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STATIONARY RESPONSE OF A MILITARY TRACKED VEHICLE

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Abstract

In this paper, stationary response of a '2+N' (9) degree of freedom (dof) tracked vehicle model fitted with conventional torsion bar suspension system is investigated for ground excitations resulting from rough off-road terrain operations. The vehicle is assumed to be moving with constant velocity on a randomly profiled terrain which is modeled as a homogeneous random process, being the output of a linear first order shaping filter to white noise. The equivalent road wheel stiffness is computed taking into account the stiffness due to track pad and the spring rate due to track tension. The investigations are carried out considering different randomness of terrain profile and also different stiffness values for the suspension system. The comparison of hull bounce acceleration and hull pitch acceleration is made for different configurations of suspension system.

1. INTRODUCTION

The ride vibration environment of a typical high-speed tracked vehicle traversing rough off-road terrain is of significance due to the magnitude of ride vibrations arising from dynamic terrain-vehicle interactions. These impose a severe ride environment on driver / crew and on functioning of on board instrumentation. Ride vibrations transmitted to the driver's compartment are of high amplitude and low frequency, the conditions to which the human body is most fatigue sensitive. Prolonged exposure to such vibrations causes the operator bodily discomfort, physiological damage and reduces performance efficiency and thus the mobility performance of the vehicle is limited.

Computer simulation using an analytical vehicle model has become a very effective tool for evaluating the ride characteristics of ground vehicles, without resorting to the expensive and time-consuming process of repeated testing. Hedrick and Firouztash [1] have derived the Lyapunov equation for the propagation of covariance matrix including the correlation between the front and rear wheels of a half-car vehicle model with passive suspension system. Wheeler [3] worked on computer simulation of tracked vehicle ride dynamics and developed mathematical models incorporating the degrees-of-freedom associated with bounce and pitch motions of the sprung mass and vertical motion of each road wheel. Wong [4] has discussed simplified dynamic models of various types of ground vehicles. Rakheja *et al.* [5] have made studies on the ride dynamics of a tracked vehicle using a seven-degree-of-freedom in-plane model, incorporating kinematics of the road wheel suspension. Dhir and Sankar [6] have performed computer simulations of a military vehicle and validated their results with field-testing for specified vehicle configurations, terrain profiles and vehicle speeds.

2. MATHEMATICAL MODELING

High-speed tracked vehicles although varying widely in shape, size and general physical appearance, share many common characteristics in the track and suspension assembly. From the point of view of analytical modeling, a typical tracked vehicle can be divided into track and suspension components and hull components. The former group includes the track, hull wheels (drive sprocket, idler and roller supports), road wheel assemblies and suspension components. Track and suspension components constitute the un-sprung mass of the system. The hull represents collectively all remaining components of the vehicle and has been referred to as sprung mass. The track is assumed to be a massless, continuous belt. Vehicle suspension units are modeled using independent suspension configurations and damping characteristics and are constrained to translate in the vertical direction.

The equivalent road wheel stiffness is computed taking into account the stiffness due to the track pad and wheel and the spring rate due to track tension [4]. For the present study, a simplified linear in-plane mathematical ride model (as shown in Figure 1) [7] of a typical tracked vehicle formulated as a “2+N” degree of freedom system traversing an arbitrary, non deformable random terrain profile and running at constant speed is used. Here N is the number of degrees of freedom corresponding to the N bounce modes (y_{wi}) of the N road wheels on each side (here ‘N’ is 7). The remaining 2 degrees of freedom correspond to the bounce (y_h) and pitch (θ_h) modes of the centre of gravity (C.G) of the hull.

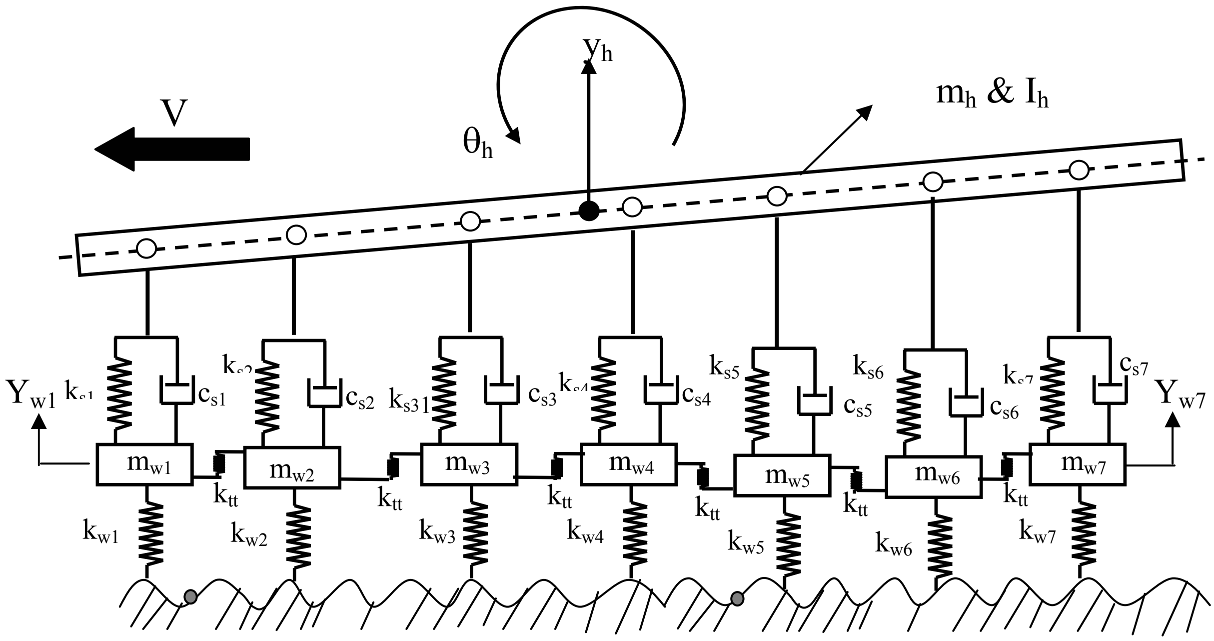

Figure 1 Equivalent dynamic model of tracked vehicle

2.1 Vehicle Parameters Used

m_h	=	Half of hull sprung mass in kg	= 25000
I_h	=	Half of hull pitch moment of inertia in kg-m^2	= 190890
m_{wi}	=	Mass of ith road wheel assembly in kg	= 450
y_h	=	Hull bounce motion in m	
θ_h	=	Hull pitch motion in radians.	
y_{wi}	=	Bounce motion of ith road wheel in m	
a_i	=	Horizontal location of road wheel centers from C.G in m (a_1= -1.727, a_2= -0.859, a_3= 0.01, a_4= 0.835, a_5= 1.67, a_6= 2.48 & a_7= 3.29)	
b_i	=	Wheel base with respect to 1st wheel in m (b_1= 0.848, b_2= 1.717, b_3= 2.542, b_4= 3.377, b_5= 4.187, b_6= 4.997)	
t_{wi}	=	Time lag between the adjacent road wheels	
k_{si}	=	Stiffness of ith suspension unit in N/m	
k_{wi}	=	ith wheel and track pad stiffness in N/m	= 918000
k_{tt}	=	Spring rate due to track tension in N/m	= 65672
c_{si}	=	Damping due to ith suspension unit in Ns/m	= 22520
T_r	=	Total vertical force acting at the road wheel center due to adjacent track segments in N	

2.2 Equations of Motion

Differential equations of motion for the linear in-plane tracked vehicle model are derived using Newton's second law of motion. Equations (1) and (2) represent the bounce and pitch motions of the sprung mass. Equation (3) represents the bounce motion of each road wheel.

$$m_h\ddot{y}_h+\sum_{i=1}^{7}C_{si}(\dot{y}_h-\dot{y}_{wi}+a_i\dot{\theta}_h)+\sum_{i=1}^{7}k_{si}(y_h-y_{wi}+a_i\theta_h)=0 \quad (1)$$

$$I_h\ddot{\theta}_h+\sum_{i=1}^{7}a_iC_{si}(\dot{y}_h-\dot{y}_{wi}+a_i\dot{\theta}_h)+\sum_{i=1}^{7}a_ik_{si}(y_h-y_{wi}+a_i\theta_h)=0 \quad (2)$$

$$m_{wi}\ddot{y}_{wi}-C_{si}(\dot{y}_h-\dot{y}_{wi}+a_i\dot{\theta}_h)-k_{si}(y_h-y_{wi}+a_i\theta_h)+k_{wi}y_{wi}-T_r=k_{wi}h_i \quad (3)$$

Total vertical force acting at the road wheel center due to adjacent track segments is given as [2]

$$T_r = k_{tt}(y_{wi+1}-y_{wi}) \text{ ....for } i=1$$
$$T_r = k_{tt}(y_{wi-1}-y_{wi})+k_{tt}(y_{wi+1}-y_{wi}) \text{ ....for } i=2,\dots 6 \quad (4)$$
$$T_r = k_{tt}(y_{wi-1}-y_{wi}) \text{ ....for } i=7$$

In the case of vehicle models including pitch and bounce of the vehicle body, the terrain input enters the system at all the wheel stations. When the vehicle moves with constant velocity, the time delays between the wheels are constant because the wheel base is also traversed at constant velocity.

2.3 Terrain Profile Model

The power spectral density function of cross country irregularity is assumed to be of the form

$$S_h(\omega) = \frac{\sigma^2}{\pi}\frac{\alpha V}{(\omega^2 + (\alpha V)^2)} \tag{5}$$

where,

$Q = 2\sigma^2 \alpha V$ is the spectral intensity matrix of white noise
V = Vehicle velocity at time t
α = A coefficient depending on the type of the terrain profile in rad/m
σ^2 = Variance of the terrain irregularity in m^2
ω = Circular frequency in rad/s

The rough terrain process $h_i(t)$ (i = 1, 2……7) is modeled as the output of a linear first order shaping filter [2], operating on white noise. In this analysis it is assumed that the distances between the wheels of the vehicle model remain constant. Then $h_i(t)$, which are governed by the first order filters are given by

$$\begin{aligned} \dot{h}_1(t) + \alpha V h_1(t) &= W(t) \\ \dot{h}_2(t) + \alpha V h_2(t) &= W(t - t_{w1}) \\ &\cdot \\ \dot{h}_7(t) + \alpha V h_7(t) &= W(t - t_{w6}) \end{aligned} \tag{6}$$

where,

$W(t)$ = a white noise process with covariance function
$E[W(t)W^T(t_{wi})] = Q\delta(t - t_{wi})$
t_{wi} = Time lag between the road wheels with respect to the first road wheel

2.4 State Space Representation of Equations of Motion

The following state variables are defined for the vehicle and terrain input models:

$$\{X\}^T = [y_h\ \dot{y}_h\ \theta_h\ \ \dot{\theta}_h\ \ y_1\ \dot{y}_1\ y_2\ \dot{y}_2\ y_3\ \dot{y}_3\ y_4\ \dot{y}_4\ y_5\ \dot{y}_5\ y_6\ \dot{y}_6\ y_7\ \dot{y}_7] \tag{7}$$

$$\{h\}^T = [h_1\ h_2\ h_3\ h_4\ h_5\ h_6\ h_7] \tag{8}$$

The state space representation of nine dof vehicle model using equations (1), (2), (3), (4) and (6) by defining state vectors in the form given in equations (7) and (8) is

$$\{\dot{X}\} = [F_X]\{X\} + [D_X]\{h\} \tag{9}$$

$$\{\dot{h}\} = [F_d]\{h\} + [D_d]\{W\} \tag{10}$$

By defining an augmented state vector $X_a(t) = \left[X_1, X_2 .....X_{25}\right]^T$, the equations representing the dynamics of the vehicle and equations representing the terrain inputs at all the wheels can be combined to yield an augmented system equation as

$$\dot{X}_a(t) = FX_a(t) + D_1W(t) + D_2W(t - t_{w1}) + ... + D_7W(t - t_{w6}) \tag{11}$$

where,

$$F = \begin{bmatrix} F_{X_{18\times18}} & D_{X_{18\times7}} \\ 0_{7\times18} & F_{d_{7\times7}} \end{bmatrix}_{25\times25} ;\ D_1 = \begin{bmatrix} 0_{18\times1} \\ D_{d1_{7\times1}} \end{bmatrix}_{25\times1} ; \quad D_2 = \begin{bmatrix} 0_{18\times1} \\ D_{d2_{7\times1}} \end{bmatrix}_{25\times1} ;\ .\ .\ .\ D_7 = \begin{bmatrix} 0_{18\times1} \\ D_{d7_{7\times1}} \end{bmatrix}_{25\times1}$$

where,

$[F_X]$ is the system matrix and $[D_X]$ is the excitation distribution matrix

$[F_d]_{7\times7} =$ Matrix with diagonal elements = $(-\alpha V)$

$[D_d]_{7\times7} =$ Matrix with diagonal elements = 1

$[W]_{7\times1} = [W(t) \quad W(t - t_{w1})....W(t - t_{w6})]^T$

The response of the system is found by using zero-lag covariance matrix of the state vector $\{X_a\}$ that is, $\{Z\} = E[\{X_a\}\{X_a\}^T]$. It is the solution of the steady state matrix differential Lyapunov equation which is solved using MATLAB function ‘lyap’.

$$\begin{aligned} &F\,Z + Z\,F^T + D_1\,Q\,D_1^T + D_2\,Q\,D_2^T + ... + D_7\,Q\,D_7^T + \phi(t, t - t_{w1})\,D_1\,Q\,D_2^T + \\ &D_2\,Q\,D_1^T\,\phi(t, t - t_{w1})^T + \phi(t, t - t_{w2})\,D_2\,Q\,D_3^T + D_3\,Q\,D_2^T\,\phi(t, t - t_{w2})^T + \\ &\phi(t, t - t_{w3})\,D_3\,Q\,D_4^T + D_4\,Q\,D_3^T\,\phi(t, t - t_{w3})^T + \phi(t, t - t_{w4})\,D_4\,Q\,D_5^T + \\ &D_5\,Q\,D_4^T\,\phi(t, t - t_{w4})^T + \phi(t, t - t_{w5})\,D_5\,Q\,D_6^T + D_6\,Q\,D_5^T\,\phi(t, t - t_{w5})^T + \\ &\phi(t, t - t_{w6})\,D_6\,Q\,D_7^T + D_7\,Q\,D_6^T\,\phi(t, t - t_{w6})^T = 0 \end{aligned} \tag{12}$$

The mean square values of hull bounce acceleration and hull pitch acceleration can be calculated in terms of the elements of the covariance matrix from the following relations. Noting that $Z_{ij} = E[X_i\, X_j]$, we have

$$E[\ddot{y}_h^2] = \sum_{i=1}^{25}\sum_{j=1}^{25} F(2,i).F(2,j).Z_{ij} \tag{13}$$

$$E[\ddot{\theta}_h^2] = \sum_{i=1}^{25}\sum_{j=1}^{25} F(4,i).F(4,j).Z_{ij} \tag{14}$$

3. RESULTS AND DISCUSSIONS

The response of the system is obtained for different variances of terrain profile by keeping cut-off wave number of the terrain profile constant at 1.5 rad/m. Also the response is obtained for different stiffness values of the suspension. These parameters are listed in Table 1.

Table 1. Different values of σ^2 and suspension stiffness

Sl. No.	σ^2 (m^2)	Suspension stiffness (N/m)		
1.	0.01	85675	115675	145675
2.	0.02	85675	115675	145675
3.	0.03	85675	115675	145675

Figures 2 and 3 show the variation of root mean square (rms) values of the hull bounce acceleration and the hull pitch acceleration for the terrain profiles with variance 0.01, 0.02 and 0.03 m^2 and for stiffness values of 85675, 115675 and 145675 N/m. These plots have been made for vehicle speeds of 1 – 18 m/s (3.6 – 64.8 kmph).

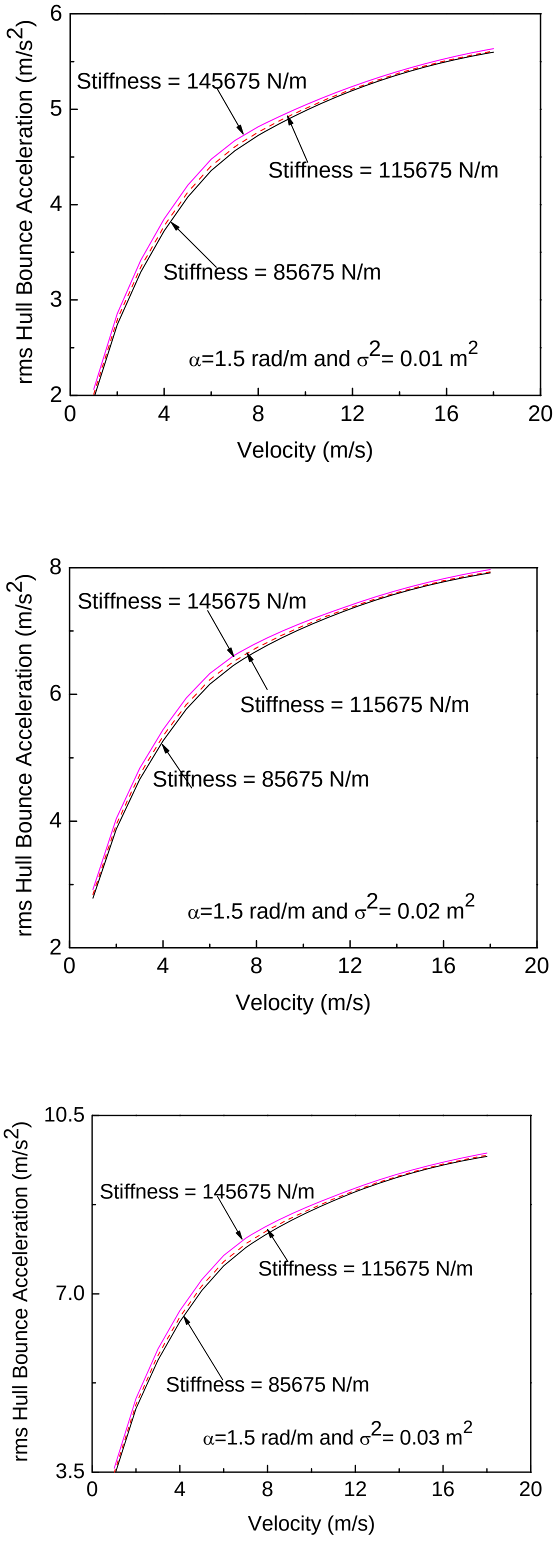

Figure 2 Variation of *rms* values of hull bounce acceleration (m/s^2) for different terrain profiles and suspension stiffness

From Figures 2 and 3 it is clear that there is an increasing trend in the rms accelerations with the increase in the vehicle speed for all values of variance of the terrain profile and also for all the stiffness values of the suspension.

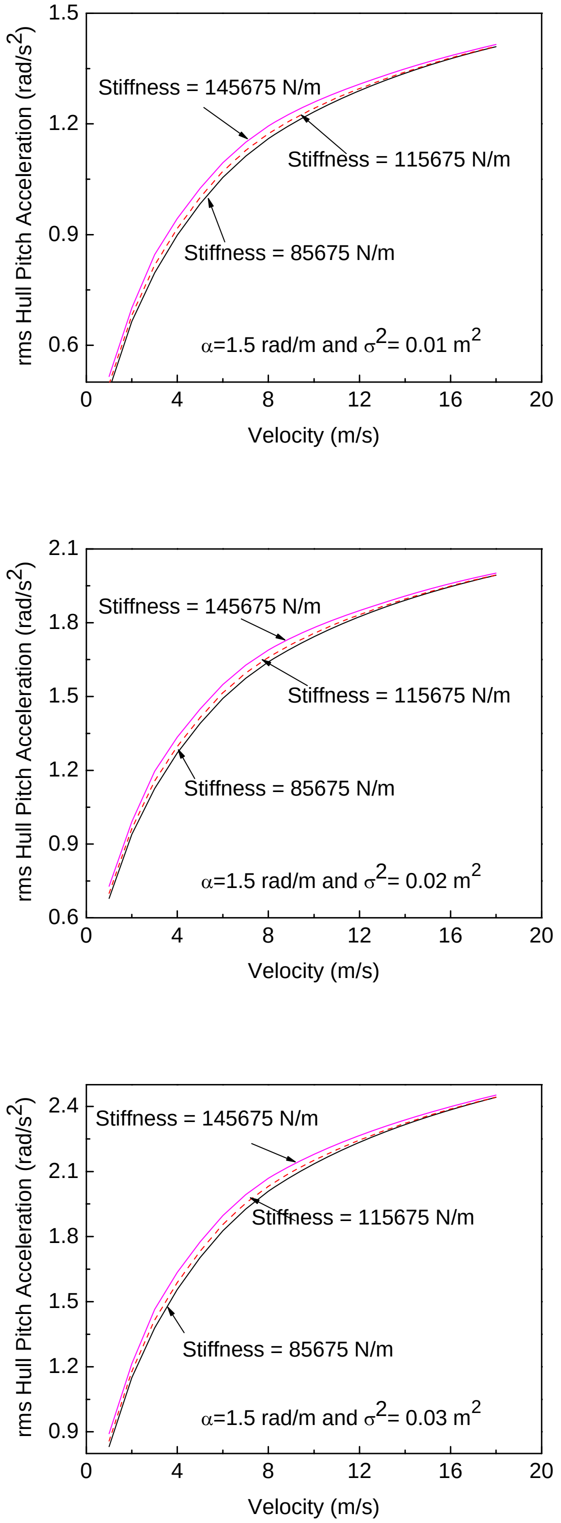

Figure 3 Variation of *rms* values of hull pitch acceleration (rad/s^2) for different terrain profiles and suspension stiffness

At vehicle velocities below 2 m/s and above 14 m/s there is hardly any difference in rms accelerations for different terrain profiles and stiffness values; for other velocities the difference is marginal.

4. CONCLUSIONS

The present study shows a comparison of bounce and pitch accelerations of hull for different values of variance of terrain profile and different values of stiffness of the suspension system. The study is conducted at constant vehicle speeds from 1 – 18 m/s (3.6 – 64.8 kmph) (stationary response). At vehicle velocities below 2 m/s and above 14 m/s there is hardly any difference in rms accelerations for different terrain profiles and stiffness values; for other velocities the difference is marginal. The study shows that the stationary response of the model can be used for investigating tracked vehicle response to different terrain conditions.

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