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## **INFLUENCE OF STIFFNESS AND MASS CHANGES ON EIGEN FREQUENCIES OF BEAM WITH PIEZO ELECTRIC PATCH**

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### **Abstract**

The ability to monitor a structure and detect small damage is of great interest for engineering community. In structural health monitoring, Smart structures with smart elements can continuously monitor their structural integrity use bonded or embedded sensors and actuators. This paper presents a method for damage detection in structure using experimental vibration data that may result from use of such sensors/actuators the structure consists of a cantilever beam and a piezoelectric patch and accelerometer as actuator and sensor respectively. The actuator is used to excite the structure with in given frequency bandwidth and the sensor measures the received signal and influence of damage induced can be analyzed to identify and quantify damage. In this paper damage as different depths of cuts are induced on the beam and here effects are analyzed by observing the changes in power spectral magnitude of vibration response. The variation of power spectrum magnitude of beam due to attached masses at distinct locations also studied.

### **1. INTRODUCTION**

The focus of this report is the detection and characterization of damage in cantilever beam like structures using smart element. Smart materials are used in a growing number of commercial applications such as the aerospace, automotive and machine-tool industries. In particular, smart structures with self actuating and sensing capabilities have potential applications in non destructive evaluation of damage. Such capabilities can be achieved by bonding piezoelectric ceramic patches to the surface of a structure to be characterized. When electric field is applied, piezo ceramic patches induce strains in the materials to which they are bonded and, conversely, they produce a voltage when a deformation occurs in the materials [1, 2]. The premise of the detection method is that damage to a structure will induce changes in the natural frequencies which can be obtained by measurement of the vibration response [3, 4]. Diaz and Soutis [5] presented a first step in developing a health monitoring system the effect of Delamination on the modal frequencies of laminated composite beams has been investigated. A PZT patch driven with a linear rapid frequency sweep is used to induce vibration on the structure and its response registered via PZT film sensors. Reaves and Horta [6] demonstrated a set of benchmark test articles were developed to validate techniques for modeling structures containing piezoelectric actuators using commercially available finite element analysis packages. Improper bounding of actuators greatly reduces the electrical to

mechanical effectiveness of the actuator producing antiresonance. The use of a piezoelectric inertial actuator affixed to a structure as a collocated sensor cum actuator for monitoring structural integrity demonstrated by Ling and Xie [7]. Later Staszewski [8] addressed the importance of intelligent signal processing for damage identification in composite structures using different monitoring techniques. A method by Worden and Manson [9] is considered a level one diagnostic based on the idea of novelty detection. The patterns used for detection were measured transmissibilities centered on a particular peak which proved sensitive to the damage. When damage occurs between two sensors, the stiffness between the sensors changes and this affects the local vibration response at high frequencies shown by Mickens et.al [10]. A project had been done to develop inexpensive non-intrusive active sensors that can be applied on existing aging aerospace structures for monitoring the onset and progress of structural damage (fatigue cracks and corrosion) has been presented Redmond and Giurgiutiu [11]. The numerical modeling of a plate structure containing bonded piezoelectric material and derived the finite element equations using the mechanical energy of structure and electrical energy of the piezoelectric material by Ribeoro et.al [12]. Sumant and Maiti [13] proposed a method to detect size and location of an edge normal crack in a beam like component by fixing discrete PZT patches at its top and bottom edges. Both theoretical and experimental results are presented. This paper presents the ANSYS modeling of the smart beam, which is composed of an aluminium beam with active PZT patch. Results of finite element model and experimental work are compared for variation of modal frequencies for different damage scenarios. Experimentally the damage at different depth of cuts for beam analyzed by observing the changes in power spectral magnitude. The variation of power spectrum magnitude response of beam due to attached masses at different locations also studied.

## 2. FINITE ELEMENT MODELING OF THE MART BEAM

In the modeling and analysis of piezoelectric crystals the typical finite elements used are the solid elements, where as in the analysis of this plate, usually shell elements are utilized. The smart beam a 0.280 x 0.025 x 0.002m aluminium beam modeled in cantilevered configuration with single 0.050 x 0.025 x 0.0009m PZT actuator placed on one surface of the beam closed to the clamped end as shown in Figure 1. The PZT is modeled by compatible solid elements SOLID5 (3-D Coupled-Field Solid) and beam with SOLID45 (3-D Structural Solid).

Table 1. Properties of aluminium beam

Property	value
Young's modulus, $E_{11}$ , N/m <sup>2</sup>	68.9e9
Poisson's ratio	0.33
Density, $\rho$ , Kg/m <sup>2</sup>	2710

Table 2. Properties of Piezoelectric material (PZT) used as actuator.

Property	Value
Young's modulus, $E_{11}$ , N/m <sup>2</sup>	63.9e9
Young's modulus, $E_{22}$ and $E_{33}$ N/m <sup>2</sup>	63.9e9
Poisson's ratio	0.3
Density, Kg/m <sup>2</sup>	7800
Piezoelectric charge constant, $d_{31}$ and $d_{32}$ , m/V or C/N	-179e <sup>-12</sup>
Piezoelectric charge constant, $d_{33}$ , m/V or C/N	354e <sup>-12</sup>

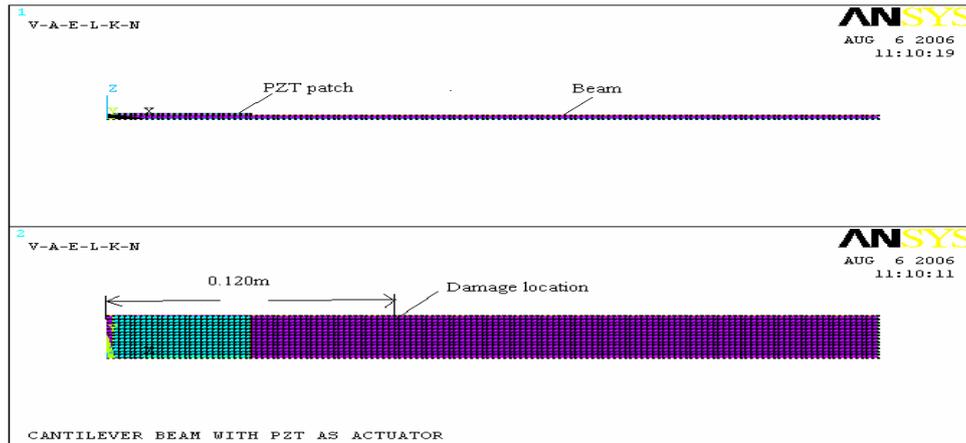


Figure 1. ANSYS model of the smart beam with thin PZT attached

The element SOLID5 has eight nodes with up to six degrees of freedom at each node also has large deflection and stress stiffening capabilities. The element SOLID45 is defined by eight nodes having three degrees of freedom at each node: translations in the nodal x, y, and z directions also the element has plasticity, creep, swelling, stress stiffening, large deflection, and large strain capabilities. The Length of beam is divided into 140 elements and width is 25 elements. So the simulated model has total of 3500 elements. Damage is simulated at a distance of 0.120m from fixed end by removing elements along the width of beam (i.e. for each damage case 5 elements are removed) The material properties of the beam and actuator (PZT) are listed in Table 1 and Table 2. Figure 1 shows the finite element model of the smart beam used for study. Modal analysis is carried out for different damage scenarios and compared the natural frequencies with experiment.

## 2. Experimental set-up

The beam width and thickness were 0.025 m and 0.002 m, respectively, while its length is 0.280 m and its material properties are shown in Table1. The piezoelectric actuators utilized were PZT 5A with dimensions of 0.0506 m length, 0.025 m width and 0.0009 m thickness as shown in Figure 2. The experimental set-up consists of a cantilever aluminium beam with one PZT actuator bonded to the structure as shown in Figure 3. The actuator was bonded on the upper surface at 0.005 m from the fixed end of the beam while the sensor here used is B&K Accelerometer is on the free end. These two locations were chosen such that all of the structure's mode shapes were excited by the actuator and sensed by the sensor.

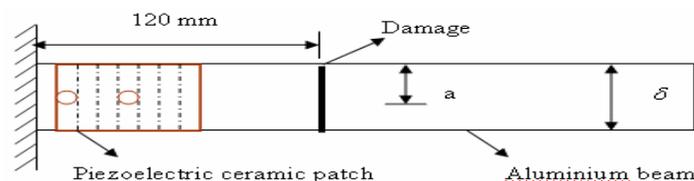


Figure2. Beam with bonded piezoelectric ceramic patch.

As depicted in Figure 2, the damage is located at a distance of 0.120 m from the fixed end produced by cutting the beam with help of hexa blade with different depth along the width of beam. A crack depth ratio of 0 is considered no damage and a ratio  $a/\delta$  of 0.8 is considered as failure of the member. The PZT actuator receives an amplified sweep white noise signal with 10 V at frequency range 5 Hz to 1000 Hz from the function generator and provokes

bending like vibration in the beam. Here a white noise signal is used to drive one PZT actuator because it has distinct advantages in identifying closely spaced natural frequencies.

Table 3. Damage scenario's with different depth of cut.

Damage scenario (CD: Cantilever damaged)	Crack depth ratio ( $a/\delta$ )
UD	0
CD1	0.2
CD2	0.4
CD3	0.6
CD4	0.8

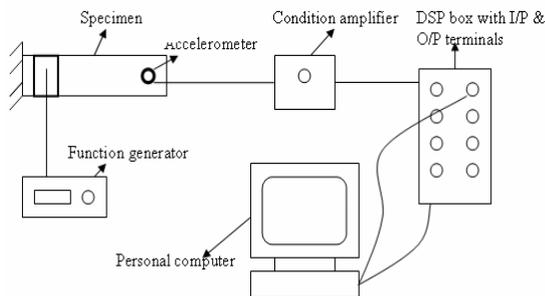


Figure 3. Schematic of the experimental set-up.



Figure 4. DSP box

This is important because different damages can affect different frequency ranges of a structure, and the resonant and anti resonant characteristics of a structure are very good indicator of damage. Other end of beam accelerometer senses the induced vibration in the beam and that signal amplified through condition amplifier and fed into the DSP box. The DSP box as shown in Figure 4 communicates with the PC running MATLAB through an RS232 serial port. It can perform data acquisition and generate signals like sine, random and arbitrary data from a '.mat' file. It can also perform sine dwell or random noise excitation (band limited) to identify the FRF characteristics of the plant, While data collection and interpretation is performed with a computer.

### 3. RESULTS AND DISCUSSIONS:

#### 3.1 Effect of crack size on power spectrum magnitude:

When the specimen is subjected to frequency white noise ranging from 5 Hz to 1 KHz with an amplitude level 5 V. The influence of different damage cases can be seen on the power spectrum magnitude plot as shown in Figure 5a. Where one can notice that as damage size increase shift in the characteristic frequency peaks before and after the damage conditions also increases. Figure 5b the zoomed view in frequency range between 540 Hz to 680 Hz which is the sensitive mode for damage detection. The shift becomes even more significant as the white noise amplitude level is increased from 5 V to 10 V as shown in Figure 6a. The spectral magnitude shows an increase with increased signal level. Figure 6b the zoomed view in frequency range between 540 Hz to 680 Hz which is the sensitive mode for damage detection. Significant increase in amplitude of the power spectrum peaks for undamaged case by variation of excitation voltage from 5 V to 10 V a shown in Figure 7. If excite the system even at different amplitude there is no effect on frequency of system.

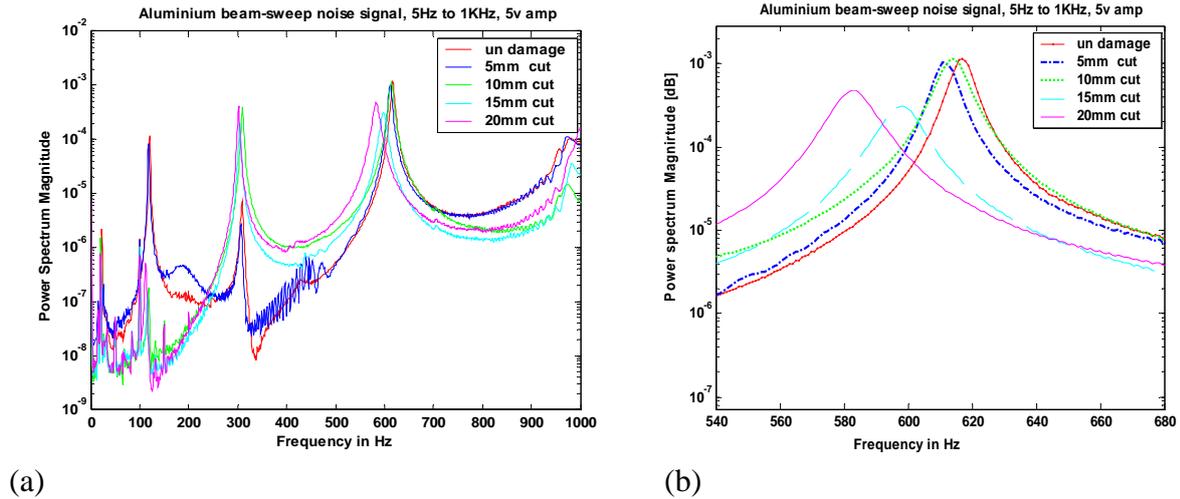


Figure 5. Power spectral magnitude plots. (a) Shift in frequency for all damage cases at 5 V amplitude of signal (b) Zoomed view in frequency range 540 Hz to 680 Hz.

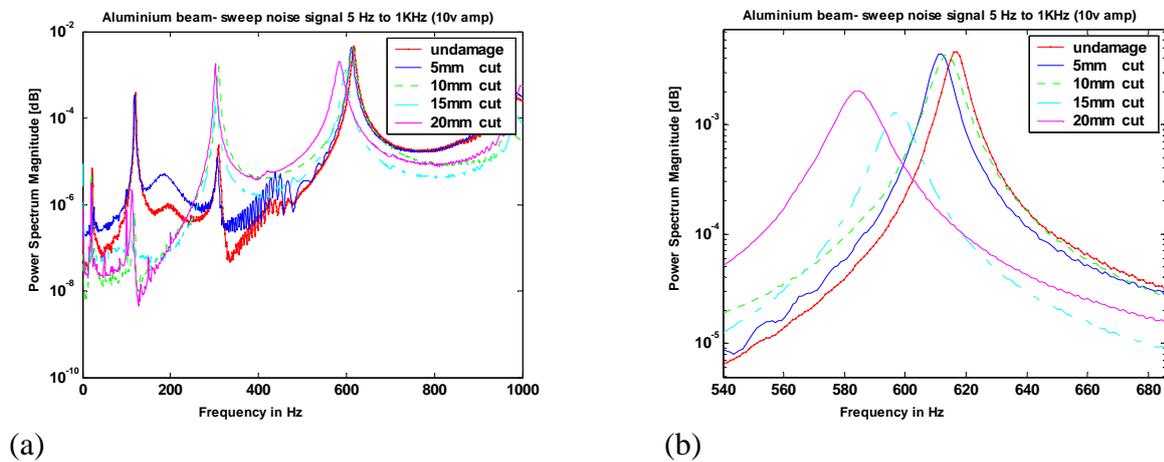


Figure 6. Power spectral magnitude plots. (a) Shift in frequency for all damage cases at 10 V amplitude of signal (b) Zoomed view in frequency range 540 Hz to 680 Hz.

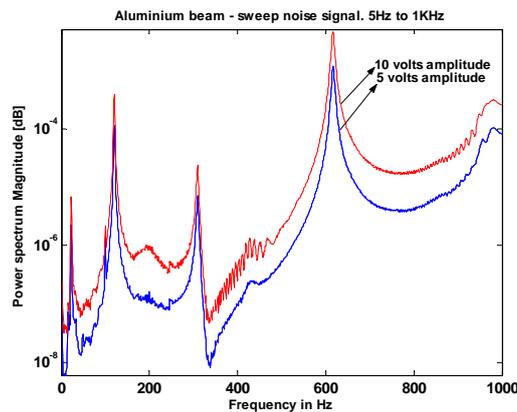


Figure 7. Power spectral magnitude plot for undamaged case with shift in frequency at 5 V and 10 V amplitude of signal

Modal data from both test cases (ANSYS and experiment) compared in terms of initial five frequencies for different damage scenarios as shown in Figure 8. Initial four modes are almost coincide in both the cases but in higher mode i.e. fifth mode the variation of frequencies is more.

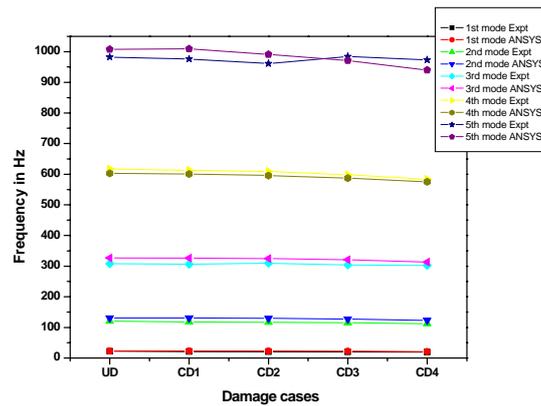


Figure 8: Comparison of initial five modal frequencies from ANSYS and experiment for different damage scenarios.

### 3.2 Quantification of damage:

A qualitative estimation of the structural health can be rapidly achieved through the damage index. The damage index is a scalar quantity that serves as a metric of the damage taking place in the structure. A convenient damage index (DI) is:

$$DI = \sum_N [\text{Re}(Y_{i1}) - \text{Re}(Y_{i0})]^2 \quad (1)$$

Where N - Number of sample points in the spectrum.

$Y_{i1}$ ,  $Y_{i0}$  --- Power spectrum, the scripts 0 and 1 signify the initial and the present state of the structure.

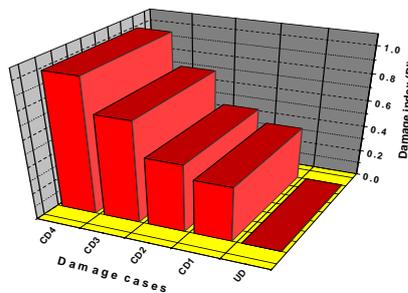


Figure 9. Quantification of different damage scenarios

This damage metric is normalized with respect to a reference damage to which all other damage will compare against. Accordingly, a crack depth ratio of 0 is considered no damage and a ratio  $a/\delta$  of 0.8 is considered failure of member.

The damage quantification bar graph is plotted in Figure 9. It could be observed that when crack depth ratio  $a/\delta$  exceeds 0.2 i.e. (CD1), the damage sharply increase and replacement of the cracked member would be necessary to avoid its failure.

### 3.3. Effect of adding mass on eigen frequencies of the beam at distinct location

Similar experimental has been repeated for the same cantilever beam by positioning the mass on top of the beam at its mid and free end of the span as shown in Figure 10. The standard weights in grams (10 and 20 gms) are used by applying on the beam with a very thin layer of wax.

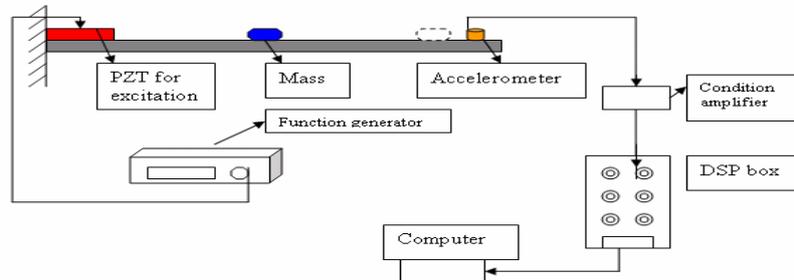


Figure 10. Schematic of the experimental set-up with position of mass at different locations.

The Table 4 shows the eigen frequencies without and with mass

Case	Frequency in Hz			
	1st mode	2 <sup>nd</sup> mode	3 <sup>rd</sup> mode	4 <sup>th</sup> mode
Mass at free end of beam				
With out mass	19	123	321	603.13
10 gms	18	100	256	512
20 gms	11	91.88	239.4	450.6
Mass at mid span of beam				
With out mass	19	123	320	602.2
10 gms	18.57	80.15	247.31	435
20 gms	17	69.404	242.4	406.65

From Table 4 clearly shows that very large shift in frequency when mass is at mid span comparing to mass at free end of beam. But in previous case shift in frequencies is very less when damage simulated by a saw cut. From this effect of variation on eigen frequencies of the structure is more when mass added rather than a crack.

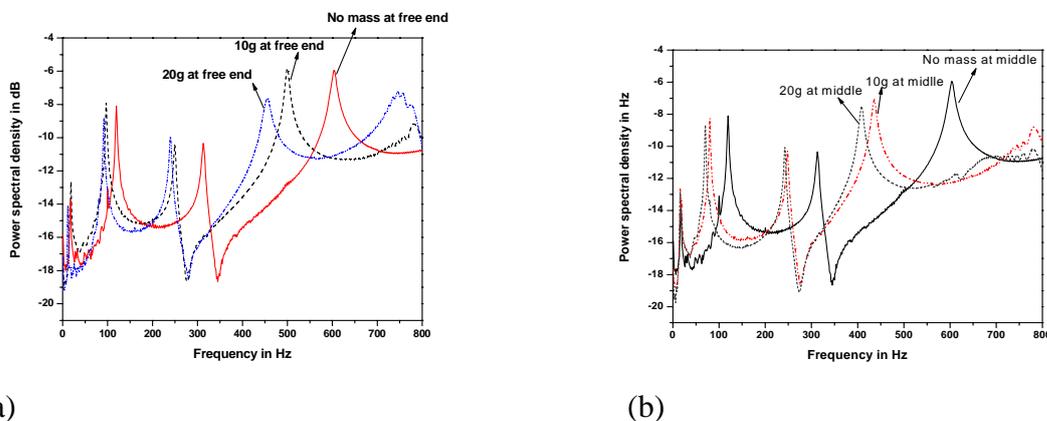


Figure 11. (a) Comparison of power spectrum magnitude with and with out additional mass at free end of the beam. (b) Comparison of power spectrum magnitude with and with out additional mass at mid span of beam.

It can be seen from the experimental results that power spectrum magnitude of the beam varies when the different masses attached at different locations. Since power spectrum magnitude is a characteristic of the structures which is independent of the applied force, observing the outstanding variations near the natural frequencies can clearly indicate the changes of structural properties due to the addition of 10g and 20g masses as shown in Figure 11a and 11b. The structure integrity is changed due to crack and masses the power spectrum magnitude will alter due to the stiffness changes of the structure.

#### 4. Conclusions:

This paper presented the modeling of the smart beam in ANSYS. Comparison of simulated and experimental modal data for different damage scenarios is presented. The experiment results show that if damages are present in the structure, the physical variations of the structures will cause the deviation in mode frequencies. Also presented the behaviour of power spectrum magnitude is more sensitive at higher frequency for damaged structures.

The variation of power spectrum magnitude response of beam due to attached masses at distinct locations. It shown that with increases in the amplitude of excitation signal from 5 V to 10 V there is no effect on the system frequencies but amplitude of spectrum getting changes.

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