ICSV14
Cairns • Australia
9-12 July, 2007

14th International Congress on Sound & Vibration

EXPERIMENTAL IDENTIFICATION OF THE TRANSFER MATRIX OF THE MUFFLER

Ke Liu[1], Tao Feng[2] , Rui Wu[1], Qijun Zhou and Lijian Ye[1]

[1] Institute of Acoustics, Chinese Academy of Sciences
P. O. X 2712 Beijing 100080 China
[2] Beijing Technology and Business University
NO.11, Fu-cheng-lu Road, Beijing, China
kevine@mail.ioa.ac.cn

Abstract

The transfer matrix, which is independent with the boundary conditions, is one of the most important parameters of the muffler. An experimental identification method obtaining the transfer matrix based on the least square method (LSM) is developed here. The quantitative relationship between the parameters in the transfer matrix and the sound pressure measured at the certain position in experimental system is researched. A muffler and a measurement pipe system are designed and established to validate the identification method. The experimental procedures according to the identification theory are also referred. The experimental identification results match the analysed ones very well.

1. INTRODUCTION

Transfer matrix can express the acoustic transmission property of muffler. Compared with insertion loss, it describes the inherent acoustic property of mufflers, which is independent to its acoustic boundary conditions. It can be used to calculate the transmission loss of complicated silencer system composed of different mufflers, or predict the sound radiation from the end of the duct with sound source connected with it.

C.W.S. To and A. G. Doige [1, 2] described the sound system with Two-port Model, and researched the calculating method of the transfer matrix both theoretically and experimentally. J. Charley [3] studied the property of sound component in pipe filled with water, using Two-port Model to describe the property of pump. They all took volume velocity and sound pressure as the port parameters, and got the matrix parameters by experiments. In this article, two travelling waves in opposite directions are considered as the port parameters, and experimental identification expressions are referred. After that, an experimental system is designed and established so parameters can be identified and the results are compared with theoretically calculated results.

2. EXPERIMENTAL IDENTIFICATION METHOD FOR THE TRANSFER MATRIX OF MUFFLER'S

In the experimental system as Fig. 1 shows，a pipe muffler is connected with two testing pipes at both sides. When getting the sound pressure signals at certain places in the testing pipes, the transfer matrix of this pipe muffler can be obtained. p_i^+, p_i^- and p_o^+, p_o^- each denotes the travelling wave in opposite directions of both testing pipes. And the parameters between the input port and output port have such relationship:

$$\begin{Bmatrix} p_o^+ \\ p_o^- \end{Bmatrix} = \begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix} \begin{Bmatrix} p_i^+ \\ p_i^- \end{Bmatrix} \quad (1)$$

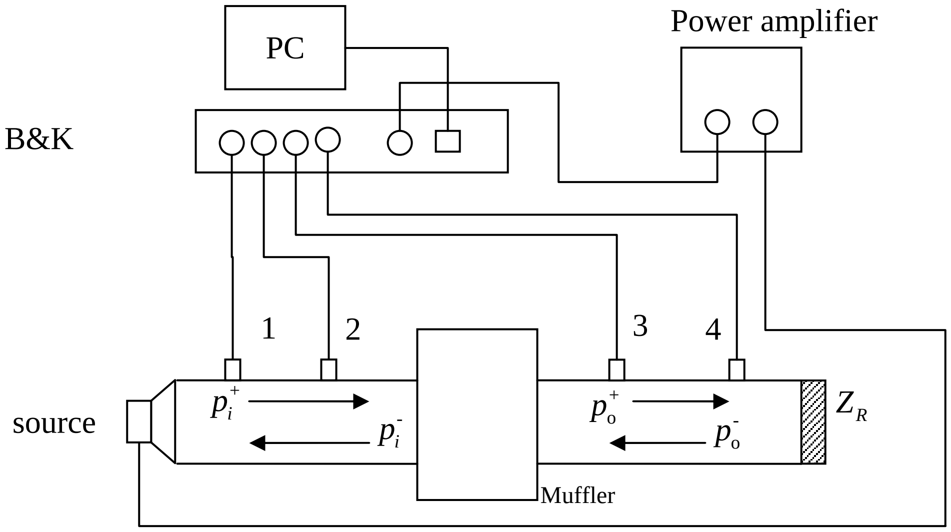

Figure 1. Experimental system of measuring muffler's transmission loss.

In Eq. (1), $[S]$ is the transfer matrix, while p_i^+, p_i^- and p_o^+, p_o^- , describing the port parameters of the transfer matrix, is in fact the result of the decomposition of the standing waves in certain pipes, so it cannot be measured directly from measurement. And how to get the transfer matrix of the muffler from the experiment data needs to be studied. Equation (1) can be changed into another form:

$$\begin{aligned} p_o^+ &= S_{11} p_i^+ + S_{12} p_i^- \\ p_o^- &= S_{21} p_i^+ + S_{22} p_i^- \end{aligned} \quad (2)$$

Then Eq. (2) can be written as

$$\begin{aligned} \frac{p_o^+}{p_i^+} &= S_{11} + S_{12} \frac{p_i^-}{p_i^+} \\ \frac{p_o^-}{p_i^+} &= S_{21} + S_{22} \frac{p_i^-}{p_i^+} \end{aligned} \quad (3)$$

where

$$U = \frac{p_o^+}{p_i^+}, V = \frac{p_o^-}{p_i^+}, R = \frac{p_i^-}{p_i^+} \tag{4}$$

p_i^+, p_i^- and p_o^+, p_o^- in Eq.(4) cannot be obtained from direct measurement. What can be measured directly by acoustic sensors is the sound pressure p_1, p_2, p_3 and p_4 at position 1, 2, 3 and 4. Then the relationship between the parameters U,V,R and the measurable parameters p_1, p_2, p_3 and p_4 needs to be studied. If

$$H_{13} = \frac{p_3}{p_1}, H_{14} = \frac{p_4}{p_1}, H_{12} = \frac{p_2}{p_1} \tag{5}$$

After applying decomposition equation[4]5 in duct here

$$\begin{aligned} p_o^+ &= \frac{e^{jks} p_3 - p_4}{e^{jks} - e^{-jks}} \\ p_o^- &= \frac{-p_3 e^{jks} + p_4}{e^{jks} - e^{-jks}} \end{aligned} \tag{6}$$

$$\begin{aligned} p_i^+ &= \frac{e^{jks} p_1 - p_2}{e^{jks} - e^{-jks}} \\ p_i^- &= \frac{-p_1 e^{jks} + p_2}{e^{jks} - e^{-jks}} \end{aligned} \tag{7}$$

In Eqs. (6) and (7), k is the wave number, s is the distance between the acoustic sensors 1 and 2 as well as 3 and 4. With respect to the available identified frequency band here, it commonly satisfies this relationship

$$0.1 < fs/c_0 < 0.4 \tag{8}$$

In Eq. (8), f denotes the signal frequency, while c_0 is the velocity of sound. While applying Eqs. (5), (6), (7) to Eq. (4), the result is

$$\begin{aligned} U &= \frac{e^{jks} - H_{34}}{e^{jks} - H_{12}} H_{13} \\ V &= \frac{H_{34} + e^{-jks}}{e^{jks} - H_{12}} H_{13} \\ R &= \frac{H_{12} - e^{-jks}}{e^{jks} - H_{12}} \end{aligned} \tag{9}$$

H_{34}, H_{13} and H_{12} stands for the transfer function between the sensors in Eq. (9), and can be measured by current equipments. If s is given a certain value, e^{jks} and e^{-jks} can be determined. So with the measured data, U , V and R can also be determined according to Eq. (9). Substituting Eq. (9) into Eq. (3), it comes to a simple result

$$\begin{aligned} U &= S_{11} + S_{12}R \\ V &= S_{21} + S_{22}R \end{aligned} \tag{10}$$

With Eqs.(10) solved, the parameters of the transfer matrix can be determined. That is, if two groups of linear independent parameters U,R as well as V,R are obtained, S_{11},S_{12} and S_{21},S_{22} can then be determined according to Eqs.(10). In this experiment, the sound field can be changed by changing the acoustic impedance of the right end of the pipe which depends upon the boundary conditions in this system. The different sound field caused by different acoustic boundary conditions would be linear independent. So groups of linear independent data can be obtained due to the change of the sound field. Then Eq. (10) becomes two systems of equations. The equations would be over determined if the equations' number is more than two, then a least square method (LSM) can be used to seek the proper value of the parameters in transfer matrix $[S]$.

$$\begin{bmatrix} U_1 \\ U_2 \\ \cdots \\ U_N \end{bmatrix} = \begin{bmatrix} 1 & R_1 \\ 1 & R_2 \\ \cdots & \cdots \\ 1 & R_N \end{bmatrix} \begin{bmatrix} S_{11} \\ S_{12} \end{bmatrix} \tag{11}$$

$$\begin{bmatrix} V_1 \\ V_2 \\ \cdots \\ V_N \end{bmatrix} = \begin{bmatrix} 1 & R_1 \\ 1 & R_2 \\ \cdots & \cdots \\ 1 & R_N \end{bmatrix} \begin{bmatrix} S_{21} \\ S_{22} \end{bmatrix} \tag{12}$$

3. EXPERIMENTAL IDENTIFICATION RESULTS

As Table 1 shows, adjusting the acoustic boundary conditions at the right end of the pipe, four different groups of experimental data can be achieved. After calculating the transfer matrix H_{12}, H_{34}, H_{13} of sound pressure between the certain measuring points, Eq. (9) can be used to computer the intermediate variables U, V, R . S_{11}, S_{12} and S_{21}, S_{22} can then be calculated by Eqs. (11) and (12). Figure 2 shows the measurement and analysis system, while Fig.3 experimental duct system. To check the validity of the data obtained by this identification method, the muffler is designed with well-regulated shape so it has theoretical expression of the transfer matrix. The identified results are provided in Fig. 4 and Fig. 5. In these Figures, continuous real lines represent identified results from experiment, while square hollow ones represent theoretical results. The experimental identification results show close agreement with the theoretical data. Figure 6 shows the residual error of the identification results. The result is sufficiently precise, with residual error less than 0.003 during the frequency from 50Hz to 550Hz.

Table 1. Acoustical boundary condition of the right end of the duct.

State No.	Z_R
1	Rigid plate
2	Open end
3	1 block of foam
4	2 blocks of foam

Figure 2. Measurement and analysis system.

Figure 3. Experimental duct system and the muffler.

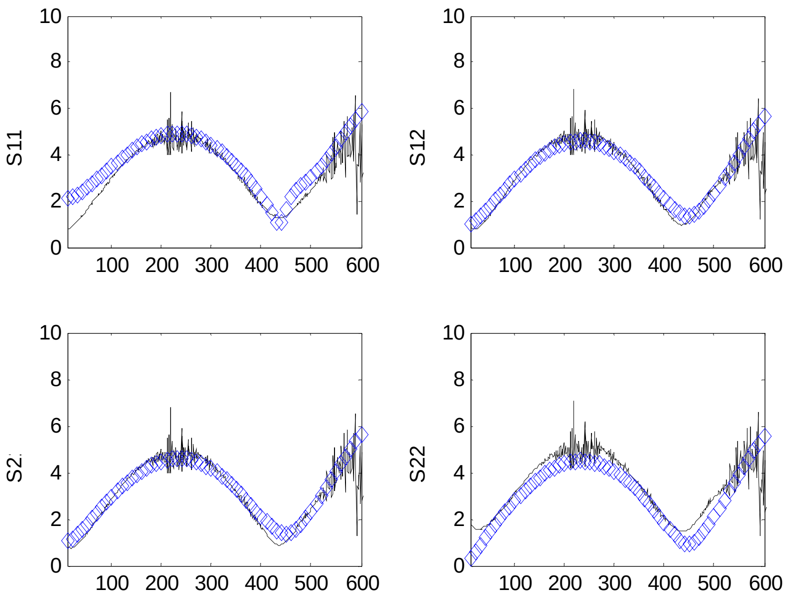

Figure 4. The modulus of the elements in the transfer matrix.

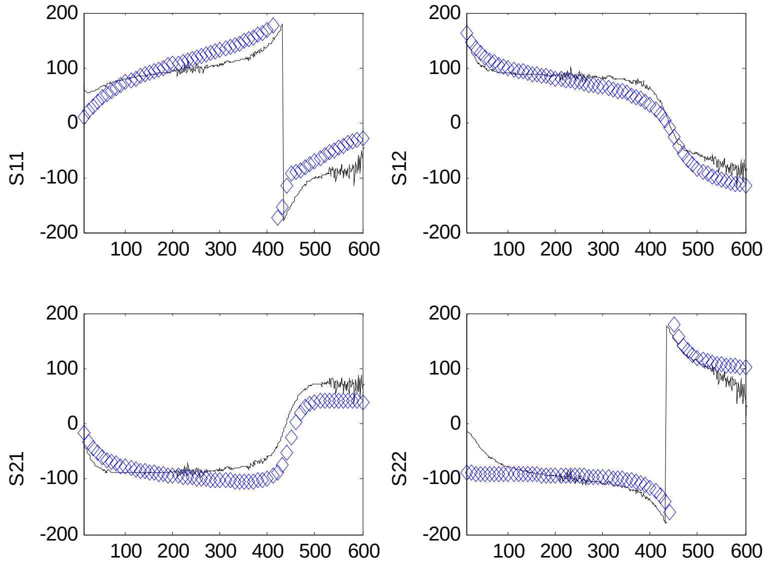

Figure 5. The phase angle of the elements in the transfer matrix.

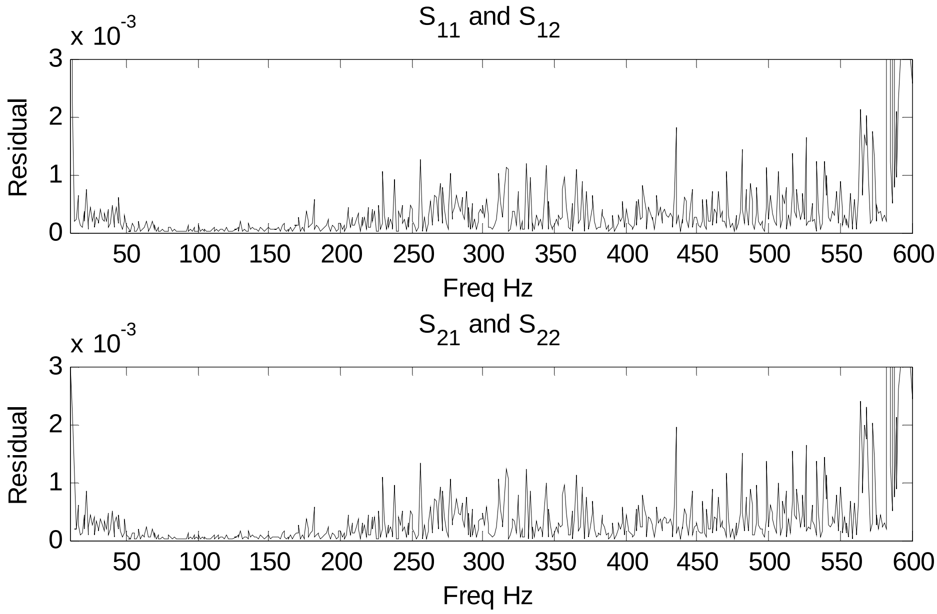

Figure 6. The residual of the identification results.

4. CONCLUSION

In conclusion, after getting the adequate groups of measurable sound field data in different boundary conditions, parameters of the transfer matrix of the muffler can be identified by the LSM, the transfer matrix is the inherent acoustic property of the muffler, independent to the boundary conditions of the experimental system. The result obtained by this identification method is precise enough, and matches the theoretical result very well. So this method has solved the problem and can be used to determine the parameters of muffler's transfer matrix in experimental measurements.

REFERENCES

[1] C. W. S. To and A. G. Doige, "A transient testing technique for the determination of matrix parameters of acoustic systems. I: Theory and principles", *Journal of Sound and Vibration*, **62**, 207-222 (1979).

[2] C. W. S. To and A. G. Doige, "The application of a transient technique to the determination of acoustic properties of unknown systems", *Journal of Sound and Vibration*, **71**, 545-554 (1980).

[3] J. Charley and F. Carta, "Application of the auto-and cross-power spectra to hydro- and aeroacoustics", *Mechanical system and signal processing*, **15**(2), 299-417 (2001).

[4] J. Y. Chung and D. A. Blaser, "Transfer function method of measuring in-duct acoustic properties. I. Theory", *Journal of the Acoustical Society of America*, **68**(3), 907-913 (1980).

[5] J. Y. Chung and D. A. Blaser, "Transfer function method of measuring in-duct acoustic properties. II. Experiment", *Journal of the Acoustical Society of America*, **68**(3), 914-921 (1980).