned} \tag{4}$$

is the element of piezoelectric stiffness matrix.

$$\mathrm{M}_{mn,kl} = \frac{1}{\overline{\phi}_{kl}}\int_{x_0^p}^{x_0^p+L_a^p}\int_{y_0^p}^{y_0^p+L_b^p}\rho_p t_p \phi_{mn}\phi_{kl}dxdy + \frac{1}{\overline{\phi}_{kl}}\int_0^{L_a}\int_0^{L_b}\rho_b t_b \phi_{mn}\phi_{kl}dxdy \tag{5}$$

is the element of the system mass matrix, and

$$\begin{aligned}\mathrm{K}^b_{mn,kl} = \frac{D_b}{\overline{\phi}_{kl}}\int_0^{La}\int_0^{L_b}&\left[\frac{\partial^2\phi_{mn}}{\partial x^2}\frac{\partial^2\phi_{kl}}{\partial x^2}+\frac{\partial^2\phi_{mn}}{\partial y^2}\frac{\partial^2\phi_{kl}}{\partial y^2} - 2\sigma_b\frac{\partial^2\phi_{mn}}{\partial x^2}\frac{\partial^2\phi_{kl}}{\partial y^2}\right.\\ &\left.+2(1-\sigma_b)\frac{\partial^2\phi_{mn}}{\partial x\partial y}\frac{\partial^2\phi_{kl}}{\partial x\partial y}\right]dxdy\end{aligned} \tag{6}$$

is the element of the plate stiffness matrix.

When the plate is backed with air cavity, which is simply modelled as a soft spring in low frequency, the governing equation becomes

$$(j\omega \mathrm{M} + \frac{1}{j\omega}\mathrm{K}^b + \frac{1}{j\omega}\mathrm{K}^p + \frac{1}{j\omega}\mathrm{K}^a)\overline{\dot{W}}(t) = \mathrm{N}(t) \tag{7}$$

where K^a is the cavity equivalent stiffness matrix. Now we can calculate the surface impedance of the plate, and then the sound absorption coefficient. As can be seen in equation (4), the piezoelectric stiffness is a function of the equivalent compliance s_{ii}^{su}. So the surface impedance of the plate correlates with the shunting RL circuit. Numerical results and experimental measurements will show the influence of the shunting RL to the sound absorption coefficient.

3. EXPERIMENTAL SETUP

The sound absorption coefficients are measured with two microphones using transfer function method [8]. Fig.3 shows the sketch of the experimental set-up, where the impedance tube has a rectangular cross section with size of 149.4×109.6mm and a length of 1630mm. The aluminium plate attached with a PZT-5 wafer is clamped all edges in the tube wall and with a 240mm distance from the tube end. A loudspeaker is mounted in the other end of the tube. The distance between the two microphones is 100mm. The cut frequency of the impedance tube is about 1137Hz. Tables 1 and 2 show the parameters of the aluminium plate and the PZT-5 patch, respectively. The PZT patch is adhered to the center of the plate. The first mode resonant frequency of the system is 200-300Hz, and therefore a large inductor L is required. As shown in Figure 4, two operational amplifiers are required to construct the simulated large inductor.

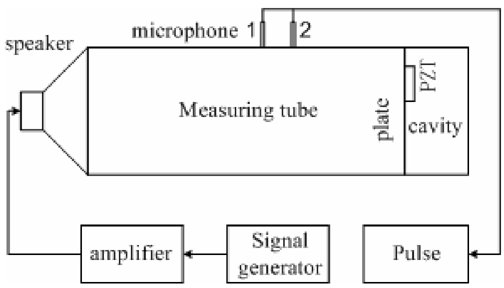

Figure.3. Experimental setup sketch

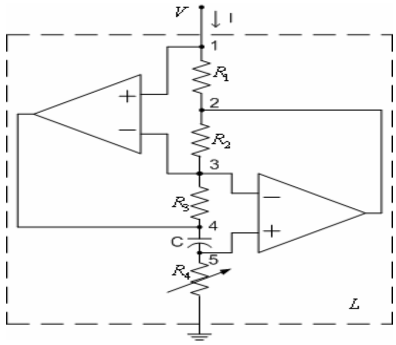

Figure.4. Simulated inductor

Table 1. Aluminium plate parameters

Parameter	Value
width (mm)	109. 6
length (mm)	149. 4
thickness (mm)	0.46
Young's modulus, E (GPa)	71.5
density, ρ (kg/m^3)	2790
Poisson's ratio, v	0.34
Loss factor	0.005

Table2. PZT-5 patch parameters

Parameter	Value
width (mm)	24. 8
length (mm)	64. 6
thickness (mm)	0.46
compliance s_{11}^{E} (m^2/N)	16. 4×10. ˆ−12
density, ρ (kg/m^3)	7750
Poisson's ratio, v	0.35
Coupling coefficient k_{31}	0.34

4. NUMERICAL AND EXPERIMENTAL ANALYSIS

In this section, numerical calculation of the sound absorption of the plate is presented. Two cases, a R shunting circuit and a series RL shunting circuit, are discussed. At the same time, the measurements are conducted to compare with numerical analysis.

4.1 Numerical results

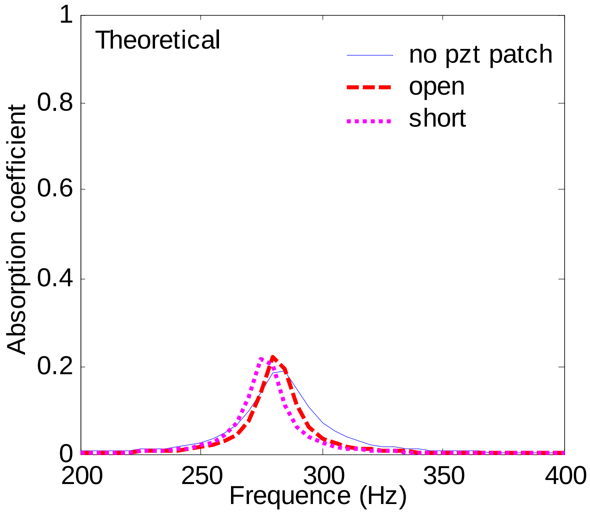

Fig.5.Absorption coefficient-shorted and open circuit (theoretical)

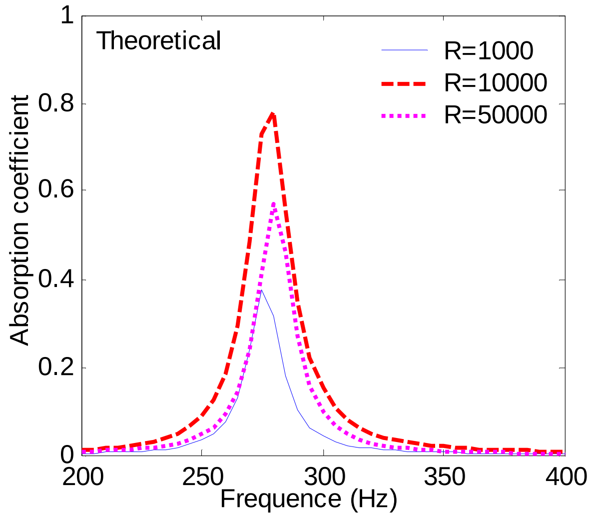

Fig.6. Absorption coefficient-various R (theoretical)

In the theoretical analysis, two factors of energy dissipation are taken into account. Those are the loss factor of the plate and the damping of the piezoelectric shunting circuit. The sound absorption of the plate with the circuit shorted or open is shown in Fig.5. It is obvious that the sound absorption coefficients are small in both cases, indicating that the loss factor of the plate is too small to provide satisfactory damping [9]. In addition, the first resonant mode of shorted circuit is shifting few *Hertz* in comparison with that of open circuit, the reason is due to $s_{ii}^D < s_{ii}^E$ (the superscript 'D' means open circuit), and a detailed explanation could refer to reference [1]. The variation of the sound absorption as the shunting resistor(R shunting circuit case) is presented in Fig.6. The maximum sound absorption peak can reach to 0.8 when the resistance value is 10kΩ. Compared to Fig.5, it is evident that the absorption coefficient

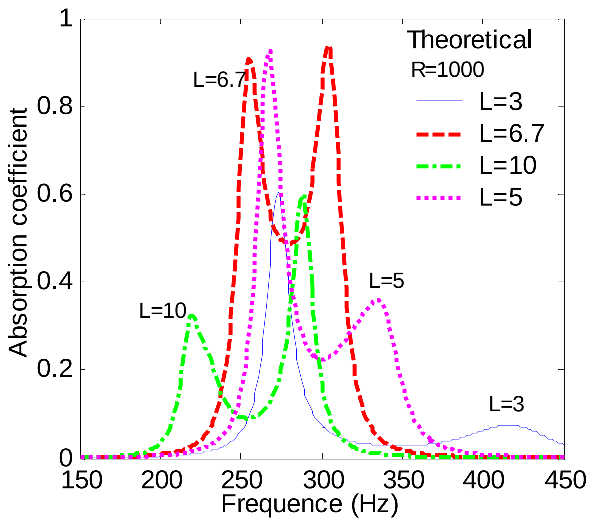

Figure.7. Absorption coefficient varies with L (theoretical)

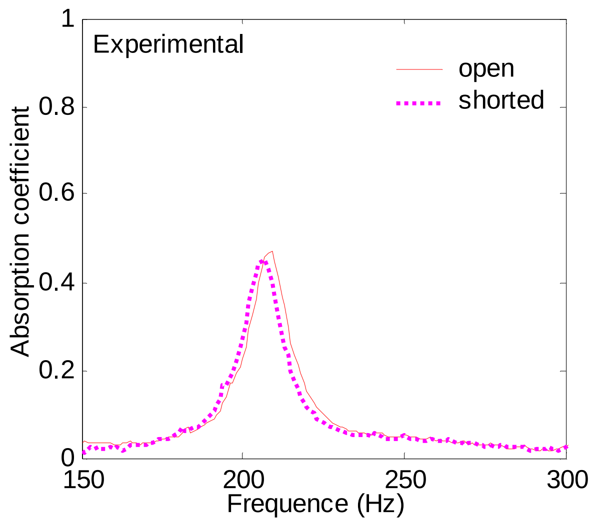

Figure.8. Absorption coefficients: shorted and open circuit (experimental)

significantly increased by shunting optimal resistance. Further investigation illustrates that the shunting PZT patch provides additional structural damping by transferring the mechanical energy to Joule heat.

Fig.7 shows the variation of sound absorption coefficient *vs* the shunting inductor(series RL shunting circuit case). The absorption coefficient is calculated when the series R is invariable at 1kΩ, while the inductor L varies among 3H, 5H,6.7H and 10H. Two absorption peaks appear in this case. One peak is resulting from the structure first structural resonance mode, and another peak is due to the shunting electrical resonance. The two absorption peaks improve each other when the electrical resonance is tuned to the structural resonance artificially.

4.2 Experimental results

Fig.8 illustrates that the experimental test of the sound absorption coefficient when the shunting circuit is shorted or open. Compared with Fig.5, two small differences may be found: the measured resonant frequency is lower than the theoretical prediction and the absorption peaks are higher. The discrepancy of resonant frequency could result from many reasons, and one possible reason is that in theoretical analysis, Equation (4) is derived under the assumption that the PZT patch makes no influence on the mode function shape. However, in practice the PZT patch can significantly stiffen the structure locally and adversely affect the strain energy distribution [10]. In other words, the theoretical calculation of the PZT patch strain energy could higher than the real case. So the predicted resonance frequency may be higher than the measured resonance frequency. The higher value of measured sound absorption is possibly due to the boundary damping. These discrepancies, however, can not dim the inspection of the shunting RL parameters' influence on the sound absorption.

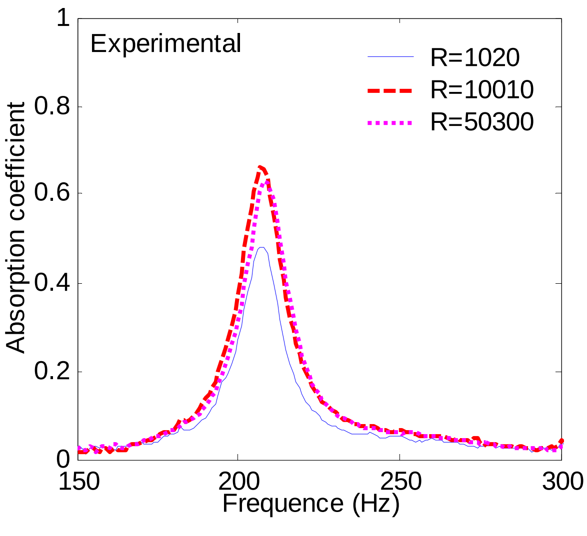

Figure.9. Absorption coefficient-various R (experimental)

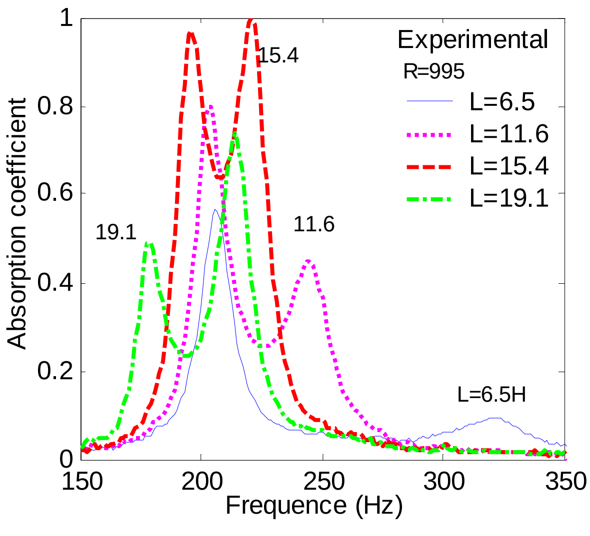

Figure.10. Absorption coefficient-various L (experimental)

Measurements of the sound absorption of the plate with a piezoelectric patch are shown in Figs.9 and 10. The experimental results show a good agreement in the trend with theoretical predictions. It should be noted that the tuning inductor L in the measurements is larger than it does in the theoretical analysis because of the structural resonant frequency variance.

5. CONCLUSIONS

A theoretical model has been developed to predict the sound absorption coefficient of a thin plate with a shunting PZT patch. The predicted and measured sound absorption coefficients agree with each other very well. Both theoretical and experimental results conclude that the sound absorption of a thin plate can be significantly improved near the first resonance mode, if choosing the shunting resistance R and inductor L appropriately. This is due to the additional damping added to the panel absorber. This work illustrates a potential that passive shunt damping technique may be used to control the sound absorption and reflection.

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