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FREQUENCY RESPONSE OF FLAT POST-TENSIONED CONCRETE FLOORS: FREQUENCY COEFFICIENT-ROOT FUNCTION METHOD

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Abstract

Vibration is a serviceability limit-state for the design of suspended floor systems in buildings that is not well understood by many structural engineers, and is often ignored. Dynamic response is an important design consideration for slender, two-way floors, particularly for those of post-tensioned concrete construction. At present, there are no reliable design guidelines that deal with this problem. This paper describes a research program, which will enable the development of much needed design guidance on the dynamic behavior of suspended post-tensioned concrete floors. Results from this parametric investigation have led to the preliminary development of new approach for predicting the natural frequency of flat, post-tensioned concrete floor structures. This new method has been named, the Frequency Coefficient-Root Function (FCRF) method. The FCRF method is a revolutionary and convenient tool structural engineers can use to design for the vibration serviceability limit-state of cast-insitu, post-tensioned concrete floor systems.

1. INTRODUCTION

Floor vibration is typically characterized by cyclic, vertical motion usually resulting from transient human induced loads. Among a number of studies that address this issue for composite, steel-framed floors, two very successful design guides have been published in the United Kingdom and North America that are commonly referred to and used in practice [1,2]. The reason these guides are successful in accessing the dynamic serviceability for composite floor construction is that they provide reasonably accurate methods for calculating the natural frequency of a floor panel. Research focused on the dynamic behaviour of cast-insitu concrete floors is limited, particularly for post-tensioned systems. The only available formal guideline for the dynamic analysis and design of post-tensioned systems is the Concrete Society Technical Report 43 (CSTR43) of 1994 [3]. Since its publication, there have been reports that the CSTR43 produces over conservative designs when used for assessing vibration serviceability, because the method suggested in this guide has the tendency to underestimate natural frequency [4-6].

This paper describes a state-of-the-art method for determining the dominant frequency response of post-tensioned concrete floors, which was empirically developed using finite element and experimental techniques. This method will be known as the Frequency Coefficient-Root Function (FCRF) method, and will deliver efficient dynamic serviceability designs.

2. THEORETICAL BACKGROUND

For single degree-of-freedom (SDOF) structural systems subjected to free vibration, it can be shown that for low to moderate damping the natural frequency of the solution to the equation of motion is approximately equal to the undamped, natural frequency:

$$f = \frac{1}{2\pi}\sqrt{\frac{k}{m}} \tag{1}$$

Where, f (Hz) is the natural frequency of the system, k is the system dynamic stiffness and m is the system mass.

Cast, in-situ, post-tensioned concrete floors are immensely complicated, multiple- degree-of-freedom (MDOF) systems that are not easily simplified and demand special attention. Because of the complexity of boundary conditions, material properties, geometry and internal loading, vibration problems in post-tensioned floors are very difficult. The most common approach to these problems is the employment of finite element analysis (FEA) to perform eigenvalue natural frequency analyses. In this case, the modal frequencies may take the form:

$$f_i = \frac{1}{2\pi}\sqrt{\frac{k_i}{m_i}} \tag{2}$$

Now, f_i (Hz) is the modal frequency, k_i represents the modal dynamic stiffness and m_i is the modal mass, each for the i^{th} mode of vibration. Recognizing that the modal frequency is proportional to the natural frequency of a SDOF system, we have:

$$f_i = C_i\sqrt{\frac{k}{m}} \tag{3}$$

In this case, C_i (*cyc/rad*) is a proportionality coefficient for the ith mode of vibration. In current practice, engineers generally assume the first mode, i = 1, is the governing case as calculated from an eigenvalue natural frequency analysis. This assumption is not necessarily true and may result in over conservative and uneconomical designs. For plate structures, like flat, post-tensioned concrete floors, the stiffness-to-mass ratio 'λ (rad/s^2) ' is calculated as follows:

$$\lambda = \frac{k}{m} = \frac{E_{dyn}I}{L_x^4 m} \tag{4}$$

Here, E_{dyn} (MPa) is the dynamic, elastic modulus of the concrete, I (mm^3) is the second moment of area per unit width of the slab, L_x (mm) is the short span dimension of the floor panel and m ($tonne/mm^2$) is the mass of the floor per unit area.

This paper will show that the primary response frequency of a given panel within a flat post-tensioned floor structure is not always proportional to the square-root of 'λ'. The findings of this research show that the proportionally coefficient, *C*, and the root of 'λ' are functions of the floor panel aspect ratio, 'α'. This paper proposes a new expression for the primary natural frequency of a floor panel as follows:

$$f_p(\alpha,\lambda) = C_p(\alpha)\ \sqrt[R_p(\alpha)]{\lambda} \tag{5}$$

Now, f_p is the dominant response frequency of the panel as a function of 'α' and 'λ'. $C_p(\alpha)$ is the panel coefficient function, $R_p(\alpha)$ is the panel root function, each of which are functions of the panel aspect ratio, $\alpha = L_y/L_x$, where L_y (*mm*) is the long span dimension of the panel. Equation 5 represents the basic form of the Frequency Coefficient-Root Function (FCRF) method. This new, empirical method will assist engineers in the economic design of flat, post-tensioned concrete floors for vibration serviceability.

3. FINITE ELEMENT ANALYSIS

3.1 FEA Panel Configuration Methodology

The objective of this phase of the investigation was to determine the response frequency of a variety of floor panel configurations. In plan, these configurations were based on the bending moment and deflection coefficients derived from yield-line theory, which account for edge continuity conditions for wall supported floor plates [7]. A plan sketch and of list of these edge conditions are shown in Figure 1.

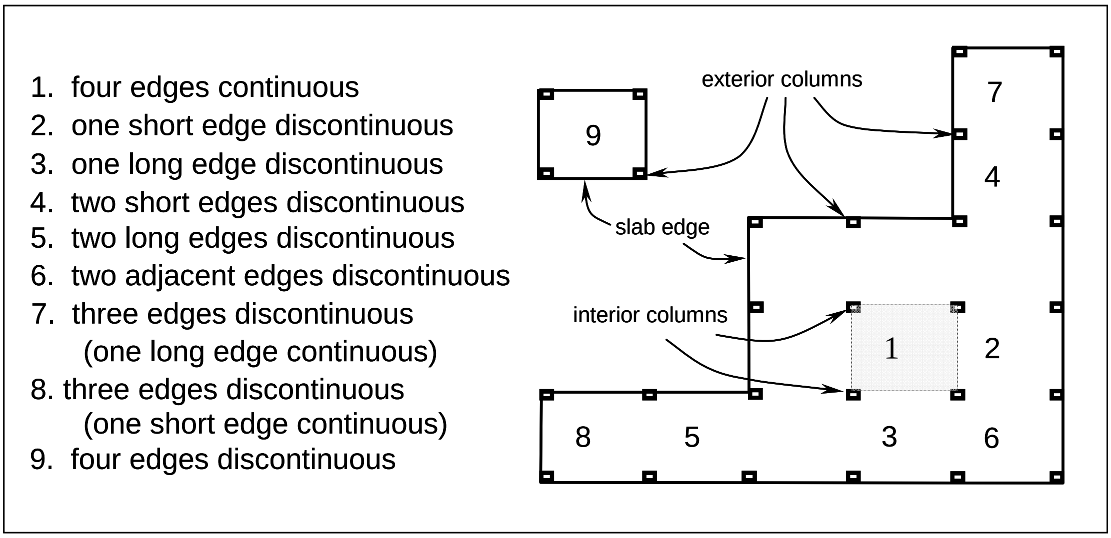

Figure 1 –Floor panel edge continuity conditions

This phase of the study has investigated the dynamic behavior of floors with exterior and interior columns. Using the FEA material model for the floor structure, which was calibrated from laboratory tests as described by Jetann et.al.[8], e.g., E_{dyn} = 33.3*GPa* and ζ=1.2%, a series of three-dimensional (3-D) FEMs was established. This series involved, three sets of three models. For each set, the long spans, L_y, of the floor panels were held constant at nine meters (9*m*) while the short spans, L_x, were adjusted to vary the aspect ratio, $\alpha = L_y/L_x$, at 1, 1.5 and 2. For each model in a set, the parameter 'λ' was investigated by adjusting the span-to-depth ratio, L_y/d, at 25, 35 and 45. To simulate the effect of support stiffness in real buildings, columns were modeled above and below the slab with eight-node solid elements having an elastic modulus of 35*GPa*. Columns heights were three meters (3*m*) above and below mid-depth of the slab elements, and all translational degrees of freedom at the end nodes were restrained to provide fixed supports. Column cross-sections were dimensioned at 5.0% of the panel span in each direction at the panel corner to provide reasonable geometry for nominal punching shear considerations. These models were analyzed for eigenvalue natural frequencies and for the panel response to a 'heel-drop' load function using non-linear, transient dynamic analysis.

3.2 FEA Results

For clarity of discussion, only the series of results related to Panel Edge Condition '1', as depicted in Figure 1, will be described initially in this section. A complete set of results for all other panel edge conditions will be summarized at the end of this paper. A graphic representation of one FEM floor structure analysed during the course of this investigation is shown in Figure 2. This particular FEM has the following parameters: $\alpha = 1$, $L_y/d = 45$ and $\lambda =$

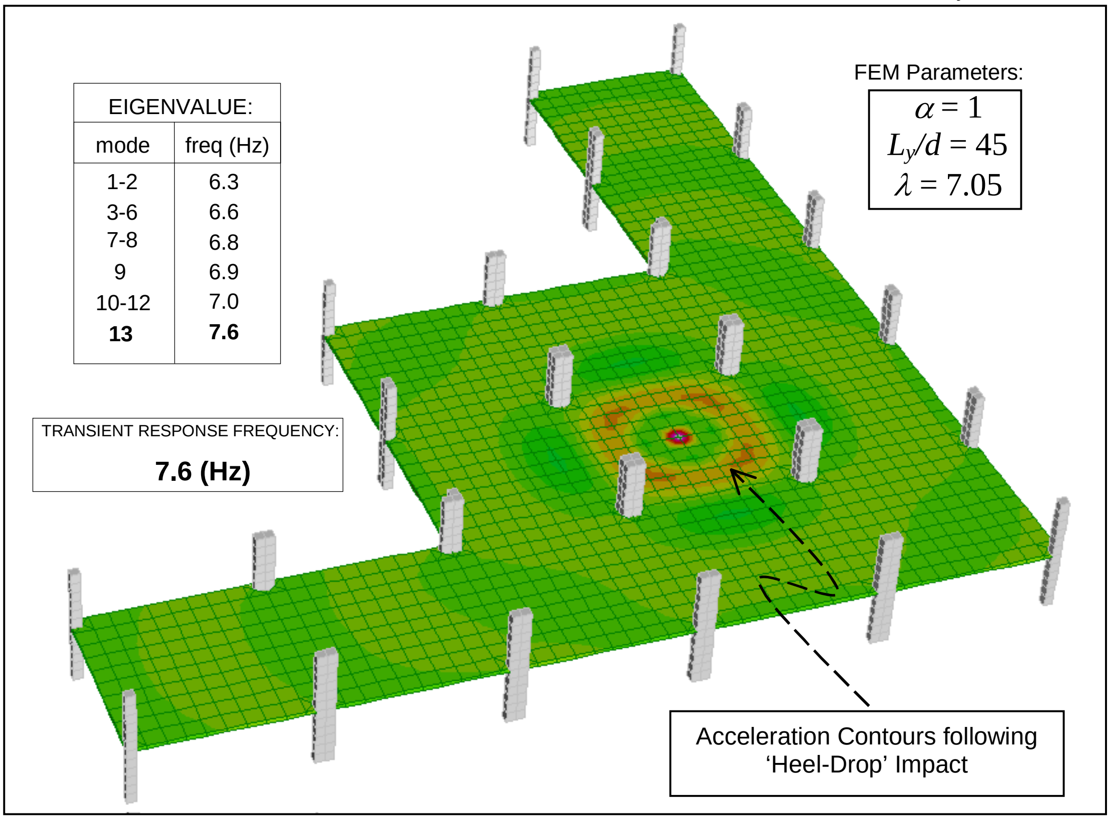

Figure 2 – FEM Floor Structure: Results for Panel Edge Condition '1'

7.05. The FEA results provided in Figure 6 are the set of eigenvalue natural frequencies for the first through the thirteenth modes, the acceleration contours immediately following a heel-drop excitation and transient dynamic response frequency resulting from a heel-drop excitation. It can be observed that the transient response frequency, of Panel 1, corresponds to the thirteenth eigenvalue frequency of 7.6*Hz*. It should be emphasized here that in current practice, engineers generally assume that the first mode frequency as calculated from an eigenvalue analysis is the governing frequency. Results from this investigation prove that this assumption is not necessarily true. The results given in Figure 2 show that the panel primary natural frequency of 7.6*Hz* would be underestimated by 17% if the first mode eigenvalue natural frequency of 6.3*Hz* were assumed to be the governing response for this panel, which could lead to an over conservative dynamic serviceability design.

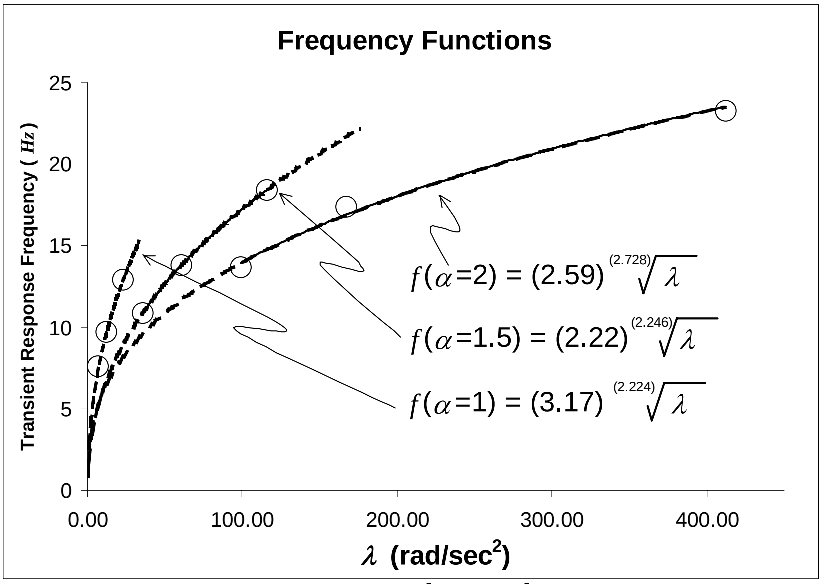

Figure 3 – Frequency Function for Panel 1: α = 1, 1.5 & 2

To obtain the transient response frequency, a time-history record of acceleration was extracted from the nonlinear, transient dynamic FEA output. From the time-history record, a power-spectrum was then analyzed. Further analyses were conducted by adjusting the FEM span-to-depth ratio, L_y/d, from 45, to 35 and 25. Now expanding this approach, when we study the frequency response behaviour of Panel 1 for aspect ratios of α = 1, 1.5 and 2 in addition to variable values of 'λ', it is obvious that the coefficient of proportionality and the root of 'λ' change with 'α'. These three cases are plotted together in Figure 3. By plotting the coefficient of proportionality 'C' and the root of 'λ' against the aspect ratio 'α', and performing a regression analysis curve-fit to these data, we can obtain expressions for each as a function of 'α'. These functions are $C_1(\alpha)$, the panel coefficient function, and $R_1(\alpha)$, the panel root function, where the subscript of '1' indicates the special case of Panel 1. Plots of $C_1(\alpha)$ and $R_1(\alpha)$ are provided in Figure 4.

Applying these functions to the form of Equation 5 gives the Frequency Coefficient-Root Function (FCRF) expression, $f_1(\alpha,\lambda)$, which can be used to estimate the primary response frequency, of a floor with the edge continuity conditions of Panel 1, as depicted Figure 5, for any value of 'α' or 'λ' as follows:

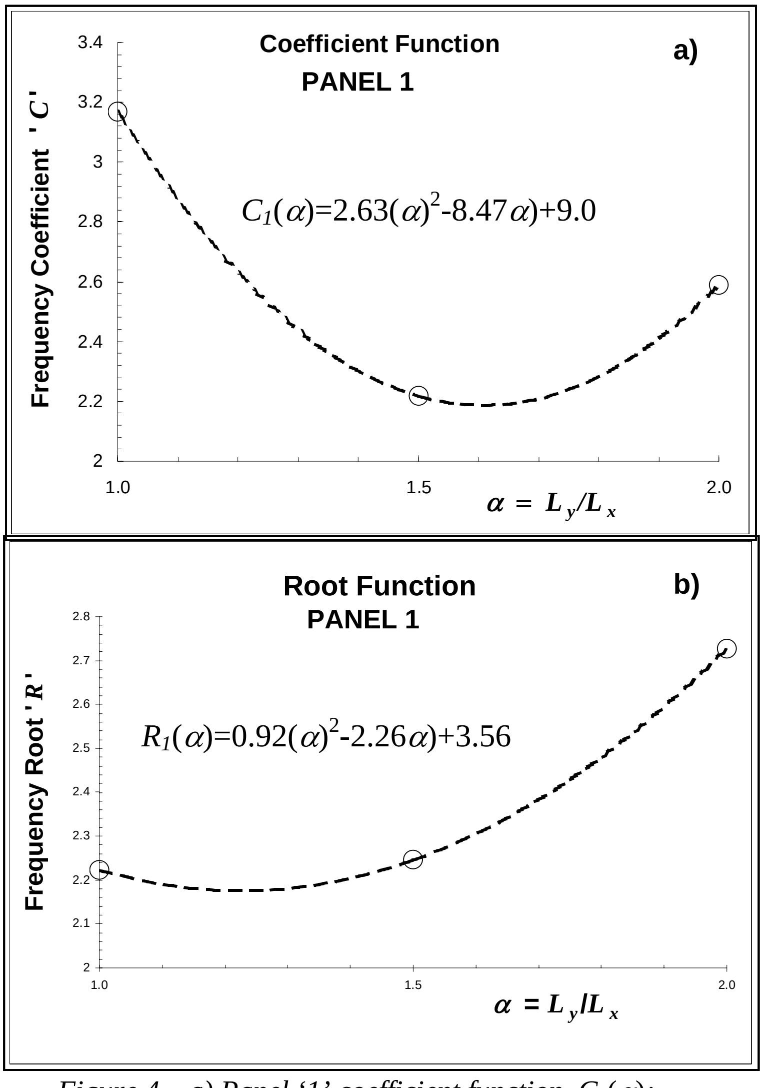

Figure 4 – a) Panel '1' coefficient function, $C_1(\alpha)$;
b) Panel '1' root function, $R_1(\alpha)$

$$f_1(\alpha,\lambda) = \left(2.63(\alpha)^2 - 8.47(\alpha) + 9.0\right)\left[\sqrt[\left(0.92(\alpha)^2-2.26(\alpha)+3.56\right)]{\lambda}\right]$$

4. FIELD TESTING AND FCRF CORRELATION

The aim of this phase of the study was to measure the natural frequency of post-tensioned concrete floor structures in real buildings, and to compare the measured natural frequencies with the frequencies predicted by the FCRF method. This section of the paper will briefly discuss the correlation of natural frequency measurements and the FCRF predicted frequency response for a floor structure having edge continuity conditions corresponding to Panel 1 as depicted in Figure 1.

Vibration measurements were gathered from a suspended, post-tensioned concrete floor panel at Charlotte Tower in Brisbane, Australia, which is a 34 story residential building. An accelerometer was rigidly fixed to the concrete surface of the slab at the center of the panel. Heel-drop tests were conducted on the panel, and a portable data acquisition system was used to obtain the acceleration time-history record and power-spectrum on site. The measured response frequency for this floor structure was 8.5*Hz*. The power-spectrum and acceleration time-history record for the tests conducted on this panel are given in Figure 5.

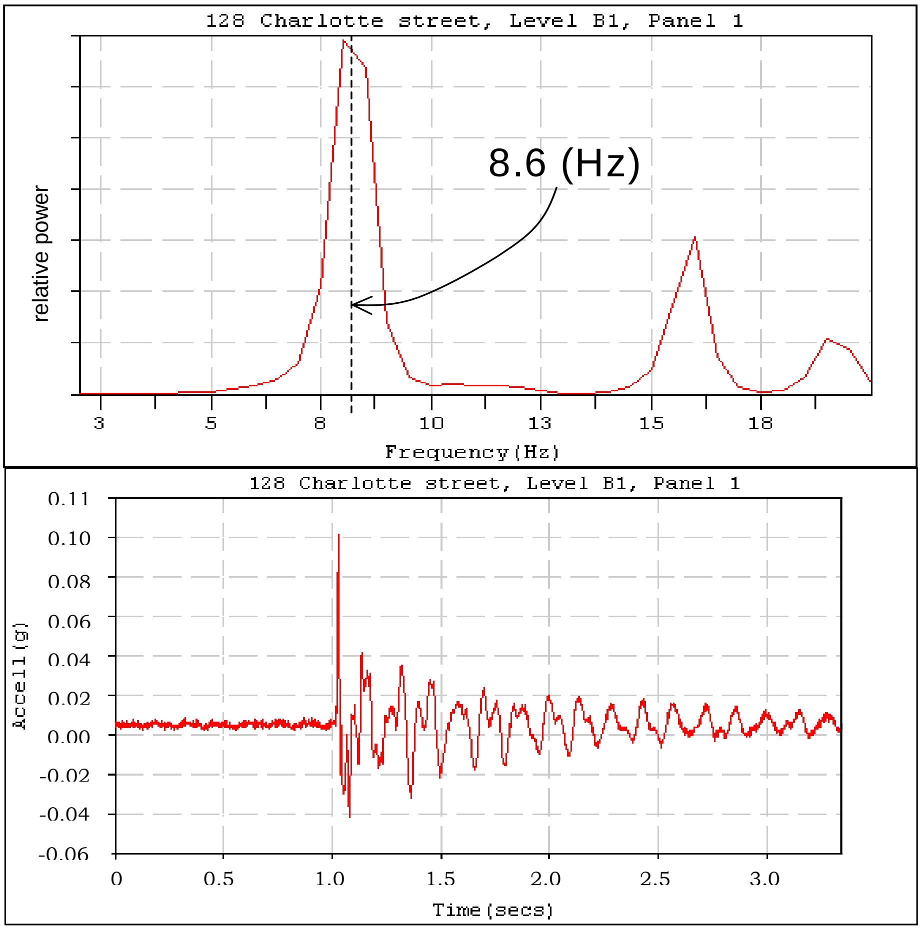

Figure5 –Charlotte Tower, Brisbane, Australia, Test Panel 1

The geometric and material properties of this panel are as follows: L_x = 7200*mm*, L_y = 8500*mm*, d = 180*mm*, I (mm^3) = $d^3/12$ = 486$x10^3$, m ($tonne/mm^2$) = 432$x10^{-9}$, f'_c = 32*Mpa*, E_{dyn} = 1.04(5055.75)$(f'_c)^{.5}$ = 29.7*GPa* . These properties correspond to the FCRF method parameters: α = 1.18 and λ = 12.95. By substituting the values of 'α' and 'λ' into Equation 6, the FCRF predicted natural frequency of the Charlotte Street floor panel shown in Figure 11 would be calculated as follows:

$$f_1(\alpha,\lambda) = \left(2.63(1.18)^2 - 8.47(1.18) + 9.0\right)\left[\sqrt[\left(0.92(1.18)^2 - 2.26(1.18) + 3.56\right)]{12.45}\right]$$

$$= 2.7(3.2)$$

$$= \underline{\mathbf{8.6}\ Hz}$$

The FCRF method frequency is exactly the measured frequency of the floor panel.

5. SUMMARY

The research discussed in this paper illustrates that dominant response frequency of a post-tensioned concrete floor panel resulting from a transient dynamic excitation does not necessarily correspond to the first-mode eigenvalue natural frequency. Furthermore, it has been demonstrated through a complete parametric investigation that the primary natural frequency of a post-tensioned concrete floor panel can be accurately predicted by a new method: The Frequency Coefficient-Root Function (FCRF) method. The basic form of the FCRF method is given by Equation 5. The development of the panel coefficient function, $C_p(\alpha)$, and the panel root function, $R_p(\alpha)$, for a floor with edge continuity conditions corresponding to Panel '1' of Figure 5 has been thoroughly explained. Table 1 summarizes these functions for all of the panel edge continuity conditions. The numerical subscript substituted for 'p' denotes the respective panel of Figure 1:

Table 1:

PANEL TYPE	*$C_p(\alpha)$ panel coefficient function and $R_p(\alpha)$ panel root function*
1	$C_1(\alpha) = 2.63(\alpha)^2 - 8.47(\alpha) + 9.0$
	$R_1(\alpha) = 0.92(\alpha)^2 - 2.26(\alpha) + 3.56$
2	$C_2(\alpha) = 6.2(\alpha)^2 - 20.3(\alpha) + 23.4$
	$R_2(\alpha) = 0.9(\alpha)^2 - 2.26(\alpha) + 3.68$
3	$C_3(\alpha) = 5.6(\alpha)^2 - 20.6(\alpha) + 24.3$
	$R_3(\alpha) = 1.06(\alpha)^2 - 2.66(\alpha) + 3.92$
4	$C_4(\alpha) = 5.8(\alpha)^2 - 18.9(\alpha) + 22.2$
	$R_4(\alpha) = 1.02(\alpha)^2 - 2.65(\alpha) + 4.01$
5	$C_5(\alpha) = 6.6(\alpha)^2 - 22.9(\alpha) + 25.4$
	$R_5(\alpha) = 1.3(\alpha)^2 - 3.25(\alpha) + 4.33$
6	$C_6(\alpha) = 7.8(\alpha)^2 - 26.5(\alpha) + 27.8$
	$R_6(\alpha) = 1.23(\alpha)^2 - 3.14(\alpha) + 4.29$
7	$C_7(\alpha) = 5.4(\alpha)^2 - 18.7(\alpha) + 22.1$
	$R_7(\alpha) = 1.3(\alpha)^2 - 3.38(\alpha) + 4.58$
8	$C_8(\alpha) = 7.8(\alpha)^2 - 26.5(\alpha) + 27.8$
	$R_8(\alpha) = 1.3(\alpha)^2 - 3.38(\alpha) + 4.58$
9	$C_9(\alpha) = 5.4(\alpha)^2 - 18.7(\alpha) + 22.1$
	$R_9(\alpha) = 1.05(\alpha)^2 - 2.5(\alpha) + 4.01$

The FCRF method calculations described in this paper apply to floor structures supported by external and internal columns. Future work will include analyses for floor panel configurations having external and internal wall supports and those with external wall and interior columns.

In conclusion, this research will exploit an opportunity to develop empirical guidelines for the dynamic behavior of post-tensioned floors, partially through the use transient dynamic analysis.

6. ACKNOWLEDGEMENTS

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