



# TWO OBSERVATIONS ON THE WAVE APPROACH TO SEA

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## ABSTRACT

First, it is shown that the use of SEA coupling factors derived for the coupling of semi infinite systems is consistent with coupling power proportionality. This demonstration is axiomatic, relying on a set of postulates. It is useful in teaching SEA, as it illustrates concepts and assumptions commonly made. It might be useful for research aiming for a better set of postulates upon which a statistical energy method can be built. Second, the wave motion in double walls is investigated. A new SEA formulation is presented in which each element describe one kind of coupled cavity-wall wave motion. This formulation obsoletes the non-resonant transmission paths and compared to classical formulations, it improves results at frequencies around and a bit above the double wall resonance.

## **1. INTRODUCTION**

It is this author's belief that Statistical Energy Analysis (SEA) has not reached its full potential, yet. This method for vibroacoustic prediction could be further developed for 'virtual prototyping' of highly developed products. It could be more used in engineering practise where often decisions are based on analysis that need not be too precise. Most important, the results of dynamic calculations and measurements could be better used if the understanding of the potential flow model for vibroacoustic energy transport was more widespread. To quote Ove Bennerhult: "there is nothing more practical than a good theory".

To promote and advance SEA there is an educational need, less expensive software need reach the market and more competent software and modelling procedures are wanted. This note aims for a contribution to the first and third of these objectives. To that end, Section 2 presents an axiomatic demonstration of the SEA power balance equations, which is useful for illustrating concepts and assumptions commonly made in SEA. Section 3 presents a new double wall formulation, where the elements are identified from a wave analysis.

SEA is built upon the Coupling Power Proportionality (CPP) hypothesis stating that the

coupling power between two directly connected elements is proportional to the difference in their modal energy. Most often, the constant of proportionality is equated to its travelling wave estimate (TWE), which is calculated for the coupling of infinite or semi infinite structures. Some fourth year students of the MWL/KTH found this practise to be a gross approximation. To improve their understanding of SEA the demonstration in Section 2 was made.

Double walls are frequently used in buildings, vehicles and aircraft and continue to be of interest for research [1-4]. The classical SEA double wall formulation describes the walls and the cavity as three separate elements and includes a direct, non-resonant, path from a room to the cavity [5-6]. Here, the wave motion of the coupled cavity-wall structure is investigated. In a frequency region between the double wall resonance and a frequency at which the cavity depth is approximately half an acoustic wavelength, there is only one kind of wave in the fluid and two flexural waves in the walls which induces some near field motion in the fluid. The fluid wave includes mass impeded wall motion and is super sonic so it radiates well into a connecting room. Based on these findings, a new double wall SEA element is derived, which, compared to the classical formulation, improves results at frequencies around and a couple of octaves above the double wall resonance.

## 2. AN AXIOMATIC DEMONSTRATION OF SEA

It is well known that SEA cannot be but an approximate method. Woodhouse shows that Lyon and Scharton's two-oscillator result cannot be extended to three oscillators [7]. Langley shows that an exact energy balance model is similar to SEA only if the dependent variables (the modal energies) are re-defined and indirect couplings are allowed [8]. Similarly, Finnveden shows that analytically calculated ensemble averaged energies in a three-element structure are related as in SEA only if indirect coupling is allowed [9]. One strand of SEA research has therefore been focused on approximating exact results for 'weak coupling', e.g., [9-11]. A review of various coupling strength definitions is presented in reference [11].

Another strand derives the equations based on postulates; e.g., in the modal approach to SEA it is sometimes postulated that two-oscillator results apply on a mode-to-mode basis for the coupling of sets of oscillators [12-13]. A similar approach is taken here: based on a set of postulates for the vibroacoustic fields of coupled structures, it is shown that the use of travelling wave estimates is consistent with CPP.

#### 2.1 The 'standard' SEA formulation

Figure 1 illustrates a structure sub divided into three elements, of which Element 1 is excited by an external source that provides a steady state vibroacoustic power  $P_{in,1}$  in a frequency band of width  $\Delta \omega$ . An SEA model of this structure's response is based on the energy conservation principle, which states that the power injected to Element *i* by external sources equals the power dissipated within this element  $P_{d,i}$  plus the net power transmitted to other elements  $P_c^{(i,j)}$ :

$$P_{in,i} = P_{d,i} + \sum_{j \neq i} P_c^{(i,j)}.$$
 (1)

The dissipated power is normally for linear losses described by

$$P_{d,i} = \eta_i \,\omega \, E_i, \tag{2}$$

where  $\omega$  is frequency,  $\eta$  is loss factor and E is the vibroacoustic energy in the considered frequency band. This engineering approximation is quite alright for homogenous structures in which the response is 'reverberant'. An example of a structure for which Equation (2) does not immediately apply is a radial car tyre. The flexural motion of the tyre wall mainly strains the rubber, which is highly damped, while the in plane motion engages the steel wires, which are lowly damped and dependent on the mode shape, losses vary in between  $\eta = .01$  and  $\eta = .25$  [14]. Even so, if the material characteristics are known, for any given wave form, the dissipated power is as in Equation (2) proportional to the wave's energy. The same applies for any given mode shape if, furthermore, the mode has resonance in the considered frequency band and we can assume that the strain and kinetic energies are approximately equal.

The law of energy conservation is exact and Eq. (2) for the dissipated power is an established engineering approximation. The formulation of the coupling power is therefore the large obstacle in any SEA. It is assumed proportional to the difference in energy per mode in directly connected elements. This Coupling Power Proportionality (CPP) is here written as

$$P_{c}^{(i,j)} = C^{(i,j)} (\hat{e}_{i} - \hat{e}_{j}), \qquad (3)$$

where the 'modal power' is given by

$$\hat{e}_i = E_i / n_i, \quad n_i = \Delta N_i / \Delta \omega, \quad (4)$$

*n* is the 'modal density',  $\Delta N$  is the number of modes in the considered frequency band and *C* is the 'modal power conductivity', or in any given context, simply the conductivity. This non dimensional parameter is defined by Eq. (3) and is related to the standard SEA coupling loss factor,  $\eta_{c,ii}$ ,

$$C^{(i,j)} = \eta_{c\,ii}\,\omega n_i\,. \tag{5}$$

Given the definition of the modal power above, the dissipated power is

$$P_d = M \hat{e}, \quad M = \eta \, \omega n \,. \tag{6}$$

The non-dimensional modal overlap factor, M, is a very important system characteristic. One reason for using the conductivity, and not the coupling loss factor, is that the conductivity relates directly to the modal overlap factor. Another reason is that it saves ink.

In possibly all practical applications of SEA, the conductivity is equated to its travelling wave estimate. This estimate is evaluated, as illustrated in Fig 1, for one junction at a time while disregarding the rest of the structure. Moreover, one element is directly excited and the receiving elements are extended towards infinity. Thus, the TWE of the conductivity is given by

$$C_{e}^{(i,j)} = \left( P_{c}^{(i,j)} / \hat{e}_{i} \right)_{C^{(i,j)} / M_{j} \to 0}.$$
(7)

It is also assumed that following power reciprocity relation holds

$$C_{e}^{(i,j)} = C_{e}^{(j,i)}.$$
 (8)

In parallel to the damping loss factor in Eq (2), the TWE can always can always be evaluated for any given vibroacoustic field in the first element. Moreover, it relates easily to measures of vibroacoustic transport such as radiation efficiency, transmission loss and reduction index and can therefore be calculated or measured in a standardized manner; this is a great advantage.

Methods such as the one indicated by Eq (7) are below termed 'one-way' methods and are commonly used in engineering practise, see, e.g., [15-16]. In a one-way procedure, first the response of one element is calculated, possibly accounting for coupling losses, and, then, the responses of connected elements are evaluated considering the given vibroacoustic field in the first element.



Figure 1. Left, illustration of three-element structure; right, evaluation of TWE.

It seems as if the TWE is a reasonable approximation of the conductivity, defined by Eq. (3), if the 'Smith-type' criterion  $C^{(i,j)}/M_j \ll 1$  applies. In what follows, however, it is demonstrated that the TWE is fully consistent with the CPP, if the response of the external excitation is not correlated with the scattered response of the elements. This criterion is more likely to be fulfilled for random structures but the required degree of randomness is somehow related also to the coupling strength, as will be discussed.

## 2.2 Demonstration of CPP for Two-element structures

Consider the two-element structure illustrated in Fig 2. The first element is excited by an external forcing  $F_1$  which provides a steady state input power  $P_{in,1}$  in the considered frequency band. The structures response is given by linear equations of motion and is thus

equally given by the superposition of the responses of two structures as seen in Fig 2.

It might be difficult to find the 'scattering' force field  $F_2$ , giving rise to the boundary reflections in element 2. In some instances it is, however, possible to make a statement of the scattered power. To this end, the following postulates are made:

1) The response is reverberant, meaning that the kinetic and potential energies are on average equal. Also, the vibroacoustic energy is roughly equal across the element, which requires that the element is quite homogenous and that  $\eta \omega L/c_g < 1$ , where L is a typical dimension and  $c_g$  is the group velocity.

2) The coupling is conservative and the amount of power dissipated in the near fields of the excitations, boundaries, and other irregularities is not significant. Also, the direct fields of the excitation and the coupling do not dissipate any significant amount of power.

3) The power  $P_{in,1}$  injected by the external forcing  $F_1$  is not altered when the indirectly excited element is extended towards infinity.

4) The scattered force field,  $F_2$ , is uncorrelated to the external forces  $F_1$ .



Figure 2. Top, original problem (0); bottom, superimposed problems (1) and (1b)

Postulate 1 defines the kind of response that SEA might be able to describe. Postulate 2 is standard in SEA and means that all dissipation can be attributed to the reverberant fields in the elements. Postulate 3 is one instance of a weak coupling assumption: in reference [8] Langley says that coupling is weak if the real part of the point mobility (Green's function) is not altered if an element is connected to another element. This criterion is fulfilled for spring coupled structures that have random properties, if the 'modal interaction strength'

$$\gamma = \frac{2 C_e^{(i,j)}}{\pi M_i M_j},\tag{9}$$

is less than unity [11]. Finally, Postulate 4 is necessary for the analysis that follows. It might be relevant if the average response of structures with random, or unknown, properties is sought. It is immediately clear that Postulate 4 cannot be correct if the structure's response is given by global modes that are well separated in frequency, since response and scattering defined by one mode must be correlated. Thus Postulate 4 requires not only structural randomness but also 'local' response and thus weak coupling [11].

Applying the postulates and the law of the energy conservation for element 1 yields

$$P_{in,1} = \left(M_1 + C_e^{(i,j)}\right)\hat{e}_1^{(1)}, \qquad (10)$$

where  $\hat{e}_i^{(j)}$  is the modal power in element *i* given by the vibroacoustic problem *j*. Similarly, the energy conservation for element 2 yields

$$\hat{e}_{2}^{(1)} = 0, \quad P_{in,2}^{(1b)} = C_{e}^{(i,j)} \hat{e}_{1}^{(1)}, \quad (11)$$

since the energy transferred to element 2 in problem (1) is finite while the modal density is infinite. Accordingly, the power injected by the scattering forces  $F_2$  is given by energy conservation and the postulates.



Figure 3. Power superposition, step 1. Top original problem (0); bottom, problem (1) and (1b).

It is, not yet, possible to solve the vibroacoustic problem (1b). So, to proceed, as in Fig. 4, Postulates 3 and 4 need be amended and a fifth postulate is needed:

3b. The power injected by the boundary force field  $F_2$  is not altered when element 1 is extended towards infinity.

4b. The boundary force field  $F_1^{(2)}$ , needed to make step 2 a valid superposition, is uncorrelated both to the external force field  $F_1$  and the boundary forces  $F_2$ .

5. The power reciprocity (8) holds.

Applying these postulates, energy conservation yields

$$P_{in,2}^{(2)} = \left(M_2 + C_e^{(i,j)}\right)\hat{e}_2^{(2)},\tag{12}$$

$$P_{in,1}^{(2b)} = C_e^{(i,j)} \hat{e}_2^{(2)}.$$
(13)



Figure 4. Power superposition. Top problem (1b); bottom, (2) and (2b).

The vibroacoustic problem (2b) is similar to the original problem (0) except for that the power is injected by another set of sources ( $F_1^{(2)}$  in stead of  $F_1$ ) that provides a smaller amount of input power.

It is conceivable that the force fields  $F_1$ ,  $F_2$  and  $F_1^{(2)}$  may be uncorrelated if the structure has random properties and the elements' responses are given by diffuse wave fields or local modes. If, additionally, the power injected by the scattered force field  $F_1^{(2)}$  is much smaller than the one injected by the external force  $F_1$ , the equations above provide the required solution. If this is not the case, the vibroacoustic problem (2b) needs to be solved and one option is then to extend the scheme of successive approximations, outlined above, *ad infinitum*. A sixth postulate is then needed

#### 6. The postulates 3-5 are valid also for further successive approximations.

This postulate is rather suspicious. If, nevertheless, it is accepted, the following scheme applies

$$\hat{\mathbf{e}} = \hat{\mathbf{e}}^{(1)} + \hat{\mathbf{e}}^{(3)} + \hat{\mathbf{e}}^{(3)} + \dots; \quad \hat{\mathbf{e}}^{(n)} = \begin{bmatrix} \hat{e}_1^{(n)} & \hat{e}_2^{(n)} \end{bmatrix}^{\mathrm{T}}$$
 (14)

$$\hat{\mathbf{e}}^{(n)} = \mathbf{K} \, \hat{\mathbf{e}}^{(n-1)} \tag{15}$$

$$\mathbf{K}_{ii} = 0; \ \mathbf{K}_{ij} = C_e^{(i,j)} / D_i; \ D_i = M_i + C_e^{(i,j)}$$
(16)

$$\hat{\mathbf{e}} = \left[\mathbf{I} + \mathbf{K} + \mathbf{K}^2 + \dots\right] \hat{\mathbf{e}}^{(1)} = \left[\mathbf{I} - \mathbf{K}\right]^{-1} \hat{\mathbf{e}}^{(1)}$$
(17)

where **I** is an identity matrix and the last equality is valid since the magnitudes of **K**'s eigen values are both less than unity. Finally, Eq. (17) is first pre multiplied by the matrix  $[\mathbf{I} - \mathbf{K}]$  and then by the diagonal matrix **D** with entries,  $\mathbf{D}_{ii} = D_i$ , and it follows that

$$\mathbf{D}[\mathbf{I} - \mathbf{K}]\hat{\mathbf{e}} = \mathbf{D}\,\hat{\mathbf{e}}^{(1)},\tag{18}$$

which is

$$\begin{bmatrix} M_1 + C_e^{(1,2)} & -C_e^{(1,2)} \\ -C_e^{(1,2)} & M_2 + C_e^{(1,2)} \end{bmatrix} \begin{bmatrix} \hat{e}_1 \\ \hat{e}_2 \end{bmatrix} = \begin{bmatrix} P_{in,1} \\ 0 \end{bmatrix}.$$
 (19)

This is the standard SEA equation for a two-element structure and it is thus demonstrated that the application of the travelling wave estimate (7) is fully consistent with the coupling power proportionality (3), if the postulates 1-6 are fulfilled.

#### 2.3 Multi-element structures

The successive approximation scheme described above applies also for multi-element structures; a three element structure is illustrated in Fig 5.



Figure 5. Power superposition for three elements

The superposition scheme indicated in the figure is applied for a structure with m elements, upon which the successive modal powers are given by

$$\hat{\mathbf{e}}^{(n)} = \mathbf{K} \, \hat{\mathbf{e}}^{(n-1)}; \quad \hat{\mathbf{e}}^{(n)} = \begin{bmatrix} \hat{e}_1^{(n)} & \hat{e}_2^{(n)} & \dots & \hat{e}_m^{(n)} \end{bmatrix}^{\mathrm{T}}, \tag{20}$$

$$\mathbf{K}_{ii} = 0; \quad \mathbf{K}_{ij} = C_e^{(i,j)} / D_i; \quad D_i = M_i + \sum_{j \neq i} C_e^{(i,j)}$$
(21)

where it is understood that  $C_e^{(i,j)}$  is zero unless elements *i* and *j* are directly connected. By definition:  $\sum_{j \neq i} |\mathbf{K}_{ij}| < 1$ , and it follows from Gerschgorin's theorem [17, p 209] that the magnitudes of **K**'s eigenvalues are all less than unity. The Eqs (17) and (18) therefore apply equally for many-element structures. In particular, for a three element structure, we have

$$\begin{bmatrix} M_1 + C_e^{(1,2)} + C_e^{(1,3)} & -C_e^{(1,2)} & -C_e^{(1,3)} \\ -C_e^{(1,2)} & M_2 + C_e^{(1,2)} + C_e^{(2,3)} & -C_e^{(2,3)} \\ -C_e^{(1,3)} & -C_e^{(2,3)} & M_3 + C_e^{(1,3)} + C_e^{(2,3)} \end{bmatrix} \begin{bmatrix} \hat{e}_1 \\ \hat{e}_2 \\ \hat{e}_3 \end{bmatrix} = \begin{bmatrix} P_{in,1} \\ 0 \\ 0 \end{bmatrix}$$
(22)

#### 2.4 Discussion

The postulate 6 cannot be valid *ad infinitum*, as was assumed above. It isn't conceivable that the successive scattering force fields are incoherent for ever. Therefore, CPP is valid only if the successive approximation scheme converges rather quickly. One might conclude from the analysis above that the convergence is quick if the Smith criterion  $C^{(i,j)}/M_j \ll 1$  is fulfilled for all connections and all elements. This conclusion, however, is not certain, since we cannot infer all characteristics of an ensemble from the characteristics of the ensemble average.

In a dynamic analysis of spring coupled structures with random properties, it was found that, for all members of the ensemble defined and for all frequencies of vibration, a similar successive approximation scheme converges if the modal interaction strength,  $\gamma$ , of Eq. (9) is less than unity [11]. Also, Finnveden presented exact ensemble averaged coupling powers and modal powers in one-dimensional systems with random properties [18]. If the modal interaction strength,  $\gamma$ , is small, the CPP (3), using the one-way conductivity, applies for these ensemble averages and, furthermore, the indirect couplings in three-element structures are negligible. These findings were later repeated in [9, 19] showing that the Smith criterion [20] is not the decisive criterion for the usefulness of standard SEA.

## **3. THE SEA OF A DOUBLE WALL**

One of the most drastic examples of an SEA based on travelling wave estimates is Price and Crocker's double wall formulation [5]. This model relies on Smith's wonderful result [21] that the radiation of sound from a mode in a structure to an infinite acoustic space is related to the mode's reception of vibroacoustic energy from a diffuse sound field by the modal power reciprocity relation (8). The model also relies on Price and Crocker's radical assumption that the radiation into a cavity of depth d, which is of the order of the acoustic wave length, or even smaller, is described equally as the radiation into semi-infinite space.

The double wall formulation considers a partition of size  $S = L_x \times L_y$  consisting of two plates separated by a distance  $L_z = d$ . For both plates, the coincidence frequencies, at which the flexural wavelength of plate *i* equals the acoustic wavelength, occurs at a rather high frequency,  $f_{c,i}$ . In a lower frequency regime, the plates' impedances to acoustic forcing are therefore predominantly of mass character. The double wall resonance,  $f_{dw}$ , is mainly determined by the wall's masses and the compressional air stiffness of the cavity, all per unit area; it is approximately given by

$$f_{dw} = \frac{c}{2\pi d} \sqrt{\frac{1}{\mu_1} + \frac{1}{\mu_2}}$$
(23)

where c is the sound speed and  $\mu_i = \rho_i t_i / \rho_0 d$  and  $\rho_i$  and  $t_i$  are the density and wall

thickness of plate *i* and  $\rho_0$  is the air density. At frequencies lower than  $f_{dw}$ , the plates are assumed to move in phase and the transmission loss is approximately given by the mass law employing the total mass of the partition. An improved low frequency SEA model would also account for resonant motion in the frame-plate structure but this will not be pursued here.

At frequencies above  $f_{dw}$ , Price and Crocker define five SEA elements, each describing the response in one of the sub structures: 1) sending room; 2) plate 1; 3) cavity; 4) plate 2; 5) receiving room. The sound transmission from a room, through a plate, to the cavity, or vice verse, is given by the resonant transmission, where the sound field excites the plate resonances which in turn radiate into the cavity. It is also given by the mass law, defined by the response of the non-resonant, mass-impeded, modes of the plate. This latter transmission path does not exist above the coincidence frequency.

Craik and Smith [6] present measurements and calculation of the sound transmission through double wall partitions. The measurement data of Fig 15 in reference [6] are used here for illustration and for validation of a new model. The walls studied are made of plasterboard, and in one case of chipboard, which are joined by wooden frames; material and geometrical data are given in [6, Table 2]. Craik and Smith base their SEA model on the Price and Crocker formulation and include the plate-plate coupling via the frames. These frames are 3 m long and are modelled as Euler beams. They are erected in the vertical direction at 40 cm cc distance and the plates are nailed to the frames at 30 cm distance. The frame-plate transmission is modelled as point coupling.

Figures 6 shows measured and calculated sound reduction indexes for double walls with a cavity depth of 100 mm; also 50 mm and 150 mm cavities are considered. The double wall resonances occur at 123 Hz, 87 Hz and 66 Hz, respectively, while the cavity depth equals half an acoustic wave length at 3.4 kHz, 1.7 kHz and 1.1 kHz. The coincidence frequency is in the 3.15 kHz third octave band. As can be seen, the Price and Crocker model, as interpreted by Craik and Smith, makes a god job at lower frequencies, where the partition is modelled as one



Figure 6. Sound Reduction Index, 100 mm double wall. Solid blue line, measured [6]; Dotted black line, old model [6]; Dashed red line, new model.

single plate; particularly so as the mode count is rather low at these frequencies. The model is also very god at higher frequencies when the cavity depth is not too small compared to the acoustic wave length. At intermediate frequencies, there are consistent errors, which increase with decreasing frequency. (These errors were reduced in reference [1].)

The double wall resonance is not seen in the measured sound reduction index (SRI) while there is a plateau, extending approximately an octave above this frequency. The dip in the 160 Hz band, seen for all cavity depths, might be caused by an increase in the plate mobility as in this band the distance between the frames is a bit more than half a flexural wave length in the plate. After the plateau, the reduction index starts increasing by 9 dB per octave. Similar characteristics have been observed by the current author for other building structures as well as trimmed vehicle and aircraft structures. In the next sub section, the wave motion in a double wall structures is investigated in more detail to gain understanding of the vibroacoustic motion of double walls.

#### 3.1 Wave motion in double walls

The waveguide Finite Element (FE) method is a versatile tool for the investigation of wave motion in structures that have constant material and geometrical properties along one direction [22-26]. Using this method, the motion's dependence of the cross sectional coordinates is approximated with FE polynomial shape functions, upon which follows a set of



Figure 7. Waveguide FE mesh for 100 mm double wall

coupled ordinary differential equations describing wave propagation and decay along the structure. Figure 7 shows the FE mesh used for the study of the wave propagation in the 100 mm double wall. The plaster boards are described by three-node quadratic deep shell elements and the fluid by nine-node Lagrange type elements. All motion in the y-direction is blocked so the model is, in effect, two dimensional.

Figure 8 shows the dispersion curves for the double wall. The two straight curves at the bottom are the quasi longitudinal waves in the two plates. These have slightly different wave numbers as the plate have different material properties. (This is an example of good design; it is also an example of professor Craik's good understanding of SEA.) The two straight lines at the top are the flexural waves of the plate. Perhaps surprisingly, the flexural waves' dispersion relations are close to those for the uncoupled plates, also at frequencies below the double wall resonance. This is so because the fluid mass loading is low compared to the plates' mass and because the fluid cannot transfer any shear stress. The wave forms, however, exhibit coupling



Figure 8. Dispersion curves for 100 mm double wall. Blue dots, wave guide FE solutions; dashed red line, free acoustic wave; solid black line, uncoupled structural waves, top flexural and bottom longitudinal waves.

between the plates and the plates and the fluid, as can be seen in Figs 9. At low frequencies, one of the flexural waves is anti symmetric and the other is symmetric while the plate amplitudes are roughly equal. Above the double wall resonance, the waves are localised to either of the plates while the amplitude of the other plate decreases rapidly with frequency.

The waves that 'cut-on' (start propagating) at 87 Hz and 1.7 kHz are predominantly fluid waves that as frequency increases approach the dispersion curve for free acoustic waves. The wave form for the first of the predominantly fluid waves is shown in Fig 10. For all frequencies, the fluid motion is almost plane with some motion of the walls, which decreases with frequency. Most important for the formulation of a new SEA element: the fluid wave does not exist below the double wall resonance and it is super sonic, so the associated wall motion radiates well.



Figure 9. Wave forms for flexural waves. Top left, f = 2 Hz, k = 1.7 1/m; top right, f = 2.9 Hz, k = 1.7 1/m; bottom left f = 116 Hz, k = 11.2 1/m; bottom right, f = 136 Hz, k = 11.2 1/m;



Figure 10. Wave form of cavity-wall wave. Left, 87 Hz; middle 126 Hz; right 840 Hz

## 3.2 A new double-wall element

#### 3.2.1 Modal density

The waveguide FEM describes most accurately and conveniently the double wall wave motion. It is, however, possible to describe the waves analytically in a lower frequency domain where the wall motion is mass impeded. These analytic expressions are detailed here for reference. The structure is isotropic so the wave motion at an arbitrary wave heading in the x-y plane of the wall are described by the following equations

$$\frac{\partial^2 \tilde{p}}{\partial x^2} + \frac{\partial^2 \tilde{p}}{\partial z^2} + k_a^2 \tilde{p} = 0; \quad \tilde{p}(x, z_j) = (\pm) i \,\omega \,\rho_0 \,d\,\mu_j \,\tilde{u}(x, z_j) \tag{24}$$

where  $z_1 = 0$ ;  $z_2 = d$ ,  $k_a = \omega/c$ ,  $\tilde{u}$  is the rms particle velocity and the minus sign applies at  $z_2$ . The wave solutions to Eqs (24) are given by

$$\tilde{p} = \tilde{p}_0 \left( \sin \gamma_r z + \mu_2 \gamma_r d \sin \gamma_r z \right) e^{i\kappa_r x}, \qquad (25)$$

$$\kappa_r(\omega) = \operatorname{Re}\left(\sqrt{k_a^2 - \gamma_r^2}\right).$$
(26)

The  $\gamma_r$  are solutions to a transcendental eigenvalue problem, defined by Eqs (24), which for the double walls considered here have solutions that are well approximated by

$$\gamma_0 = 2\pi f_{dw}/c; \ \gamma_r = r\pi/d, \ r \ge 1,$$
(27)

so, in this approximation  $\gamma_r$  does not depend on frequency. For each of these eigenvalues, there is a two-dimensional motion in the x-y plane. The number of modes below a frequency  $\omega$  is given by

$$N(\omega) = \kappa_r^2 S / 4\pi.$$
<sup>(28)</sup>

It follows, the average modal density for the plane cavity-wall modes, in a band of width  $\Delta \omega = \omega_u - \omega_l$ , is given by

$$n_{cw} = \left(\kappa_0^2(\omega_u) - \kappa_0^2(\omega_l)S\right) / 4\pi \Delta \omega, \qquad (29)$$

while the total modal density for the oblique cavity modes is

$$n_{co} = \sum_{r=1} \left( \kappa_r^2(\omega_u) - \kappa_r^2(\omega_l) S \right) / 4\pi \Delta \omega \,. \tag{30}$$

#### 3.2.2 Conductivity

The power radiated from the resonant vibrations of one of the walls to a semi-infinite room is

$$P_{rad} = \rho_o c \, S \, \sigma^{(p,r)} \left\langle \tilde{v}^2 \right\rangle = \rho_o c \, n_p \, \hat{e}_p \, \sigma^{(p,r)} / m_p \tag{31}$$

where  $n_p$ ,  $\hat{e}_p$  and  $m_p$  are the modal density, modal power and the mass per u.a. of the wall and  $\sigma^{(p,r)}$  is the radiation efficiency from a plate to an acoustic volume. The corresponding conductivity then follows from Eq. (7). The same formulation is used for the radiation from the resonant plate vibration into the cavity, though, the radiated power is distributed to the oblique cavity modes and the plane cavity-wall modes according to their modal density, since the radiation efficiencies for these different mode groups have not been evaluated. Thus, the conductivities are given by

$$C^{(p,r)} = \rho_o c n_p \sigma^{(p,r)} / m_p$$

$$C^{(p,co)} = \rho_o c n_p \sigma^{(p,r)} n_{co} / (m_p n_{ct}); \quad C^{(p,cw)} = \rho_o c n_p \sigma^{(p,r)} n_{cw} / (m_p n_{ct}),$$
(32)

where  $n_{ct} = n_{cw} + n_{co}$ . To evaluate the conductivity for the energy flow from the cavity-wall modes to a room, the wall's mean square vibration velocity need be related to the modal power and the radiation efficiency for these modes need be evaluated.

In the simplified model considered, all strain energy is in the fluid and for sinusoidal mode shapes in the x-y plane we have

$$\hat{e}_{cw} = \frac{S |p_0|^2}{4 n_{cw} \rho_0 c^2} \int_0^d (\sin \gamma_r z + \mu_2 \gamma_r d \sin \gamma_r z)^2 dz, \qquad (33)$$

if the modal density is non-zero and  $\hat{e}_{cw} = 0$ , otherwise. The ms vibration velocity of wall *j* is similarly from Eqs. (24) and (25)

$$\left\langle \tilde{v}_{j}^{2} \right\rangle = \frac{\left| p_{0} \right|^{2} \left( \sin \gamma_{r} z_{j} + \mu_{2} \gamma_{r} d \sin \gamma_{r} z_{j} \right)^{2}}{4 \left( \omega \rho_{0} d \mu_{j} \right)^{2}}, \qquad (34)$$

and is thus linearly related to the modal power (33).

In reference [1], Craik improves the model for non-resonant transmission in double walls upon assuming that the plane cavity modes are sonic so that the forced wall vibration will radiate as if the considered frequency was the critical frequency. The plane cavity-wall modes considered in this work are, as seen in Fig x, super sonic. The radiation efficiency is therefore [27, p 503]

$$\sigma_r^{(cw,r)} = k_a / \sqrt{k_a^2 - \kappa_r^2} = k_a / \gamma_r \,. \tag{35}$$

though, it should not exceed the maximum value for sonic modes in a finite plate [27, p533]:  $\sigma_{\max}^{(cw,r)} = 0.45\sqrt{U f/c}$ , where U is the plate perimeter. Additionally, around the coincidence frequency and above this frequency, the cavity pressure does not excite mass impeded wall motion but resonant modes only. A dodge to account for this effect is to multiply the radiation

efficiency with the factor  $(1 - \sigma^{(p,r)})$  or zero, whichever is larger. Thus the radiation efficiency for the wall motion of the cavity-wall modes is here given by

$$\sigma_r^{(cw,r)} = \min\left(k_a/\gamma_r, \sigma_{\max}^{(cw,r)}\right) * \max\left(1 - \sigma^{(p,r)}, 0\right).$$
(36)

Upon this basis, the conductivity for the coupling of cavity-wall modes and a room, and similarly for the oblique cavity waves and the room, is given by

$$C^{(cw,r)} = \rho_0 c S q_0 \sigma_0^{(cw,r)}; \quad C^{(c0,r)} = \rho_0 c S q_r \sigma_r^{(cw,r)}$$
(37)

where  $q_r = \langle \tilde{v}_j^2 \rangle / \hat{e}_{cw}$  is given by Eqs (33) and (34).

## 3.3 Comparison with measurements

The new double wall formulation is illustrated in Fig 11. It is simular to Craik and Smith's model [6] except for that the cavity element is subdivided into two elements and these describe not only the fluid motion but also the mass impeded motion of the walls. The first element models the almost plane motion of the cavity-wall and the second element describe the oblique cavity modes. Their modal densities are given by Eq. (29) and (30). The conductivities, for the coupling to the resonant vibrations of the walls and to the rooms, are described by the conductivities in Eqs (32) and (37). The other parts of the new model is described similarly to reference [6], as far as it has been possible.

A particularly difficult problem in both dynamic and energy based modelling of vibroacoustic response of real life structures is the estimation of damping. Here, the frequency independent loss factors reported in [6] for the plates and frames of  $\eta = 0.01$  and  $\eta = 0.015$  are adopted. The equivalent absorption areas of the rooms are chosen so large that their precise values do not influence the calculated SRI. The reverberation time for the 100 mm cavity, with no extra damping, is reported in Fig 7 of reference [6]. The cavity damping



Figure11. SEA model of double wall.

loss factors are derived, equally for the plane cavity-wall modes and the oblique cavity modes, from these reverberation times. The damping loss factors for the 50 mm and the 150 mm cavities are assumed to be given by the one for the 100 mm cavity in inverse proportion to the cavity depth, since, it is believed, most of the damping arises, because of viscosity and heat conduction, at the walls and frames and, because of air pumping, at the interface of these structural components.

The radiation efficiency,  $\sigma^{(p,r)}$ , for the radiation of a thin-walled, simply supported and baffled, plate to a semi infinite acoustic volume is calculated using the equations given by Leppington, Broadbent and Heron [28]. The multipliers suggested in Fig. 5 of reference [6] are applied, with the point coupling condition assumed.

The calculated SRIs for the three cavity depths are shown in Figs 6 and 12 together with the calculated and measured values of Craik and Smith. The new model underestimates the SRI at the coincidence frequency, in the 3.15 kHz band, by some 4-8 dB, which is unexplained, as the new and old models should be similar at such high frequencies. The agreement in the low frequency region, below, around and an octave above the double wall resonance, is good but somewhat erratic, probably so as the mode count is rather low for most of the elements. Additionally there is a consistent overestimation in the 160 Hz band. Other than this, the agreement between calculations and measurements is excellent for the 50 mm and the 100 mm cavities and good for the 150 mm cavity. Given the expected uncertainty of an SEA model in general, and the damping estimates in particular, the results are rather too good.



Figure 12. Sound Reduction Index. Left, 50 mm cavity; Right 150 mm cavity. Solid blue line, measured [6]; Dotted black line, old model [6]; Dashed red line, new model.

## 6. CONCLUSIONS

The title of this note promised two observations on SEA. These observations are not originally presented here but are substantiated. They are: 1) The use of travelling wave estimates for evaluating the coupling parameters is consistent with the Coupling Power Proportionality hypothesis, if the postulates in Section 2 are valid. 2) The elements of an SEA are not substructures but elements of vibroacoustic response.

The demonstration in Section 2 shows that SEA can be applied successfully whenever a

one-way calculation of energy transport is considered useful. Examples of such one-way calculations uses, e.g., radiation efficiency, reduction index or insertion loss to quantify the energy transport from one structure to another. The SEA formulation has the advantage that it is symmetric so that one model can be built and then used for predictions of response at different locations from different excitations. Additionally, the analysis in Section 2 indicates that the SEA provides better results, than the one-way approaches, whenever receiving elements are not fully absorbent so that some re-radiation occurs.

The current author has found the demonstration in Section 2 useful for teaching SEA to students that have some knowledge of Engineering Noise Control and the use of one-way energy methods. The students understand that SEA is, at least, as good as the methods they already know and has, at least, the same range of validity. When they also learn that SEA software include many of formulas and procedures, which they have painstakingly learnt and quickly forgot they become interested. Moreover, the axiomatic demonstration illustrates the postulates or assumptions that 'standard' SEA is built upon and this motivates further studies of 'reverberant motion', 'weak coupling' and 'random structures'.

Most good theories are axiomatic, e.g., Newton's first law, the Euler-Bernoulli beam theory and the continuum hypothesis. The beam theory is an example of a theory for which the correct set of assumptions was for a long time elusive. (Stephen Timoshenko enjoyably describes the evolution of beam theories in "History of strength of materials" [29].) SEA cannot be but an approximate method and we are still seeking for an effective set of assumptions upon which to build the method. Section 2 tentatively presents a list of such assumptions. It is believed, better sets of postulates are found upon a better understanding of: the proper identification of the elements of response, the weak coupling concept and the response of random structures. To that end recent works by Langley, Brown, Cotoni and Shorter, e.g. [30-32], Guayder and Totaro [33], Le Bot [34] and Finnveden [11] are interesting.

The second observation is illustrated for a double-wall structure. An analysis of the wave motion shows that there are no modes in the cavity at frequencies below the double-wall resonance. At lower frequencies there are, however, near fields that decay away from one wall towards the other. These near fields couple the walls' motion and an improved low frequency model, of particular interest for shallow cavities, would describe this coupling but this was not attempted here.

The plane acoustic waves that cut-on at the double-wall resonance involve quite large, mass impeded, motion of the walls. The modes that correspond to these waves define one SEA element for the cavity; the other element is defined by the oblique cavity waves that cut-on when the cavity depth approximately equals half an acoustic wave length. This element formulation obsoletes the non-resonant transmission paths used in earlier double-wall formulations [1, 5-6]. Instead there is a direct coupling between the acoustic volumes and the resonant cavity-wall modes. As already noted by Craik [1] the plane cavity-wall waves are super sonic and have radiation efficiency greater than unity, which was the value used in earlier works for the non-resonant transmission [5-6]. The radiation efficiency, however, is close to unity at the double-wall resonance and then increases towards the value assumed by Craik which applies for sonic motion. The new element attributes the frequency dependent relation

between modal power and wall motion. This new formulation explains why the reduction index does not have a distinct minimum at the double-wall resonance but instead have a rather constant value in a frequency region, which extends roughly an octave above this resonance.

The new double-wall formulation provides results that agree with Craik and Smith's measurements [6]. In fact, the agreement is for the 50 mm and 100 mm double-walls much better than warranted by the uncertainty in the damping loss estimates; instead the results for the 150 mm wall seems more representative for the precision that one could expect. The new formulation has, compared to earlier formulations, the additional advantage that it applies through the double-wall resonance so there is no need to shift between a 'low frequency' and a 'high frequency' model.

Finally the double-wall example emphasises that the SEA elements are elements of response that need not be localised to substructures of the whole structure. The successful application of SEA to a new kind of structure, therefore, requires diagnostic measurements and diagnostic calculations so that the proper elements can be identified. The same observation is probably valid for the newly developed hybrid method that models some substructures with the FEM and others by SEA [35].

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#### REFERENCES

- 1. R.J.M. Craik. Non-resonant sound transmission through double walls using statistical energy analysis. *Applied Acoustics* **64**, 325-341 (2003).
- 2. V. Hongisto, M. Lindgren, R. Helenius. Sound insulation of double walls an experimental parameter study. *Acta Acoustica united with Acustica*, **88**, 904-923 (2002).
- 3. J. Brunskog. The influence of finite cavities on the sound insulation of double-plate structures. *JASA*, **117**, 3727-3739 (2005).
- 4. J-D Chazot, J-L Guayder. Prediction of transmission loss of double panels with a patch-mobility method *Journal of the Acoustical Society of America* **121**, 267-278 (2007).
- 5. A.J. Price, M.J. Crocker. Sound transmission through double panels using statistical energy analysis. *Journal of the Acoustical Society of America*, **47**, 683-693 (1970).
- 6. R.J.M. Craik, R.S. Smith. Sound transmission through double leaf lightweight partitions, part I: airborne sound transmission. *Applied Acoustics* **61** 223-245 (2000).
- 7. J. Woodhouse. An approach to the theoretical background of statistical energy analysis applied to structural vibration. *Journal of the Acoustical Society of America* **69**, 1695-1709 (1981).
- 8. R. S. Langley. A general derivation of the statistical energy analysis equations for coupled dynamic systems. *Journal of Sound and Vibration* **135**, 499-508 (1989).
- 9. S. Finnveden. Ensemble averaged vibration energy flows in a three element structure. *Journal of Sound and Vibration* **187**, 495-529 (1995).
- 10. R. S. Langley. A derivation of the coupling loss factors used in statistical energy analysis. *Journal* of Sound and Vibration 141, 207-219 (1990).

- 11. S. Finnveden A quantitative criterion validating coupling power proportionality in statistical energy analysis. TRITA-AVE 2006:97 (2000). (Re-re-re-submitted to the Journal of Sound and Vibration. Available at www.ave.kth.se -> Staff -> Svante Finnveden.)
- 12. R. H. Lyon and R. G. DeJong 1995 Theory and application of SEA. Butterworth-Heinemann.
- 13. L. Maxit and J-L Guayader. Estimation of SEA coupling loss factors using a dual formulation and FEM modal information. *Journal of Sound and Vibration* **239**, 907-948 (2001).
- 14. M. Fraggstedt. Power dissipation in car tyres. Licentiate thesis, TRITA-AVE 2006:26.
- 15. L.L. Beranek, I.L. Ver (eds). Noise and vibration control engineering. McGraw-Hill (1992).
- 16. D.A. Bies, C.H. Hansen. Engineering noise control. Spoon Press (2003).
- 17. G. Dahlquist, Å. Björk and N. Anderson. Numerical methods. Prentice-Hall (1974).
- 18. S. Finnveden. Energy flows within a three element structure with a statistical description of the design parameters. *Proc InterNoise* 945-948, *Göteborg* (1990).
- 19. B. R. Mace The statistical energy analysis of two continuous one-dimensional subsystems. *Journal of Sound and Vibration* **166**, 429-461 (1993).
- 20. P. W. Smith. Statistical models of coupled dynamical systems and the transition from weak to strong coupling. *Journal of the Acoustical Society of America* **65**, 695-698 (1979).
- 21. P. W. Smith. Response and radiation of structural modes excited by sound. *Journal of the Acoustical Society of America* **34**, 640-647 (1962).
- 22. L. Gavric, 1994 *Journal of Sound and Vibration* **173**, 113-124. Finite element computation of dispersion properties of thin-walled waveguides.
- 23. CM Nilsson. Waveguide finite elements applied on a car tyre. PhD thesis TRITA-AVE 2004:21.
- 24. S. Finnveden Evaluation of modal density and group velocity by a finite element method, *Journal of Sound and Vibration* **273**, 51-75 (2004).
- 25. C-M Nilsson and S. Finnveden 2004. Waves in thin-walled fluid-filled ducts with arbitrary cross sections. (Submitted to the *J of Sound and Vibration.*)
- 26. S. Finnveden TRITA-AVE 2006:38. Waveguide Finite Elements for Curved Structures.
- 27. L. Cremer, M. Heckl and E. E. Ungar Structure-borne sound. Springer-Verlag, (1988).
- F.G. Leppington, E.G. Broadbent, K.H. Heron. The acoustic radiation efficiency of rectangular panels. *Proc Royal Society A* 382 245-271 (1982).
- 29. S.P. Timoshenko History of strength of materials. Dover Publications (1983).
- 30. R. S. Langley, A.W. Brown. The ensemble statistics of the energy of a random system subjected to harmonic excitation. *Journal of Sound and Vibration* **275**, 823-846 (2004).
- 31. P.J. Shorter, R.S. Langley. On the reciprocity relationship between direct field radiation and diffuse reverberant loading. *Journal of the Acoustical Society of America* **117**, 85-95 (2005).
- 32. R.S. Langley. On the diffuse field reciprocity relationship and vibrational variance in a random subsystem at high frequencies. *Journal of the Acoustical Society of America* **121**, 913-921 (2007).
- 33. N. Totaro and J-L Guayader. SEA substructuring using cluster analysis: The MIR index. *Journal* of Sound and Vibration **290**, 264-289 (2006).
- 34. A. Le Bot. Derivation of statistical energy analysis from radiative exchanges. *Journal of Sound and Vibration* **300**, 763-779 (2007).
- P.J. Shorter, R.S. Langley. Vibro-acoustic analysis of complex systems. *Journal of Sound and Vibration* 288 669-699 (2005).