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FREE VIBRATION ANALYSIS OF SANDWICH HEMI-SPHERICAL SHELLS WITH CONSTRAINED ELECTRO-RHEOLOGICAL FLUID DAMPING

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Abstract

This paper deals with the dynamic characteristics of hemispherical sandwich shells with electro-rheological (ER) fluid core under clamped free and clamped clamped conditions. The free vibration and damping analysis is carried out by using the semi-analytical finite element method. Three noded line elements with seven degree of freedom per node are used in meridional direction and Fourier series is assumed in circumferential direction. There are two types of electro-rheological fluid cores used in the present study. The damping variation of hemispherical sandwich shell for different radius to thickness (R/t) ratio and core to facing thickness ratio (t_c/t_f) are carried out. The variation of damping properties of ER fluid with electric field is also investigated.

1. INTRODUCTION

The study of dynamic characteristics of a sandwich hemispherical shell structures has a direct and important bearing on the structural problems of missile and rockets. The use of sandwich structures with core material is an effective way of reducing the vibration, when subjected to dynamic loading. In recent past a lot of research has been done on the sandwich shell structures with viscoelastic core.

Okazaki *et al.* [1] investigated damping properties of three layered shallow spherical shells with a constrained viscoelastic layer. Natural frequencies and loss factors of shallow spherical shells are calculated for axially and non-axially symmetric modes. Gautham and Ganesan [2] studied vibration and damping characteristics of spherical shells with a viscoelastic core, based on discrete layer theory. Ravikiran and Ganesan [3] investigated a theoretical analysis of linear thermo-elastic buckling of composite hemispherical shells with a cut out at the apex. They have used the semi-analytical finite element method applicable to

moderately thick shells. Jia and Lien [4] studied the dynamic stability of a sandwich plate with a constrained layer and electro-rheological (ER) fluid core. Wilkins *et al.* [5] have studied the free vibration analysis of orthotropic conical shell with honeycomb core sandwich structures. The effect of shear deformation is accounted for the facings. There are no studies in the literature available on the sandwich hemi-spherical shell with electro-rheological fluid core structures. The present study deals with free vibration analysis of sandwich hemispherical shell having cutout at a apex with electro-rheological fluid core.

2. FINITE ELEMENT FORMULATION

Semi analytical method is used to calculate the frequency and loss factor of cylindrical shell. The finite element formulation is used in the present study is same as given by Ramasamy and Ganesan [6]. The model is made up of sandwich structure, which consists of isotropic facings with electro-rheological fluid core as shown in figure 1. The general shell element with sandwich structure, which is converted in to hemispherical shell, is shown in figure 1. Where β is the cut out angle at apex, and radius R is equal to R_ϕ and R_θ , the mid surface is discretised in to finite elements is also shown in figure 1, t_f and t_c are the facings and core thickness respectively. Two types of electro-rheological (ER) fluid cores are used in the present study. The properties of ER fluid and mild steel materials are given in the Table 1.

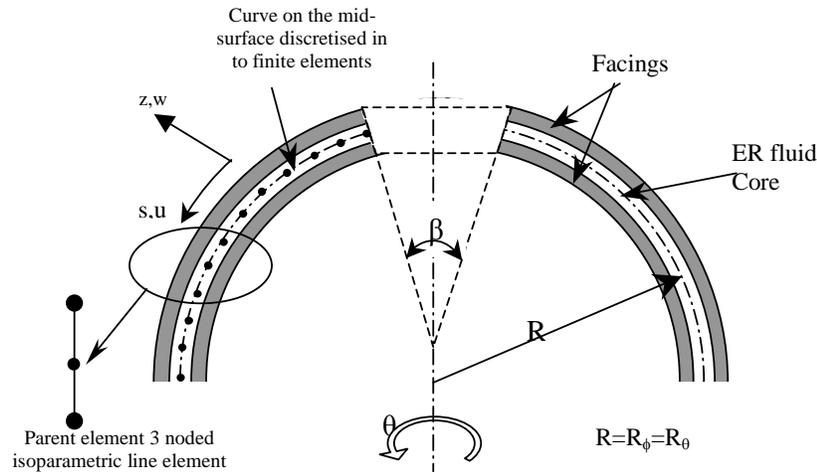


Figure 1: Cut section of hemispherical shell with cut out angle at apex.

Table: 1 Properties of ER core and mild steel facings [4]

Shear modulus (G) N/m ²	ER fluid core		Mild steel facings
	Type 1 (ER1)	Type 2 (ER2)	
Real part (G_R)	$\approx 15000E_*^2$	$\approx 50000E_*^2$	$E = 204\text{ GPa}; \nu = 0.3$ $\rho = 7850\text{ Kg} / \text{m}^3;$
Imaginary part (G_I)	≈ 6900	$\approx 2600E_*^2 + 1700$	

Where E_* is the electric field in kV/mm

The displacement field used in the present analysis is proposed by Wilkins *et al.* [5]. The strain displacement relations for core, inner and outer facings referred in the present study are given by Ramasamy and Ganesan [6]. In the present analysis three noded elements with seven degrees of freedom per node is used. The vector of displacement per element u_e is expressed as $\{u_e\} = \{u_{0,1} v_{0,1} w_{0,1} \psi_{s,1} \psi_{\theta,1} \phi_{s,1} \phi_{\theta,1} \dots \phi_{s,3} \phi_{\theta,3}\}$ where the subscripts 1,2 and 3 denote the node number.

The strain vectors can be represented as

$$\{\varepsilon\} = \{\varepsilon_{ss} \varepsilon_{\theta\theta} \gamma_{s\theta} \gamma_{\theta z} \gamma_{sz}\}^T = [B]\{u_e\} \quad (1)$$

Here [B] is the derivative of shape function matrix.

The element stiffness matrix $[K_e]$ is expressed as

$$[K_e] = \int_v [B]^T [D] [B] dv \quad (2)$$

$$[K_e] = [K]_R + [K]_I$$

The stiffness matrix $[K_e]$ consists of real part $[K]_R$ and imaginary part $[K]_I$ due to complex material properties of facings, ER1 and ER2 fluid core.

The element mass matrix $[M_e]$ is given by

$$[M_e] = \int_v [N]^T \rho [N] dv \quad (3)$$

Where ρ is density of the structure

2.1 Evaluation of frequency and loss factor

The following eigen value problem has to be solved to get the natural frequencies.

$$[K]_R - \omega^2 [M] = 0 \quad (4)$$

Where ω is the natural frequency

The Composite and ER fluid loss factor for n^{th} mode can be calculated using modal strain energy method.

$$\eta_n = \frac{\phi_n^T [K]_I \phi_n}{\phi_n^T [K]_R \phi_n} \quad (5)$$

Where $[K]_R$ and $[K]_I$ are the real and imaginary parts of the stiffness matrices $[K]$ respectively, and ϕ_n is n^{th} mode eigenvector.

3. RESULTS AND DISCUSSION

The frequency and damping of sandwich hemispherical shell is analyzed by using two types of electro-rheological (ER1 and ER2) fluid cores. The parametric study carried out for

different core to facing (t_c/t_f) thickness and radius to thickness (R/t) ratio in clamped free (C-F) and clamped-clamped (C-C) boundary condition. The results are plotted for first axial mode of a cylindrical shell.

3.1 Clamped free sandwich hemispherical shell

3.1.1 Effect of electric field on electro-rheological fluid damping

Figure 2 shows the loss factor of ER fluids (type1 and type2) with 20 circumferential modes for different electric field. In case of ER1, loss factor decreases with increasing the electric field it is observed clearly at 2nd circumferential mode but at other modes the influence of electric field is not observed as shown in figure 2(a). In case of ER2, loss factor increase with increasing the electric field, which is clearly observed from 2 to 10 circumferential modes as shown in figure 2(b). The variation of loss factor with electric field is same as given by Jia and Lien [4].

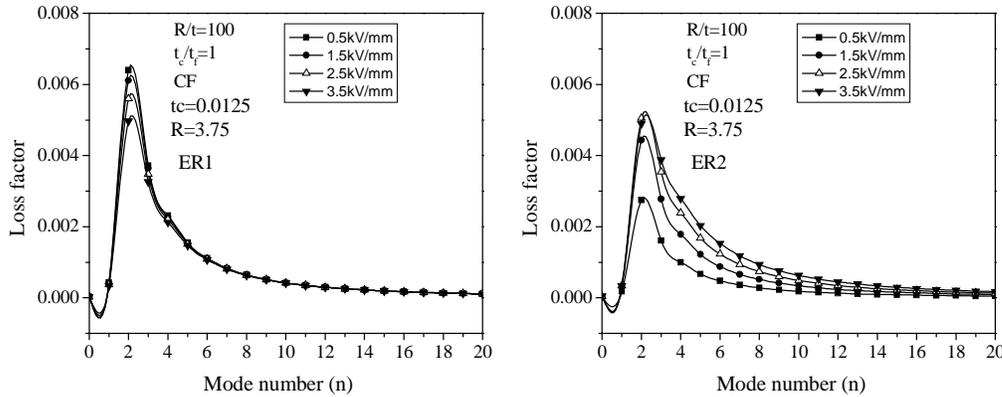


Figure 2 Effect of electric field on loss factor of hemispherical sandwich shell (a) ER1 fluid core (b) ER2 fluid core.

3.1.2 Effect of core to facing thickness ratio (t_c/t_f) on frequency and loss factor

Figure 3 shows the variation of frequency and loss factor of ER1 fluid core at R/t ratio 100 for different core to facing thickness ratio (t_c/t_f). Frequency decreases with up to 2nd mode and reaches a minimum value then increases with increase in mode number. Increasing the core to facing thickness ratio frequency decreases, which is clearly observed at higher circumferential modes. This effect may be due to increases in core to facing ratio the stiffness of the structure decreases so the frequency decreases. Increasing the t_c/t_f ratio the frequency variation is observed more at higher modes compared to lower modes this effect may happens because at the lower modes the membrane effect is not having much influence on frequency and at higher modes the bending effect is more predominant. The ER1 fluid damping increases with increasing the circumferential modes up to 2nd mode then onwards decreases with increasing the circumferential modes. From the figure 3 it is noticed that increasing the t_c/t_f ratio, ER fluid damping increases, which is observed clearly at intermediate (2 to 14) circumferential modes.

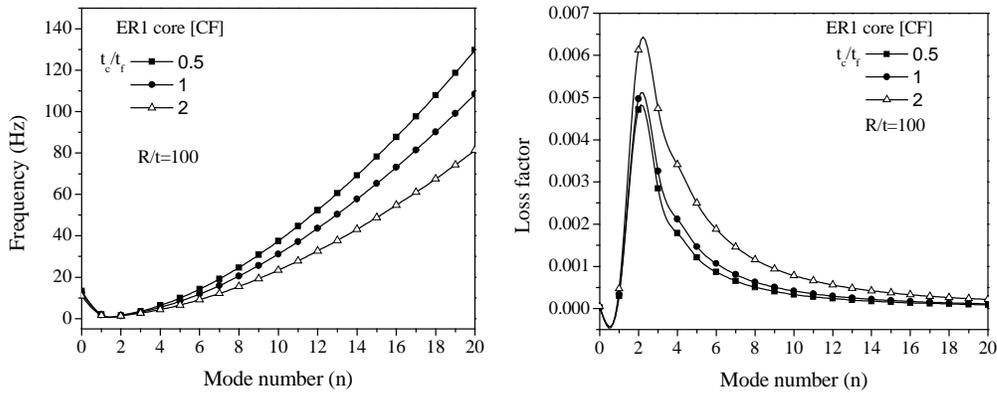


Figure 3 Variation of frequency and loss factor with harmonics of hemispherical sandwich shell for different core to facing thickness ratio (t_c/t_f).

3.1.3 Effect of radius to thickness ratio (R/t) on frequency and loss factor

Figure 4 shows the variation of frequency and loss factor with harmonics for different radius to thickness ratio (R/t) by keeping the core to facing thickness ratio constant ($t_c/t_f=1$). Increasing the R/t ratio the frequency increases at breathing mode and at higher circumferential modes, but there is no influence of R/t ratio at 2nd mode. This may be due to the membrane effect is predominant lower modes and the bending effect is more predominant at higher modes. The ER1 fluid damping increases with increasing the R/t ratio, because increasing the radius of the hemispherical shell the loss of energy also increases, which may leads to increase the loss factor of the shell.

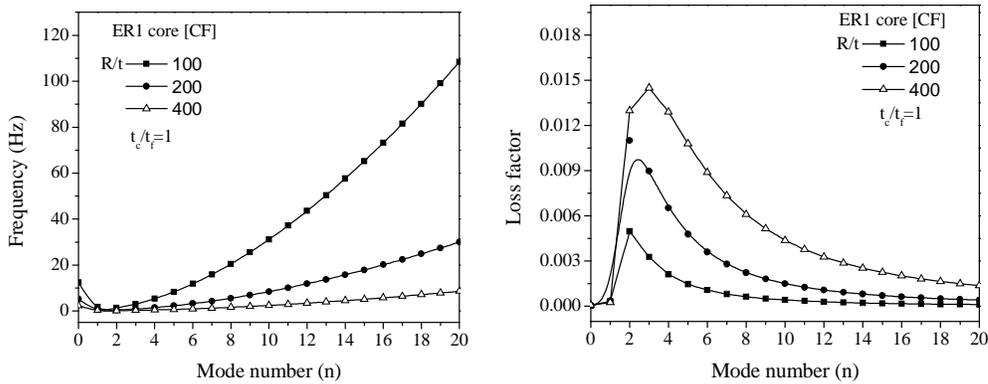


Figure 4 Variation of frequency and loss factor with harmonics of hemispherical sandwich shell for different radius to thickness ratio (R/t).

3.1.4 Influence of cut out angle on frequency and loss factor

Figure 5 shows the variation of frequency and loss factor with harmonics for different cut out angle, by maintaining the t_c/t_f ratio and R/t ratio constant. The effect of cut out angle on frequency and loss factor is not observed much in clamped free hemispherical shell. Here one end is free at which the cut out angle varied and other end is clamped, because at free end if the cut out angle is varied which is not having much influence on the vibration behavior.

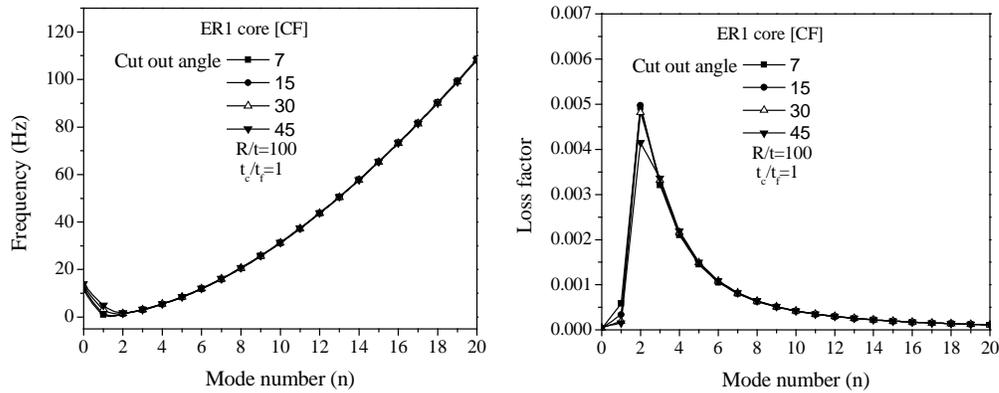


Figure 5 Frequency and loss factor with harmonics of hemispherical sandwich shell for different cut out angle.

3.1.2 Effect of core material on frequency and loss factor

Figure 6 shows the variation of frequency and loss factor with circumferential modes for ER1, ER2 fluid cores at R/t ratio 100. The comparison of properties with ER1 and ER2 core shell is carried out at t_c/t_f ratio 1 and cut out angle 15° . Figure 6 shows that the frequency remains same for both types of cores at all circumferential modes, but the loss factor of ER2 core hemispherical shell is more than the ER1 core hemispherical shell. This is due to the variation in the shear modulus of the core materials, which are referred from Jia and Lien [4] tabulated in table 1. Because the ER2 loss modulus is depends on the electric filed and ER1 loss modulus is independent of electric field, which may leads to have more loss factor in case of ER2 fluid core.

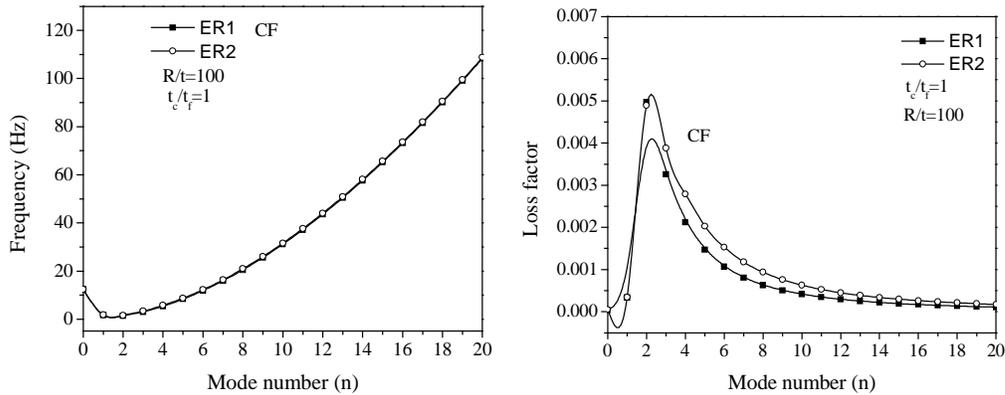


Figure 6 Comparison of Frequency and loss factor of ER1 and ER2 fluid core sandwich hemispherical shell in clamped free condition.

3.2 Clamped-clamped sandwich hemispherical shell

3.2.1 Effect of core to facing thickness ratio (t_c/t_f) on frequency and loss factor

The figure 7 shows the variation of frequency and loss factor of hemispherical sandwich shell with ER1 fluid core. The frequency increases up to 5th circumferential mode then onwards

almost remains constant. The second mode is having more influence on the damping. The effect of core to facing thickness ratio on frequency and loss factor is same as discussed in case of clamped free condition (Sec 3.1.2). The damping is very low and frequency is high in case of C-C hemispherical shell compared to C-F hemispherical shell at all t_c/t_f ratios.

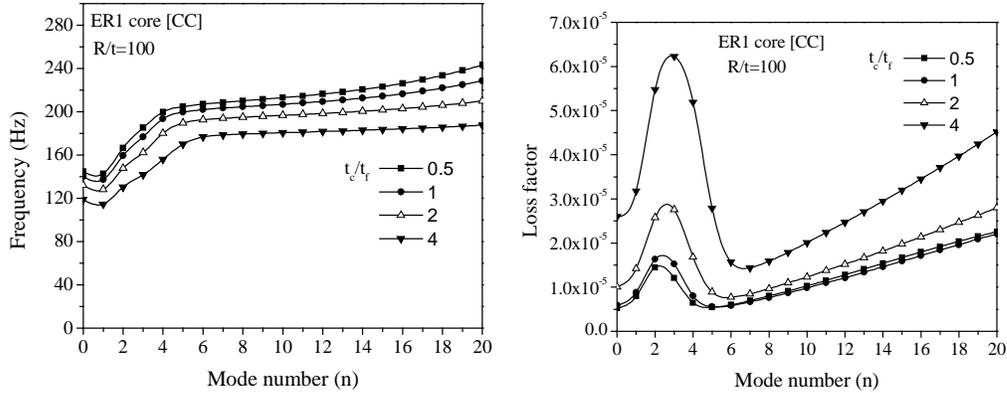


Figure 7 Variation of frequency and loss factor with harmonics of hemispherical sandwich shell for different core to facing thickness ratio (t_c/t_f).

3.2.2 Effect of cut out angle on frequency and loss factor

Figure 8 shows the variation of frequency and loss factor with circumferential modes for different cut out angle by keeping R/t and t_c/t_f ratio constant. Effect of cut out angle on frequency and damping is having more influence at lower modes, as the cut out angle increases the frequency decreases and loss factor increases. The reason for this may be in case of C-C hemispherical shell as the cut out angle increases the stiffness of the shell decreases, which leads to decrease in frequency.

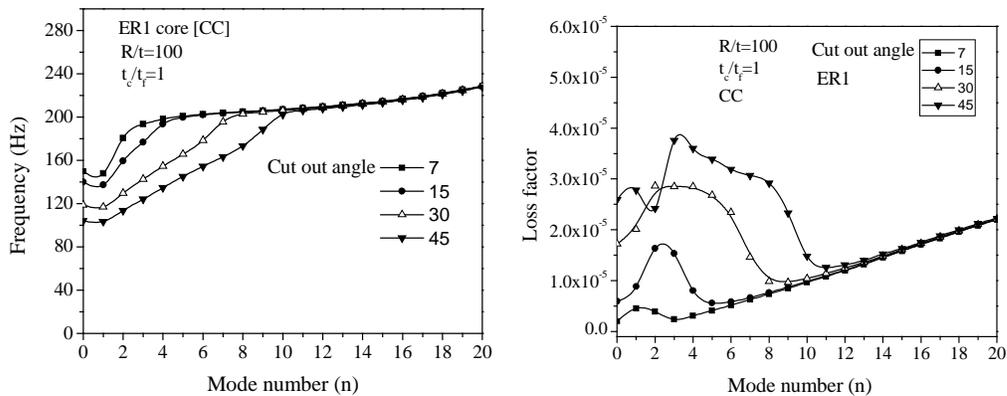


Figure 8 Frequency and loss factor with harmonics of hemispherical sandwich shell for different cut out angle.

3.2.3 Effect of core material on frequency and loss factor

Figure 9 shows the frequency and loss factor of sandwich hemispherical shell with ER1 and ER2 fluid cores under clamped-clamped condition. In this case also the core material is not

having influence on frequency but the loss factor of ER2 core is more than the ER1 core hemispherical sandwich shell.

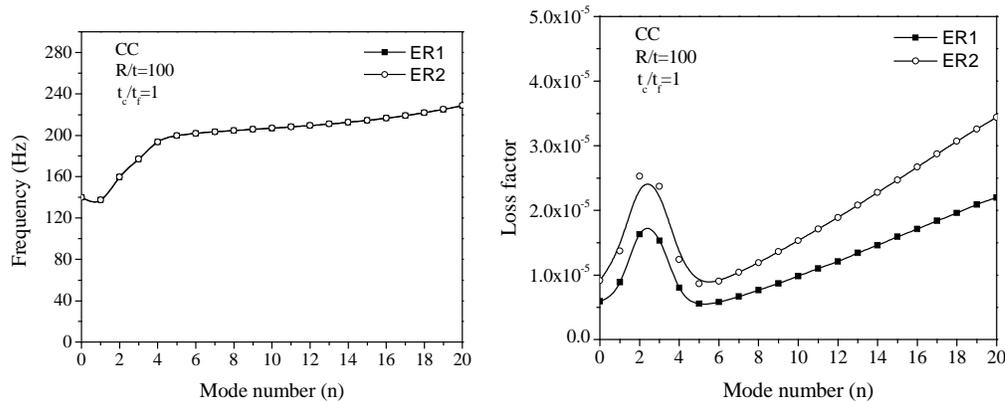


Figure 9 Comparison of Frequency and loss factor of ER1 and ER2 fluid core sandwich hemispherical shell in clamped-clamped condition.

4. CONCLUSIONS

The free vibration and damping analysis of sandwich hemispherical shell with electro-rheological fluid core in clamped free and clamped-clamped condition is investigated, the following conclusion are arrived with the study.

1. Increasing the electric field ER1 fluid damping decreases and ER2 fluid damping increases.
2. Increasing the core to facing thickness ratio (t_c/t_f) and radius to thickness ratio (R/t) the frequency decreases and loss factor increases.
3. The effect of cut out angle is observed in clamped clamped condition, as the cut out angle increases the frequency decreases and damping increases.
4. The frequency of ER1 and ER2 fluid cores are same but the ER2 fluid damping is more compared to ER1 fluid damping in case of clamped free and clamped-clamped hemispherical shell.

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