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FINITE ELEMENT MODELING OF MULTI-LAYER COMPOSITE PLATES WITH INTERNAL DELAMINATION

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Abstract

A three-dimensional finite element model for the delaminated fiber reinforced composites is developed to analyze the dynamics of multi-layer composite plates with internal delamination. Natural frequency and modal displacement are analyzed for samples with different dimensions of delamination. Numerical results showed a good agreement with the available experimental data and an enhancement of the accuracy of the results when the proposed model is adopted. The results of this study are useful for detecting delamination in multi-layer composite materials.

1. INTRODUCTION

The use of composite materials in space vehicles and various machine components has increased considerably over the past decades. Under repeated or impact loads these materials are subjected to various forms of damage, mostly delaminations and cracks. Such damage becomes an obstacle to the more extensive usage of composite materials. Therefore, the monitoring of internal or hidden damage in composite material is critical in engineering practice.. As it is difficult to obtain accurate analytical solution for multi-layer material problem theoretically, the computational approach, e.g., finite element method plays an important role in the implement of damage detection for laminated composites. There are contributions on both numerical and experimental investigations into the behavior of delaminated multi-layer composites. Sankar [1] had modeled a delaminated beam as two sublaminates by offset beam finite elements, Rikards [2] developed a model of finite superelements for sandwich composite beam and plate without delamination, each layer being considered as a simple Timoshenko beam. Zak et al. [3] developed models of finite elements for beams and plates with boundary delamination. Among most of the publications the prediction of material mechanical or dynamic behaviors is based on the classical laminated plate theory [4]. Due to the ignorance of the transverse shear deformation effect, the classical layerwise theory based on the straight normal assumption of elastic thin plate cannot provide accurate results for moderately thick laminated plates, for which the inplane elastic modulus is much higher than the transverse shear modulus [5]. Besides, the Poisson's effect is significant for angle-ply laminated plates. Therefore, threedimensional layerwise theory must be adopted in order to obtain an accurate prediction of dynamic response for multi-layer composite plates. In addition, potential dangers are often induced by hidden or internal delaminations in the in-service laminated composites. However, there is a lack of both numerical and experimental investigations on internal delaminations with different geometries in laminated composites.

In this paper, a three-dimensional finite element model for multi-layer composites with internal delamination is established, and the fiber orientations of individual lamina as well as the transverse shear effect are taken into account. Numerical calculation using ABAQUS CAE v6.3 package is carried out for different plates of free boundary conditions. Natural frequencies, modal displacements of the intact and damaged multilayer composite plates are subsequently analyzed for various samples.

2. FINITE ELEMENT MODELING FOR LAMINATED COMPOSITE PLATES

The finite element used for dynamic behavior analysis of a multi-layer composite plate is an eight-node rectangular solid thin plate as shown in Fig. 1. For each node, there are three degrees of freedom, i.e. translations along the global coordinate axes of *x*, *y*, and *z*, respectively. The element thickness is assigned to be equal to that of the corresponding individual lamina. The local element coordinate system (x_1, x_2, x_3) is arranged with the first axis being coincident with the fiber direction. All physical parameters throughout an element are assumed to be the same.

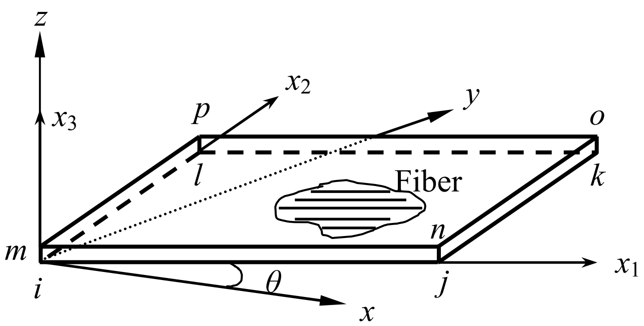

Figure 1: The cross-ply laminated plate and the coordinate system.

For an eight-node finite element with three degrees of freedom per node, the displacement field over an element is given by

$$\{\delta\} = (u, v, w)^T = \sum_{i=1}^{8} [N_i]\{\delta_i\} \tag{1}$$

where $\{\delta\} = (u_i, v_i, w_i)^T$ is the displacement vector at node *i*, $[N_i] = N_i[I_3]$, $[I_3]$ is the three-order unit matrix and N_i the shape function [8]. Then the strain vector of each element can be expressed in terms of displacement in the global coordinate system as

$$\{\varepsilon^e\} = (\varepsilon_{11}^e, \varepsilon_{22}^e, \varepsilon_{33}^e, \varepsilon_{12}^e, \varepsilon_{13}^e, \varepsilon_{23}^e)^T = \sum_{i=1}^{8} [B_i]\{\delta_i\} \tag{2}$$

Where $[B_i]=[\Delta][N_i]$ and
$$[\Delta]^T = \begin{bmatrix} \frac{\partial}{\partial x} & 0 & 0 & \frac{\partial}{2\partial y} & 0 & \frac{\partial}{2\partial z} \\ 0 & \frac{\partial}{\partial y} & 0 & \frac{\partial}{2\partial x} & \frac{\partial}{2\partial z} & 0 \\ 0 & 0 & \frac{\partial}{\partial z} & 0 & \frac{\partial}{2\partial y} & \frac{\partial}{2\partial x} \end{bmatrix}$$

Thus, the strain vector in an element can be expressed in terms of nodal displacements as

$$\{\varepsilon^e\}=[B]\{\delta^e\} \tag{3}$$

with $[B]=\left[\ [B_1],[B_2],...,[B_8]\ \right]$ and $\{\delta^e\}=\left(\{\delta_1\}^T,...,\{\delta_8\}^T\right)^T$

Therefore, the stresses of an element in global coordinate system can be expressed by the nodal displacement as

$$\{\sigma^e\}=\left(\sigma^e_{11},\sigma^e_{22},\sigma^e_{33},\sigma^e_{12},\sigma^e_{13},\sigma^e_{23}\right)^T=\sum_{i=1}^{8}[K^e]\{\delta^e\} \tag{4}$$

where $[K^e]$ is the element stiffness matrix as $[K^e]=\int_{V_e}[B]^T[A][C][A]^{-1}[B]dV$ (5)

$$[C]=\begin{bmatrix} \frac{1}{E_1} & \frac{-\upsilon_{12}}{E_1} & \frac{-\upsilon_{13}}{E_1} & 0 & 0 & 0 \\ \frac{-\upsilon_{12}}{E_1} & \frac{1}{E_2} & \frac{-\upsilon_{23}}{E_2} & 0 & 0 & 0 \\ \frac{-\upsilon_{13}}{E_1} & \frac{-\upsilon_{23}}{E_2} & \frac{1}{E_3} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{G_{12}} & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{G_{13}} & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{1}{G_{23}} \end{bmatrix}^{-1} \qquad [A]=\begin{bmatrix} \cos^2\theta & \sin^2\theta & 0 & 0 & 0 & -2\sin\theta\cos\theta \\ \sin^2\theta & \cos^2\theta & 0 & 0 & 0 & 2\sin\theta\cos\theta \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & \cos\theta & \sin\theta & 0 \\ 0 & 0 & 0 & -\sin\theta & \cos\theta & 0 \\ \sin\theta\cos\theta & -\sin\theta\cos\theta & 0 & 0 & 0 & \cos^2\theta-\sin^2\theta \end{bmatrix}$$

Here, $[C]$ is the matrix of material constants with E_1, E_2, E_3, G_{12}, G_{13}, G_{23}, υ_{12}, υ_{13} and υ_{23} being the orthotropic elastic constants of individual lamina, and $[A]$ is the transform matrix between the local and global coordinate systems. After assembly of the nodal displacements of all elements, the total strain energy of multi-layer composite plate can be represented as

$$U=\frac{1}{2}\{\delta\}^T[K]\{\delta\} \tag{6}$$

where $\{\delta\}$ and $[K]$ are the global nodal displacement vector and stiffness matrix, respectively. From the eigenvalue problem of

$$\left(\ [K]-\omega^2[M]\ \right)\{\delta\}=0 \tag{7}$$

where $[M]$ is the global mass matrix, the modal parameters such as natural frequencies ω_i, mode shapes, modal strains, etc., can be obtained. For an arbitrary intact laminated plate, in order to ensure the material continuity, the displacements and their variations of each pair of coincident nodes on the upper and lower adjacent laminae have to be equal in the whole process of computation. When the plate is delaminated, the displacements of each pair of coincident nodes just on the upper and lower surfaces within the delamination region are considered not to be connected to each other.

2.1 Model Refinement

A sensitivity analysis is carried out to study the effect of element size on the accuracy of the finite element results. In this study, a laminated square composite plate, Plate 1, is an 8-layer square plate with a side length of 178 mm and a thickness of 1.58 mm. All the ply orientations are equal to 0^o and the material constants are E_1 = 172.7 GPa, $E_2 = E_3$ = 7.2 GPa, $G_{12} = G_{13}$ = 3.76 GPa, G_{23} = 2.71 GPa, $\upsilon_{12} = \upsilon_{13}$ = 0.3, υ_{23} = 0.33 and ρ = 1566 kg m^{-3}. Table 1 shows the natural frequencies of the first six modes for different generated mesh sizes. The attained results are representing variable number of elements per layer. A remarkable change in results is obtained when increasing the total number of elements from 25 to 100 elements per layer. Meanwhile, the change in natural frequency obtained with 900 elements in comparison with 400 is negligible. This leads to applying the 400 elements per layer for each lamina model in the subsequent computations to achieve high accuracy and minimum computations time.

Table 1
Natural frequencies (Hz) of the intact Plate 1 for different meshes.

Mode	Element number for each lamina			
	25	100	400	900
1	78.51	81.23	81.48	81.56
2	100.17	107.20	109.20	109.58
3	277.31	207.72	199.50	199.51
4	316.42	294.01	300.46	301.73
5	489.29	422.93	391.40	391.12
6	581.06	523.91	533.89	535.80

2.2 Model Verification

To verify the results of the developed model, Plate 1 is compared with the numerical results of Yam [8] and both the numerical and experimental work of Lin [6]. It is obvious that the present developed model is more accurate than the numerical work by Yam [8] and the numerical work by Lin [6] in comparison to the experimental work by Lin [6] for the first six modes. In addition, another laminated composite plate is considered for model verification. This plate is called; Plate 2. It is a 16-layer plate with an area of 240 x 180 mm^2 and a thickness of 2.08 mm. The ply orientations are of $[0^o/0^o/90^o/90^o/0^o/0^o/90^o/90^o]_s$ and the material constants are E_1 = 125 GPa, $E_2 = E_3$ = 8.5 GPa, $G_{12} = G_{13}$ = 4.5 GPa, G_{23} = 3.27 GPa, $v_{12} = v_{13} = v_{23}$ = 0.3 and ρ = 1550 kg m^{-3}. The first six natural frequencies of Plate 2 are compared with the numerical and experimental work by Wei [9]. It is obvious that the present model demonstrates better agreement with the experimental results by Wei [9]. Table 2 shows the natural frequencies of the first six modes for bending vibration of Plates 1 and 2.

Table 2
Natural frequencies (Hz) of the intact Plate 1 and Plate 2 for different methods.

Mode	Plate 1				Plate 2		
	Present	Reference [8] Yam	Reference [6] Lin		Present	Reference [9] Wei	
		Numerical	Numerical	Experimental		Numerical	Experimental
1	81.48	82.26	83.57	81.50	89.16	90.52	90
2	109.20	113.10	118.42	107.40	278.97	279.17	289
3	199.50	207.29	207.79	196.60	330.35	333.59	318
4	300.46	325.28	329.41	285.50	354.92	354.22	354
5	391.40	408.51	419.83	382.50	393.26	397.62	386
6	533.89	539.92	546.93	531.00	574.50	583.71	570

3. RESULTS AND DISCUSSIONS

3.1 Samples

There are five square plates made of multi-layer carbon fiber-reinforced epoxy composites. Every sample plate has a side length of 225.5 mm and a thickness of 2.05 mm and consists of 8-layer in orientation of $[0^o/90^o/0^o/90^o]_s$. One of the plates is intact and named O, the other plates are named A, B, C, D, respectively, are damaged with delamination of different areas. The delamination location is simulated as an interspace of 0.02 mm between the third and the fourth layers counted from the top of the plate. Every delaminated plate has only one square delamination located at the position with the center as shown in Fig. 2. The delamination center coordinates are x = 163.5 mm, y = 163.5 mm and z = 1.28125 mm. The delamination areas of plates A, B, C and D are 11.275 x 11.275, 33.825 x 33.825, 56.375 x 56.375 and 78.925 x 78.925 mm^2, respectively. The material constants of the samples for FEM computations are E_1 = 37.78 GPa, $E_2 = E_3$ = 10.9 GPa, $G_{12} = G_{13} = G_{23}$ = 4.91 GPa, $v_{12} = v_{13}$ = 0.3, v_{23} = 0.11 and ρ = 1813.9 kg m^{-3}.

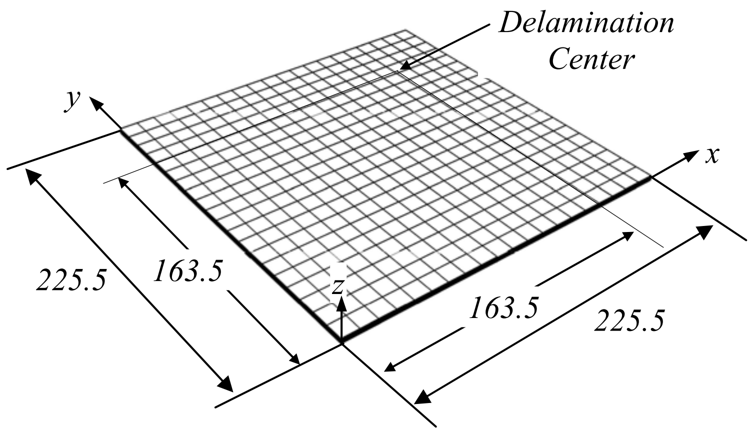

Figure 2: The finite element model of the laminated plate and the delamination region center.

3.2 Delamination effect on natural frequency

According to Eq. (7) natural frequencies are computed for the first ten modes of the plates.
It can be seen that with the increase of delamination area, the natural frequency decreases. The delamination effect on the natural frequency is however very slight for the first six modes and higher for the other modes. The relationship between the frequency and delamination size is investigated. Figures 3 and 4 show the percentage changes of natural frequencies with delamination areas, where the height of the column represents the absolute value of the percentage change of natural frequency, i.e.,

$$\kappa = \frac{|\omega_{\text{delaminated}} - \omega_{\text{intact}}|}{\omega_{\text{intact}}} \tag{8}$$

It is obvious that these absolute values increase with the delamination area. It is also seen that the decrease of natural frequency is not the same for different modes. The delamination-induced decreases of natural frequencies are relatively large for mode 3 and mode 9, and the change of natural frequency is almost negligible in mode 1. However, the variation manner of the values is not the same for each plate. When the delamination area is 11.275 x 11.275 mm^2, the delamination-induced changes of natural frequencies are nearly zero for all the considered cases, which indicates that the delamination-induced frequency change is insignificant for small delamination. Therefore, it is indispensable to analyze the delamination-induced changes of other parameters for effective detection of delamination in composite plates.

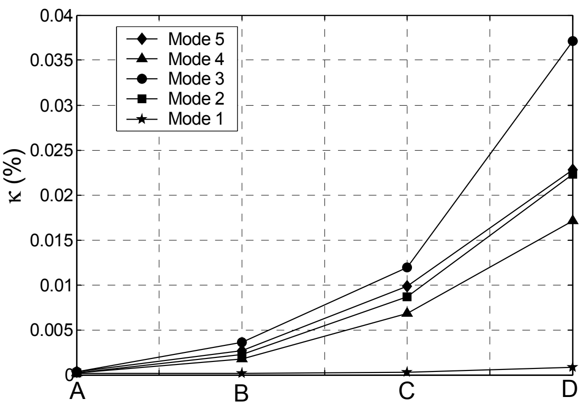

Figure 3: Percentage changes of natural frequencies

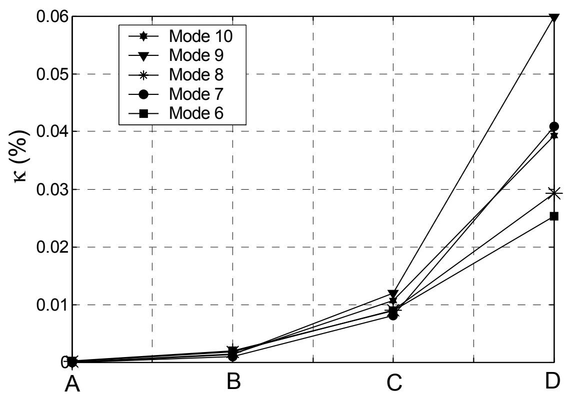

Figure 4: Percentage changes of natural frequencies

3.3 Delamination effect on mode shape

The above analysis demonstrates that the delamination-induced changes of plate parameters are mode dependent. This may imply that the delamination region exerts specific effects on the relevant modes. In order to further investigate the relationship between delamination and mode-dependent variations of energy dissipation in the plate, the unit-normalized local displacements of points within the delamination region of the plates are computed. The relative displacements are analyzed for points along the line (y = 163.5 mm, z = 1.28125 mm) just on the upper and lower surfaces within the delamination region of Plate C (the plate with delamination area of 56.375 x 56.375 mm^2). Figures (5), (6) and (7) show the differences of displacement along z-direction between the upper and lower points

$$\delta\varphi = \left(w_{upper} - w_{lower}\right) \tag{9}$$

which are assumed to be coincident with each other before plate motion. It can be seen that obvious penetrations occur in modes 3, 4, 6, 8, 9 and 10 because of the negative values of $\delta\varphi$. As there is no restriction to penetration within the delamination region in the FE model, penetration occurs in some modes. However, this is physically impossible; then, it is known that obvious impact exists within the delamination region in modes 3, 4, 6, 8, 9 and 10 during vibration of the plate. Therefore, energy dissipation variation will be larger in these modes for Plate C during vibration. Figures (8) – (14) show the mode shapes of Plate C. It is obvious that the delamination appears in mode 7 because of the high energy dissipation in this mode as it is seen in Figure (7).

Therefore, when delamination occurs somewhere in a composite plate, there may be interactive motion or impact within the delamination region during vibration of the plate. These phenomena cause the variations of energy dissipation in the plate, and they are modal dependent. Thus, the delamination can be detected according to the variation of energy dissipation in the plate during vibration.

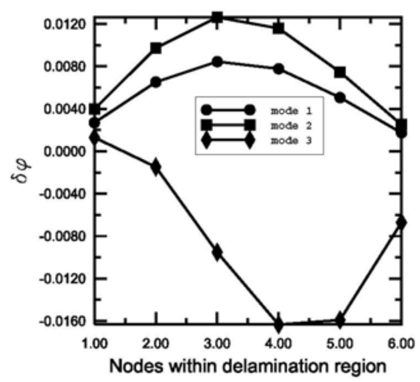

Figure 5: Displacements along *z*-direction

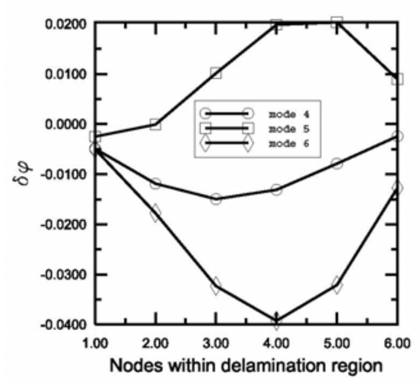

Figure 6: Displacements along *z*-direction

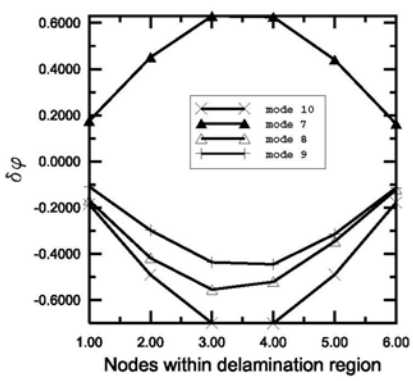

Figure 7: Displacements along *z*-direction

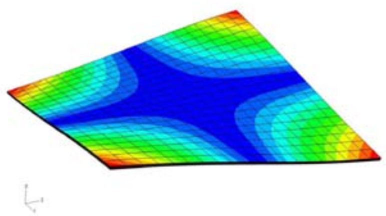

Figure 8: The first mode shape of Plate C (70.07 Hz)

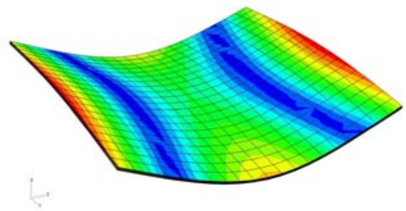

Figure 9: The second mode shape of Plate C (132.71 Hz)

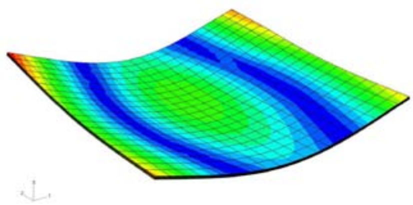

Figure 10: The third mode shape of Plate C (166.71 Hz)

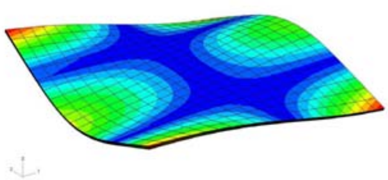

Figure 11: The fourth mode shape of Plate C (194.53 Hz).

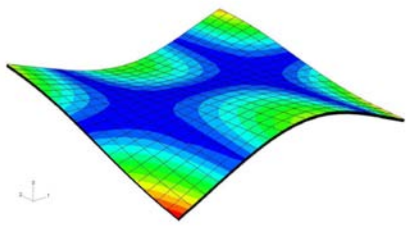

Figure 12: The fifth mode shape of Plate C. (216.15 Hz)

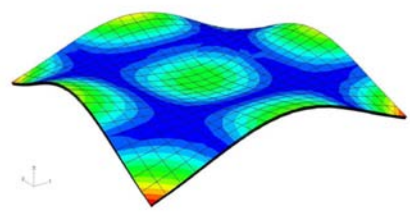

Figure 13: The sixth mode shape ofPlate C (357.12 Hz).

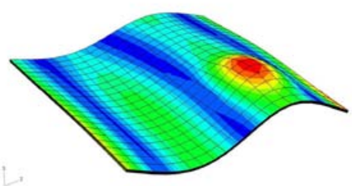

Figure 14: The seventh mode shape of Plate C (370.37 Hz).

4. CONCLUSIONS

The delamination problem for multi-layer composite plates has been analyzed using finite element method and modal analysis. To achieve accurate results for delamination detection of multi-layer composite plate, different fiber orientations, orthotropy of laminated composites and transverse shear effect are taken into account in the finite element computation. The following conclusions may be drawn from the results of numerical simulation in this study.
(1) The finite element model proposed in this paper can predict accurately the dynamic behaviors of a multi-layer composite plate with internal delamination at arbitrary location. It is not as computationally expensive as the usually used three-dimensional brick elements when the proposed element is used because there is no restriction of ratio between element length and thickness. (2) Local internal delamination has slight effect on natural frequencies of a multi-layer composite plate although the extent of natural frequency variation increases with both the delamination dimension and the order of natural frequency. (3) Delamination-induced change of deformation is more sensitive than that of frequency, and changes of displacement are mode-dependent. The most remarkable delamination-induced changes of displacement occur in the same mode as the most remarkable decrease of frequency when the delamination area is relatively small for the cases considered in this paper. (4) The results of numerical analysis in this study can be taken as guidance for arrangement of displacement measurements on the surface of specimen when experimental modal analysis is carried out to investigate local changes of structural parameters for damage detection. The finite element model, strategy and numerical method provided in this paper can be used for dynamic response analysis of damaged engineering structures, especially for multi-layer composite plates.

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