ICSV14
Cairns • Australia
9-12 July, 2007

14th International Congress on Sound & Vibration

FEEDBACK CONTROL OF PIEZO-LAMINATE COMPOSITE PLATE

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Abstract

Based on the applicability of smart materials in controlling the behaviour of engineering structures, a feedback control algorithm, which has been introduced previously by the authors [7], is implemented to control the dynamic response of composite laminates using bonded piezoelectric sensors and actuators. In order to investigate the shear transverse effect in the piezo-laminate and therefore be capable of analyzing thick plates, finite element formulation is derived based on higher order shear deformation theory of laminated plates. Finally, feedback control parameters, containing displacement and velocity gains, are changed and amplitude of dynamic response is thereby controlled. The numerical results show the effects of the different lamination angles on the vibration of plate. Furthermore, it is observed that how static deflection, natural frequencies and peak responses can be controlled by the displacement control gain and active damping can be provided by adjusting the velocity control gain.

1. INTRODUCTION

Composite structures and, in more practically used kind, laminated components are being used vastly in aerospace and automotive applications due to their high strength and stiffness to weight ratios. In addition, use of them allows the designer to choose between many possible structural layouts of the material, in order to obtain high structural performances. Piezo-laminates as smart-intelligent composites offer great potential for active control of advanced aerospace, nuclear, and automotive structural applications.

From the previous decade, many finite element models have been proposed for modeling structures along with piezoelectric actuators and sensors. Moita et al. [1] have utilized the Classical Laminted Plate Theory (CLPT) for finite element modeling of the active vibration control of a thin composite laminated plate containing piezoelectric layers. However, Chandrashekhara and Tenneti [2] and Bansal and Ramaswamy [3] have constructed their formulation on the basis of First-Order Shear Deformation Theory of laminated plate (FSDT). They have used shear flexible four and nine node elements for the same purpose to consider transverse shear effects, respectively. Furthermore, Higher-Order Shear Deformation Theory

(HSDT) exists which assume a parabolic distribution of transverse strains through the thickness, which was introduced by Reddy [4] for the analysis of composite laminated plates. This theory has been utilized by Zhou et al. [5] for modeling a composite plate with bonded piezoelectric layers (without active control). Dynamic behavior of laminated plates is important to be considered when transient loads are applied. Dynamic response of bimorph plates and beams are studied by Wang [6]. In the author’s previous works[7], finite element formulations were developed for the shape and vibration control of Functionally Graded Material (FGM) plates based on HSDT using piezoelectric sensors and actuators. The effects of constituent volume fraction (as a characteristic of the FGM material), the configuration of the sensor/actuator pairs and the velocity and displacement feedback control gains on the static and dynamic response of the structure were studied in their work.

In the present study, following the tendency to study smart composites, active control of composite laminated plates is being to be considered. Finite element method has been chosen to analyze the active control of vibration and dynamic response of laminated plates bonded to piezoelectric actuator/sensor patches. The utilized element is of four node second order type. In order to investigate thick plates and also the shear transverse effect in the piezo-laminate, finite element formulation is derived based on the higher order shear deformation theory (HSDT) of laminated plates. In order to prevent shear locking phenomenon in thin plates, C^1 continuous elements are used. A new feedback control algorithm, which is introduced by the authors [7], is implemented to control the static deflection, natural frequency and dynamic response of composite laminates using bonded piezoelectric sensors and actuators. In the previous works performed on the piezo-laminates, either no active control is implemented (i.e. Ref [3]) or only one control gain is considered in the control strategy. For instance, in the work published by Chandrashekhara and Tenneti [2], active control is achieved using only velocity control gain. However, in the feedback control system used here, both displacement and velocity gains are considered which can be adjusted with the corresponding parameters.

2. MATHEMATICAL FORMULATION

2.1 Governing Equations

According to the linear theory of piezoelectricity [8], constitutive equations for a piezoelectric material containing direct and indirect effects can be written as below:

$$\sigma_{ij} = C_{ijkl}\varepsilon_{kl} - e_{ijk}E_k \tag{1}$$

$$D_i == e_{ijk}\varepsilon_{jk} + K_{ij}E_j \tag{2}$$

where σ_{ij}, D_i, E_j, C_{ijkl}, e_{ijk}, and K_{ij} are the stress tensor, electric displacement, electric field vector , elastic modulus, electric permittivity and piezoelectric tensors, respectively. Displacement-based formulations will be made by the variational principle which is in the form:

$$\int_{t_0}^{t_1}\int_V (-\rho\ddot{u}_i\,\delta u_i - \sigma_{ij}\delta\varepsilon_{ij} + f_i\delta u_i)dvdt + \int_{t_0}^{t_1}\int_{V_p} D_i\delta E_i dV_p dt - \int_{t_0}^{t_1}\int_S T_i\delta u_i dSdt + \int_{t_0}^{t_1}\int_{S_p} Q_i\delta\phi dSdt = 0 \tag{3}$$

where S and V represent the surface area and volume of both the composite and piezoelectric materials, while S_p and V_p reply for the surface area and volume of the piezoelectric material, respectively. Parameter T indicates the applied surface traction and Q is the electrical charge applied to the surface of piezoelectric actuators.

2.2 Finite Element Formulation

In order to mathematically formulate the finite element model, the displacement field is assumed according to the Higher order Shear Deformation Theory (HSDT) [9]. By using the HSDT, a four node plate element with seven degrees of freedom containing u_0, v_0, β_x, β_y (displacements of a point in the middle or reference surface in x, y directions and the rotations of normal to the mid-surface about y and x axes, respectively, interpolated using bilinear Lagrangian interpolation functions) and w_0 (displacements in z direction), $\partial w_0 / \partial x$, and $\partial w_0 / \partial y$ (interpolated using Hermite interpolation functions) is chosen. In addition, elements containing actuator and sensor patches have two electric potential degrees of freedom. Structural and electrical continuous variables are estimated in terms of nodal displacements as:

$$\{u\} = [N_u]\{u^e\} \ ; \ \{\phi\} = [N_\phi]\{\phi^e\} \tag{4}$$

where $[N_u],[N_\phi]$ are the structural and electrical shape (interpolation) functions, respectively. Notations $\{u^e\}$ and $\{\phi^e\}$ indicate the structural and electrical generalized nodal variables vectors. With the above interpolations, strain components and the electric field can be obtained through derivations:

$$\{E\} = -\nabla \varphi = -[B_\varphi]\{\varphi^e\} \tag{5}$$

$$\{\varepsilon\} = [B_u]\{u^e\} \tag{6}$$

where $[B_\varphi] = \nabla[N_\varphi]$ and $[B_u]$ is a matrix containing the derivatives of interpolation functions.

The kinetic and strain energy of the panel can be readily found as [3]:

$$T = \frac{1}{2}\iint \left[\sum_{k=1}^{n} \int_{h_k}^{h_{k+1}} \rho_k \left\{ \frac{du^k}{dt} \ \frac{dv^k}{dt} \ \frac{dw^k}{dt} \right\} \times \left\{ \frac{du^k}{dt} \ \frac{dv^k}{dt} \ \frac{dw^k}{dt} \right\}^T dz \right] dxdy \tag{7}$$

$$U = \frac{1}{2}\iint \left[\sum_{k=1}^{n} \int_{h_k}^{h_{k+1}} \{\sigma\}^T \{\varepsilon\} dz \right] dxdy - \sum_{m=1}^{n_p} \int_{V_p} \{E\}^T \{D\} dv \tag{8}$$

where n is number of layers of the laminated plate, n_p number of piezoelectric layers, ρ_k density, h_k thickness, and $\left\{ \frac{du^k}{dt} \ \frac{dv^k}{dt} \ \frac{dw^k}{dt} \right\}$ velocity vector of the k^{th} layer. The work done by the prescribed traction T, body forces F_b and the applied electrical charge density Q on the actuators is given by:

$$W = \int_A \{u\}^T T dA + \int_V \{u\}^T F_b dv - \int_{A_p} \{\phi_a\}^T Q dA \tag{9}$$

in which ϕ_a refers to the electric potential on the actuator. Now the Hamilton principle is applied to obtain the finite element governing equation of motion:

$$\delta\int_0^t (T - U - W)dt = 0 \qquad (10)$$

By introduction of Eq.'s (7)-(9) in Eq. (10), the equations of motion of the piezo-laminated plate can be obtained as follows:

$$[M_{uu}]\{\ddot{u}\} + [K_{uu}]\{u\} + [K_{u\phi}]\{\phi\} = \{F_m\} \qquad (11)$$

$$[K_{\phi u}]\{u\} - [K_{\phi\phi}]\{\phi\}_a = \{F_q\}_a \qquad (12)$$

$$[K_{\phi u}]_s\{u\} - [K_{\phi\phi}]_s\{\phi\}_s = 0 \qquad (13)$$

where $[M_{uu}],[K_{uu}],[K_{u\phi}]$, and $[K_{\phi\phi}]$ are mass, structural stiffness, coupled structural-electric stiffness, and electric stiffness matrices, respectively. Vector $\{F_q\}_a$ is the applied electrical charge to the piezo-actuator patches and $\{F_m\}$ is the applied mechanical loading. Substituting Eq.'s (12) and (13) in Eq. (11) results in the overall finite element governing equation of motion:

$$[M_{uu}]\{\ddot{u}\} + ([K_{uu}] + [K_{u\phi}][K_{\phi\phi}]^{-1}[K_{\phi u}])\{u\} = \{F_m\} + [K_{u\phi}]_a[K_{\phi\phi}]_a^{-1}\{F_q\}_a \qquad (14)$$

The induced electric potential in sensor can be calculated from Eq. (13):

$$\{\phi\}_s = [K_{\phi\phi}]_s^{-1}[K_{\phi u}]_s\{u\} \qquad (15)$$

By using the closed loop feedback control algorithm that will be used in the active control of the structure [7]:

$$\{\phi\}_a = G_d\{\phi\}_s + G_v\{\dot{\phi}\}_s \qquad (16)$$

With G_d as the displacement control gain and G_v as the velocity control gain. Now we substitute Eq.'s (15) , (16) to obtain:

$$[M_{uu}]\{\ddot{u}\} + [C^*]\{\dot{u}\} + [K^*]\{u\} = \{F_m\} \qquad (17)$$

in which:

$$\begin{aligned} &[K^*] = [K_{uu}] + G_d[K_{u\phi}]_a[K_{\phi\phi}]_s^{-1}[K_{\phi u}]_s + [K_{u\phi}]_s[K_{\phi\phi}]_s^{-1}[K_{\phi u}]_s \\ &[C^*] = G_v[K_{u\phi}]_a[K_{\phi\phi}]_s^{-1}[K_{\phi u}]_s + [C_p] \\ &\{F^*\} = \{F_m\} \end{aligned} \qquad (18)$$

where C_p is the proportional damping matrix.

3. RESULTS AND DISCUSSION

3.1 Verification of the Finite Element Code

With the purpose of validating the solutions obtained from the developed finite element code, verifications are made by comparing the results obtained for natural frequencies and static deflections under electric and mechanical loads with the ones gained from the commercial finite element code ANSYS. It should be noted that comparisons are made for no feedback control situation, since simulating the closed loop control process in ANSYS is impossible. Each piezoelectric layer has a thickness equal to 0.00125m; however the core laminated plate is 0.005m thick with lamination angles [0/90/90/0]. The plate is of square type with length of 40cm and properties of both laminated and piezoelectric materials, considered in the present study, are mentioned in

Table 1 and

Table 2, respectively. Comparison of the centreline deflections under both uniform load of $2.5\times10^3\,N/m^2$ and electrical load of an additional voltage equal to 40v (applied on the actuator layers) is illustrated in Figure 1. The well agreement of the solutions is because of precise consideration of transverse shear deformation effects in the higher order shear deformation theory used in the present study.

Table 1. Properties of the laminated material.

Graphite /Epoxy	E_{11}=150GPa	$E_{22}=E_{33}$=9GPa	$G_{12}=G_{13}$=7.1GPa
	G_{23}=2.5GPa	$\nu_{12}=\nu_{13}=\nu_{23}=0.3$	$\rho=1600Kg/m^3$

Table 2. Properties of the piezoelectric material.

Elastic modulus	63×10^9 (N/m)
Poisson's ratio	0.3
Density	7600 (Kg/m^{3})
Piezoelectric constant	254×10^{-12} (m/V)
Dielectric coefficient	15×10^{-9} (F/m)

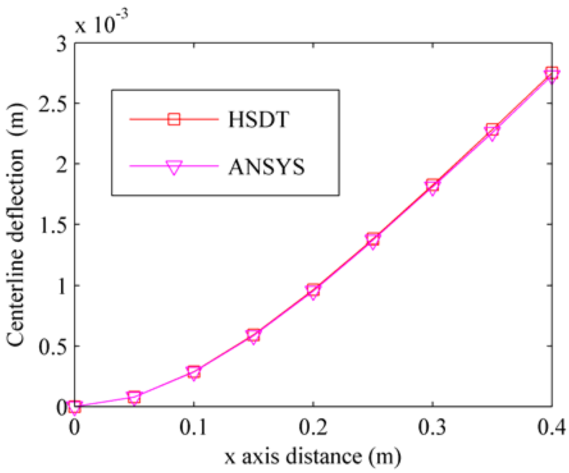


Figure 1. The centerline deflection of plate under uniform distributed load $2.5\times10^3\,N/m^2$ and actuator Voltage V = 40v.

3.2 Static Deflection Control

In this section, a cantilever plate with sensor/actuator patch locations is considered for the static analysis (Figure 2). The material properties are those given in Tables 1 and 2. Thickness of each patch and the laminated part (the lamination of the composite core [0/90/90/0]) is 0.0001m and 0.01m, respectively. Mathematical formulation of the problem indicates that the displacement control gain value is consisted in the stiffness matrices. In order to observe the effect of this control gain on the static deflection of the laminated plate, two different displacement gain values are considered in the analysis and the results for tip deflections of the cantilever plate under mechanical and electrical loads are indicated in Figure 3. From this figure, it can be concluded that reduction of the displacement gain (G_d) value results in decrease of deflection. Therefore, with varying the G_d, static deflection of the structure can be controlled.

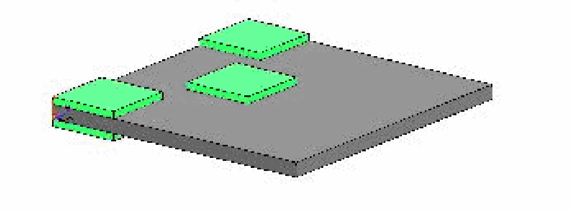


Figure 2. Sensor/Actuator patch configuration for the cantilever piezo-laminated plate.

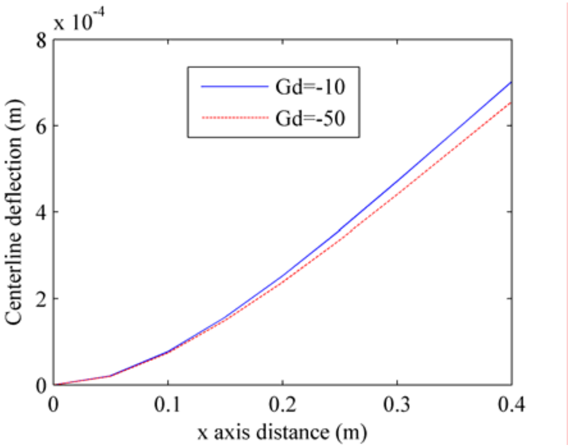


Figure 3. Tip deflection of the piezo-laminated plate under mechanical load

3.3 Dynamic Response Control

Control of the behaviour of the laminated plate in free vibration and transient dynamic conditions is considered in this section. The properties and dimension of the plate and piezoelectric patches are the same as the ones mentioned in the previous section. Table 3 indicates the effect of displacement control gain on the first five natural frequencies of the plate. As G_d is decreased, natural frequency values are increased.

Table 3. First five natural frequencies (Hz) of the plate for two different displacement control gains.

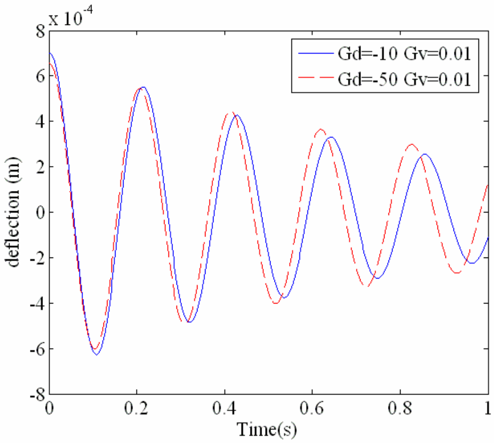
	Mode 1	Mode 2	Mode 3	Mode 4	Mode 5
G_d = -10	93.541	128.33	324.85	565.48	608.21
G_d = -50	97.106	133.35	329.57	577.51	622.21

Dynamic response of the laminated plate is also controlled with piezoelectric sensor/actuator patches. The plate is subjected to a unit force at the tip in vertical direction and then released. This would provide an initial displacement for the plate to be assumed as an initial condition for the dynamic transient problem. The results are plotted in Figure 4 in which two different values of displacement gain and a velocity gain values equal to 0.01 and 0.1 are considered. It can be observed that reducing G_d causes a reduction in the peak response of the structure. Regarding to Figure 4 and recalling the contribution of G_v to the damping matrix of the structure, variation of velocity gain value has affected the damping process of the transient response. Therefore, it can be concluded that G_d has effect on the peak response and G_v has effect on the damping of the laminated plate.

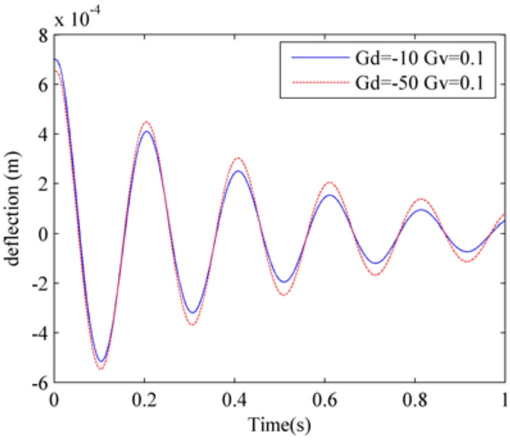
In order to observe the influence of a different orientation of fibers in the composite plate on the dynamic response, the same simulations are done for lamination angles [30/45/60/90]. Dynamic deflection of this plate is shown in Figure 5. The same effects of G_d and G_v can be concluded on the peak response and damping speed of the responses. Again, the reduction of displacement gain reduces the amplitude of dynamic response and the influence of velocity gain is observable. However, due to the relation between the fiber orientations of a laminated structure and the corresponding structural stiffness, larger deflection is produced in the new laminate (i.e. laminate [0/9090/0] has more structural stiffness). So the design of the composite plate with respect to the stacking sequence requirements can be effective on the transient response of the structure.

4. CONCLUSIONS

Finite element formulations have been presented base on HSDT for active control of a piezo-laminated plate under electric and mechanical loads. Verification of the code has been carried out by comparing the results with possible solutions made by commercial finite element package. A new feedback control algorithm, which is introduced by the authors [7], has been implemented to control the static deflection, natural frequency and dynamic response of composite laminates using bonded piezoelectric sensors and actuators. Then the influences of displacement and velocity control gains have been investigated. Finally, the effect of different orientation of fibres in the laminated plate has been observed on the solutions.

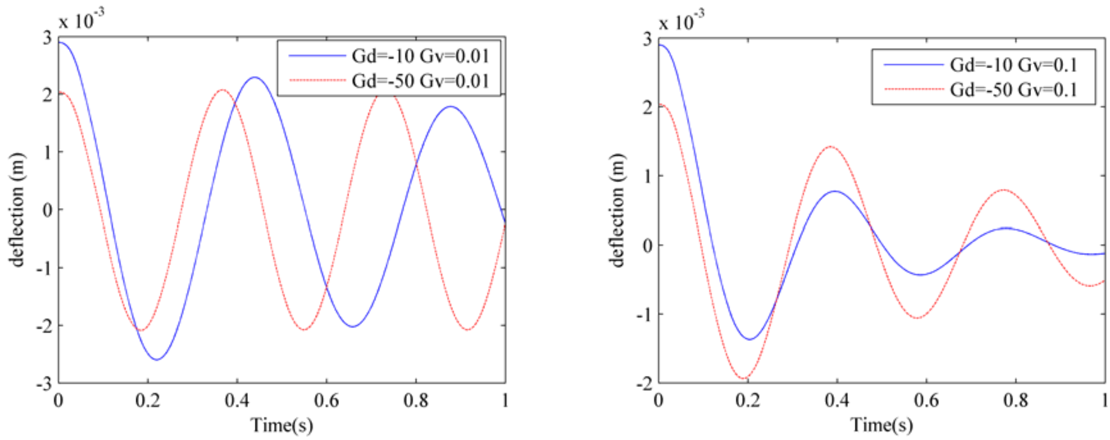


(a)



(b)

Figure 4. Dynamic deflection at the tip of the cantilever plate with different displacement and velocity gain values (with lamination angles [0/90/90/0]) - (a) G_v=0.01 ; (b) G_v=0.1.



(a) (b)

Figure 5. Dynamic deflection at the tip of the cantilever plate with different displacement and velocity gain values (with lamination angles [30/45/60/90]) - (a) G_v=0.01 ; (b) G_v=0.1.

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