IMPLEMENTATION OF MODAL FILTERS FOR ACOUSTIC SENSING OF SOUND POWER RADIATION

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Modal filtering has emerged as an enabling technique for the measurement of significant system parameters for use in active noise and vibration control. By providing a measure of only a few of the most significant parameters, reductions in the complexity of the adaptive control system can be achieved. The physical implementation of such a modal filter is discussed for a system for measurement of sound power radiated from a rectangular, simply supported panel. The effect of frequency normalisation of the modal filter on the amount of attenuation that may be achieved is examined theoretically.

INTRODUCTION

The total sound power radiating from a vibrating structure is often preferred to acoustic pressure at a point (or points) as an error function for an active noise control system[1,2,3]. One way of achieving a measure of sound power is to use a combination of modal filtering and distributed structural vibration sensors such as shaped PVDF film[4]. This is acceptable when the control source is a secondary vibration source on the structure, as used in applications of ASAC, however, on some structures it is inconvenient to apply ASAC and acoustic control sources may provide the only option. In this case, the *structural measure* of acoustic power provided by distributed PVDF film sensors no longer represents the total farfield sound power (ie including the contribution of the acoustic control source). This paper examines the use of appropriately pre-filtered microphone signals to provide a simple and instantaneous *acoustic measure* of the farfield sound power for use as the error criterion for active control of radiated noise.

THEORETICAL ANALYSIS

Considering a baffled vibrating rectangular panel with a number of point excitations and point control forces, the error criterion may be expressed as

$$J = \mathbf{w}^{\mathrm{H}} \boldsymbol{\Pi} \mathbf{w}\,, \qquad (1)$$

where J is the global error criterion, $\mathbf{w}$ is the modal displacement amplitude vector and $\boldsymbol{\Pi}$ is a weighting matrix which, for the sound power radiated by a rectangular panel, can be expressed

$$\mathbf{\Pi} = \frac{r^2}{2\rho c} \int_{\phi=0}^{2\pi} \int_{\theta=0}^{\pi/2} \mathbf{z}^{\mathrm{H}}(\mathbf{r})\, \mathbf{z}(\mathbf{r}) \sin\theta \, d\theta \, d\phi \,, \tag{2}$$

where $\mathbf{z}(\mathbf{r})$ is the modal radiation transfer function vector[5]. The weighting matrix is real and symmetric, with the diagonal terms representing the *self impedance* of the structural mode and the off-diagonal terms representing the modifications in radiation efficiency due to the co-existence of the other structural modes. The matrix is also sparse, with only the modes with like index pairs exerting a mutual influence on each other[3].

As the weighting matrix is real and symmetric it can be diagonalised by the orthonormal transformation;

$$\mathbf{\Pi} = \mathbf{Q}\,\mathbf{\Lambda}\,\mathbf{Q}^{\mathrm{T}} \,, \tag{3}$$

where $\mathbf{Q}$ is the orthonormal transformation matrix with columns representing the eigenvectors of the weighting matrix and $\mathbf{\Lambda}$ is the diagonal matrix of the associated eigenvalues, λ_i of $\mathbf{\Pi}$.

Substituting the transformation of the weighting matrix (Eq. (3)) into Eq. (1) shows that the total system sound power can be expressed as[6]

$$W = \mathbf{w}^{\mathrm{H}}\,\mathbf{Q}\,\mathbf{\Lambda}\,\mathbf{Q}^{\mathrm{T}}\,\mathbf{w} = \mathbf{u}^{\mathrm{H}}\,\mathbf{\Lambda}\,\mathbf{u} \,, \tag{4}$$

where $\mathbf{u}$ is the transformed modal displacement amplitude vector defined by

$$\mathbf{u} = \mathbf{Q}^{\mathrm{T}}\,\mathbf{w} \,. \tag{5}$$

Eq. (5) shows that each transformed mode is made up of some combination of the normal structural modes; the proportion defined by the associated eigenvector contained in $\mathbf{Q}$. Each transformed mode is an orthogonal contributor to the error criterion, in this case the total sound power.

Two important properties of the transformed modes can be exploited. The first is that the eigenvalues (representing the radiation efficiency of the transformed modes) quickly become very small, so in practice it is only necessary to include the first few transformed modes to account for most of the power radiated from the panel[1,7]. The second is that the low order transformed modes (with the highest radiation efficiency) *also* converge very quickly to their correct shape by considering a limited number of structural modes[8]. In practice then, it is possible to use the $n_m \times n_{tm}$ submatrix of the $n_m \times n_m$ orthonormal transformation $\mathbf{Q}$ where n_{tm} is the number of transformed modes considered analytically. Similarly the $n_{tm} \times n_{tm}$ submatrix of the eigenvalue matrix $\mathbf{\Lambda}$ can be used with no appreciable loss of accuracy in calculating the sound power.

ACOUSTIC SENSING OF TRANSFORMED MODES

PVDF film sensors have been implemented to detect the transformed modal displacement amplitudes, $\mathbf{u}$, directly on the structure[4,6]. The farfield sound pressure radiation patterns can also be decomposed to determine the contributions from the transformed modes.

The farfield sound pressure at n_e microphone error sensors resulting from all of the *normal* structural modes is given by the $n_e \times 1$ vector

$$\mathbf{p} = \mathbf{Z}_n\,\mathbf{w} \,, \tag{6}$$

where $\mathbf{Z}_n$ is the $n_e \times n_m$ normal mode radiation transfer function matrix given by

$$\mathbf{Z}_n = -\frac{\rho\omega^2}{2\pi}\begin{bmatrix} \int_A \psi_1(\boldsymbol{\sigma})\frac{e^{-jkr_1}}{|\mathbf{r}_1|}d\boldsymbol{\sigma} & \cdots & \int_A \psi_{n_m}(\boldsymbol{\sigma})\frac{e^{-jkr_1}}{|\mathbf{r}_1|}d\boldsymbol{\sigma} \\ \vdots & \ddots & \vdots \\ \int_A \psi_1(\boldsymbol{\sigma})\frac{e^{-jkr_{n_e}}}{|\mathbf{r}_{n_e}|}d\boldsymbol{\sigma} & \cdots & \int_A \psi_{n_m}(\boldsymbol{\sigma})\frac{e^{-jkr_{n_e}}}{|\mathbf{r}_{n_e}|}d\boldsymbol{\sigma} \end{bmatrix}. \tag{7}$$

Rearranging Eq. (6) to decompose the normal modal displacement amplitude, **w**, from the pressure field and substituting into Eqn (4) gives

$$W = \mathbf{w}^{\mathrm{H}}\,\mathbf{Q}\,\boldsymbol{\Lambda}\,\mathbf{Q}^{\mathrm{T}}\,\mathbf{w} = \mathbf{p}^{\mathrm{H}}\,(\mathbf{Z}_n^{-1})^{\mathrm{H}}\,\mathbf{Q}\,\boldsymbol{\Lambda}\,\mathbf{Q}^{\mathrm{T}}\,\mathbf{Z}_n^{-1}\,\mathbf{p} \tag{8}$$

or

$$W = \mathbf{p}^{\mathrm{H}}\,\mathbf{Z}_t^{\mathrm{H}}\,\boldsymbol{\Lambda}\,\mathbf{Z}_t\,\mathbf{p} \tag{9}$$

where $\mathbf{Z}_t$ is the $n_{tm} \times n_e$ transformed mode radiation transfer function matrix (or *modal filter matrix*) which relates the pressure in the far field to each transformed modal amplitude, given by

$$\mathbf{Z}_t = \mathbf{Q}^{\mathrm{T}}\,\mathbf{Z}_n^{-1}. \tag{10}$$

The elements of each row of this modal filter matrix represent a weighting value that, when applied to the signal from the corresponding pressure sensor and the result summed over all of the sensors, will give a measure of the transformed modal amplitude. In practice the number of error sensors (n_e) will be much less than the number of normal modes considered (n_m) and so the normal mode radiation transfer function matrix, $\mathbf{Z}_n$, will be rectangular and underdetermined. It is therefore necessary to apply the Moore-Penrose pseudoinverse of a matrix to determine $\mathbf{Z}_n^{-1}$.

The eigenvalue matrix $\boldsymbol{\Lambda}$ is highly frequency dependent, and frequency weighting each transformed mode sensor output to account for the different radiation efficiency of the transformed modes has been suggested using appropriately shaped *eigenvalue filters*[6]. The transformed mode radiation transfer function matrix, $\mathbf{Z}_t$, is a direct function of the transformed mode shapes which, although dependent on frequency, have been shown to change by only a small amount over *small* ranges for a simply supported rectangular panel[6,9]. This is fortunate as in a practical *frequency-correct* modal filter system such as that shown in Fig. 1, it may not be practical to store filters representing the matrix for a wide range of frequencies. Even so, for broadband control over a wide frequency range it would be much better to incorporate the frequency dependence of the radiation transfer function matrix into a single meta-filter representing the frequency dependence of both the transformed mode radiation efficiency (eigenvalues), $\boldsymbol{\Lambda}$, *and* the radiation transfer function matrix $\mathbf{Z}_t$.

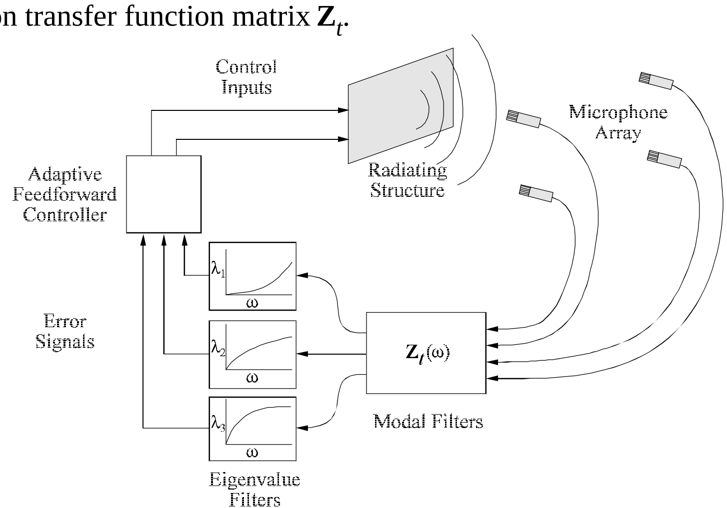

Figure 1 Modal filter and eigenvalue filter arrangement.

This can be achieved by normalising the radiation transfer function matrix to that at some fixed frequency ω_f such that

$$\mathbf{Z}_t = \mathbf{K}\,\mathbf{Z}_{t|\omega_f} \tag{11}$$

to give the $n_{tm} \times n_{tm}$ correction matrix

$$\mathbf{K} = \mathbf{Z}_t\,\mathbf{Z}_{t|\omega_f}^{-1}\,. \tag{12}$$

If ω_f is chosen such that the transformed mode shapes at that frequency are representative of the mode shapes over the frequency range of interest then the off diagonal terms in $\mathbf{K}$ can be ignored with little loss in accuracy. All of the frequency dependence of both the eigenvalue and transformed mode radiation transfer function matrices can then be grouped into one *real* $n_{tm} \times n_{tm}$ diagonal frequency weighting matrix $\mathbf{X}$ with elements

$$\chi_{ij} = \begin{cases} k_{ij}^{*}\,\lambda_{ij}\,k_{ij}, & i = j \\ 0, & i \neq j \end{cases}. \tag{13}$$

Practically, this *corrected fixed-frequency* modal filter system can be implemented by a simpler system, where the modal filter $\mathbf{Z}_t$, isn't implemented explicitly, but is replaced by a single frequency independent weighting value $\mathbf{Z}_{t|\omega_f}$, for each transformed mode. It is also possible to implement an *uncorrected fixed-frequency* filter system by not correcting the eigenvalue filter to account for the frequency dependence of the transformed mode radiation transfer function matrix (ie. $\mathbf{X} = \mathbf{\Lambda}$ rather than $\mathbf{X} = \mathbf{K}^{\mathrm{H}}\mathbf{\Lambda}\mathbf{K}$), but still replacing the modal filter with a single weighting value from a fixed frequency ω_f. In practice this would appear to be an unnecessary complication, considering the eigenvalue filter would still need to be implemented.

If these modifications are required to enable wideband control then Eq. (9) becomes

$$W = \mathbf{p}_n^{\mathrm{H}}\,\mathbf{Z}_{t|\omega_f}^{\mathrm{H}}\,\mathbf{X}\;\mathbf{Z}_{t|\omega_f}\,\mathbf{p}\,. \tag{14}$$

Note that the only difference between this approach and the exact solution of Eq. (9) is the small loss of accuracy introduced by ignoring the off diagonal terms of the correction matrix $\mathbf{K}$.

MODAL FILTER RESPONSE

The complex elements of the transformed mode radiation transfer function matrix, $\mathbf{Z}_t$, make up the modal filters. In particular, each row of the matrix represents the overall filter required for each transformed mode, and each column within that row represents the filter for the corresponding sensors' contribution to that mode. The phase and amplitude response of each element of the first row corresponding to the filters for the first transformed mode for the case where three error sensors are decomposed into the first and second transformed modes are shown in Fig. 2. It should be recognised that the form of these filters depends entirely on the position of the error sensors and as such the results presented here show only what may be expected for a typical arrangement. In particular the form of the amplitude filters shown in Fig. 2 indicate that the second error sensor contributes very little to the detection of the first transformed mode.

The phase response of the filters is linear and can be implemented in practice by introducing a group delay (corresponding to the slope of the phase response) between the signal paths of the individual sensors. In practice it would only be necessary to implement the relative (net) delay between sensors as the electronic control system would compensate for the gross delay (and corresponding phase shift).

TRANSFORMED MODE FREQUENCY CORRECTION

Further simplifications can be made in the physical realisation of the modal filter by fixing the transfer function to that at a single frequency and then lumping the frequency dependence together with the eigenvalue filter as defined in Eq. (13) above, to create a *corrected fixed-frequency* filter system, where the modal filter is not frequency dependant at all, and is replaced by a frequency independent weighting

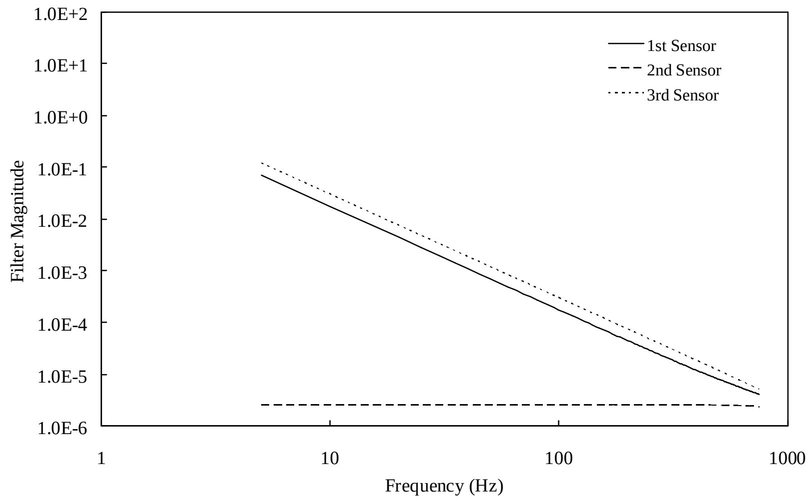

Figure 2 Amplitude response of the modal filter required to decompose the 1st transformed mode from 3 farfield sensors.

value. The meta-filter response for each transformed mode, as contained in the diagonal of **X**, is shown in Fig. 3 for a transfer function fixed at 100Hz. The sound power attenuation achieved by minimising two transformed modes determined from a *frequency-correct* modal filter system (which varies optimally with frequency) is compared to that from a *corrected fixed-frequency* filter system and an *uncorrected fixed-frequency* filter system in Fig. 4. It is observed that the corrected fixed-frequency filter performs as well as the frequency-correct filter over a wide frequency range, with a few slight deviations of both *better* and worse control. The uncorrected fixed-frequency modal filter causes severe lapses in control at some frequencies, particularly above 350Hz, though below this it performs as well as the frequency-correct filter. This suggests that a fixed-frequency modal filter could be used *without correction*, if control were limited to a small frequency range around that of the filter.

Moreover, it is of interest that the magnitude of the correction filter, **X**, is itself relatively constant over a large frequency range (Fig. 3). This indicates that it should be possible to select a single *correction factor* (say the value of the correction filter at 100Hz) which, when combined with the frequency independent weighting values of the modal filter at some fixed frequency, will produce a single weighting value for each sensor that will perform as well as a fully implemented frequency-correct filter system up to around 550Hz. In other words the corrected fixed-frequency filter *value* and correction *factor* could be combined to give a single gain factor to be applied to each sensor input (see Eq. (14)), independent of the operating frequency. In this way a "modal filter" implementation, albeit with somewhat limited performance, could be constructed by simply delaying and weighting each sensor input, without the need for any explicit "filtering" at all.

EXPERIMENTAL VERIFICATION

A rectangular steel panel of dimensions 380mm x 300mm, and thickness h=1.942mm was mounted in a heavy steel frame using spring steel shims to approximate simply supported boundary conditions. The panel was placed in the centre of a large wooden baffle in an anechoic chamber. A minishaker was used to excite the panel at (35mm,103.3mm) through a stinger. A force transducer was attached to the panel to measure the input force. A pair of piezoceramic crystals were placed on the panel at (35mm,0mm) to provide a control moment. The acoustic intensity at a distance of 70mm from the panel was measured using an intensity probe, which was mounted in an X-Y traverse such that it could be remotely positioned at any location in front of the panel. Software performed the necessary calculations to determine the sound power radiated by the panel by measuring the acoustic intensity at a large number of points in front of the panel.

An array of five inexpensive electret microphones were used as error sensors, mounted at a radius of 2.0m from the centre of the panel.

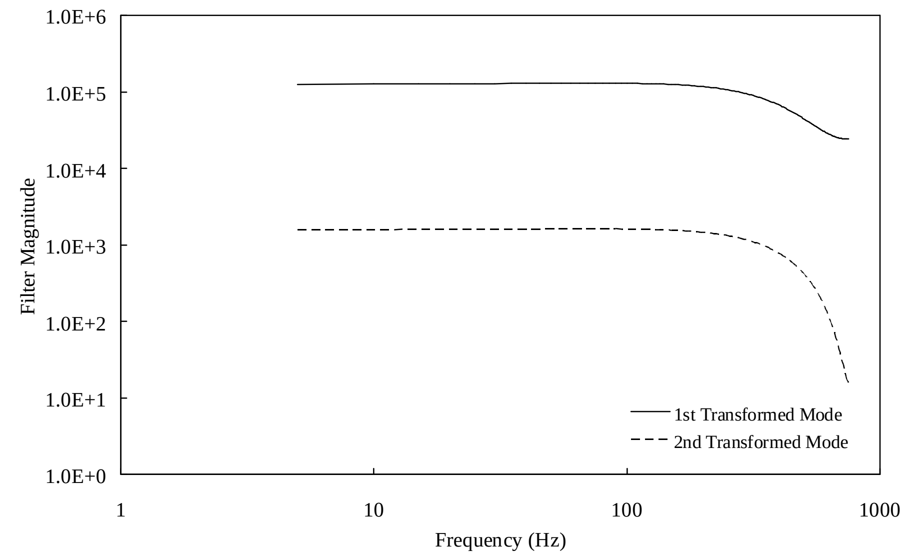

Figure 3 Amplitude response of the correction filter, **X**, for the 1st and 2nd transformed modes decomposed from 3 sensors.

The modal filters were implemented on a modified CAUSAL SYSTEMS EZ-ANC digital signal processing board. In this case a *corrected fixed frequency* filter was implemented with the transfer function frequency fixed at 100Hz. The correction factor was also assumed to be a constant, using the value at 100Hz, and multiplied by the modal filter magnitudes to give a single overall gain for each sensor input as discussed above. Custom software was written to provide the group delay, appropriate relative gains and signal summation to produce output signals representing the magnitude of the first and second transformed modes as shown in Fig. 5. The group delay was implemented by using short FIFO buffers on the first and second input signal channels, sampled at a rate of 6.25kHz. At each measurement frequency control was achieved and optimised using a CAUSAL SYSTEMS EZ-ANC Active Noise Control System.

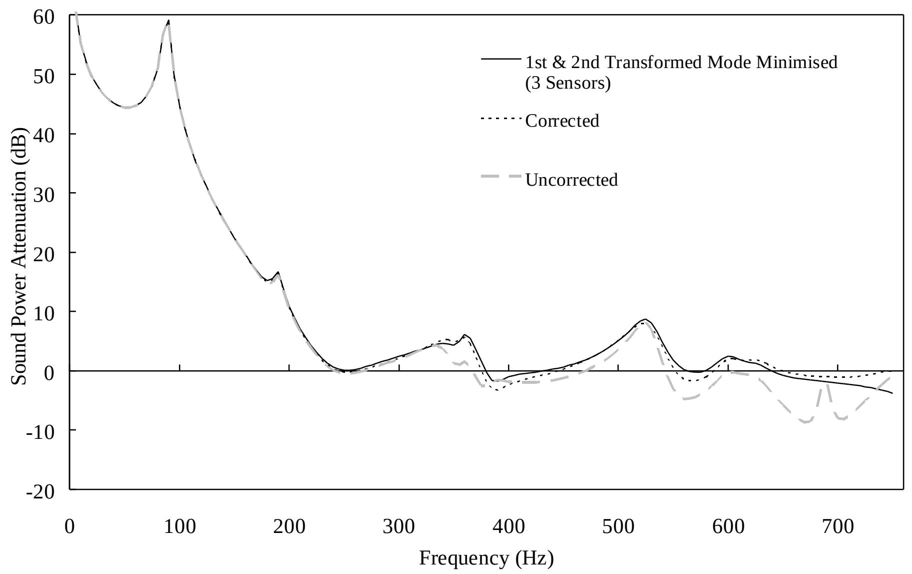

Figure 4 Attenuation by minimising the 1st and 2nd transformed modes decomposed with corrected and fixed frequency (100Hz) modal filters.

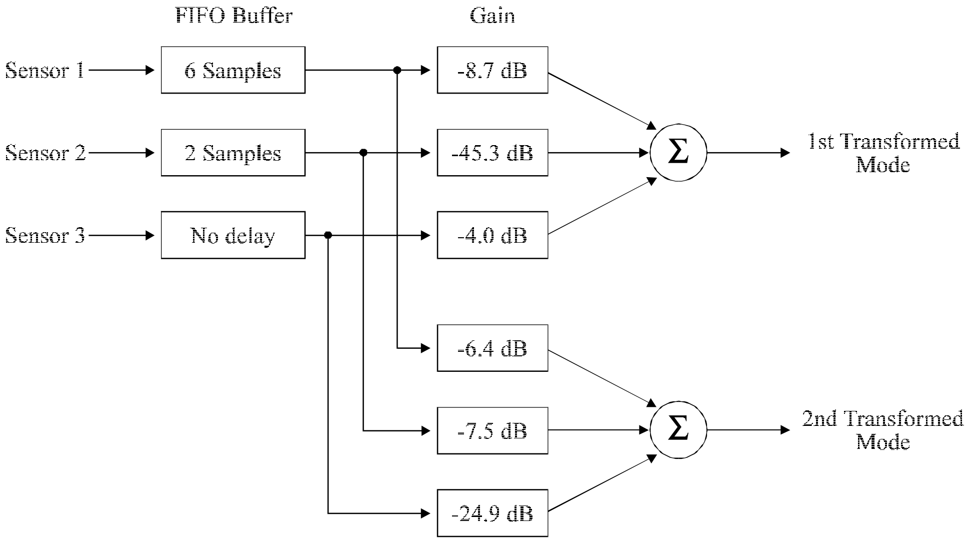

Figure 5 Modal filter implementation.

RESULTS AND DISCUSSION

A comparison of experimental results minimising one and five pressure sensors and the 1st and 2nd transformed modes is shown in Fig. 6 and, though cluttered due to the small differences in levels expected between error criteria, a number of trends are clearly evident. The sound power attenuation achievable when using five sensors or the transformed modes is clearly greater than using a single sensor. At frequencies above 550Hz, controlling the transformed modes does not perform well, as expected. Additionally, the attenuation levels achieved by controlling five sensors and the transformed modes are of a similar level. This is of interest because although the levels are comparable, the latter were achieved using only two channels on the electronic controller.

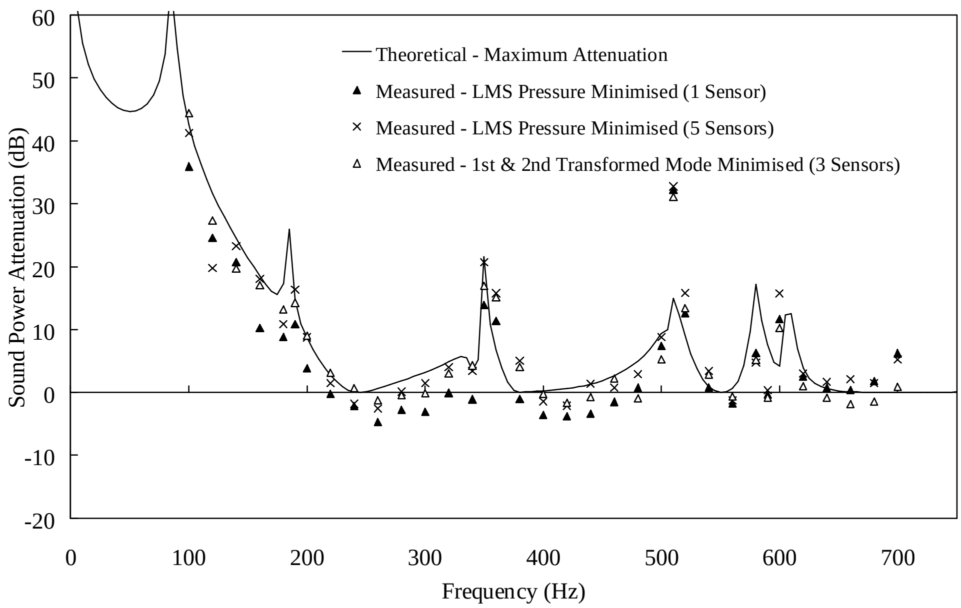

Figure 6 Experimental radiated sound power attenuation, minimising pressure at 1 and 5 sensors and minimising the 1st and 2nd transformed modes.

With this in mind, Fig. 7 shows the experimental results for the two cases where only two error channels are minimised by the control system, specifically either two farfield pressure signals or the first and second transformed modes. These results show that an increase in attenuation of between 3dB and 4dB can be expected when minimising the transformed modes at frequencies less than 500Hz.

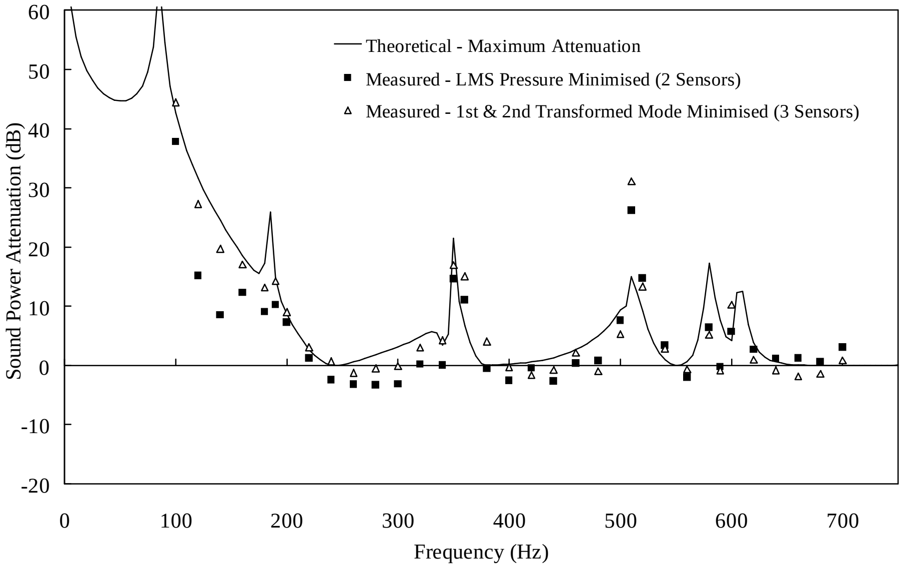

Figure 7 Experimental radiated sound power attenuation, minimising pressure at 2 sensors and minimising the 1st and 2nd transformed modes.

CONCLUSIONS

Theoretical definitions of modal filters were developed such that the transformed modal amplitudes (and, indirectly, the radiated sound power) could be measured by using a small number of acoustic sensors in the farfield. Due to the consistency of the acoustic radiation patterns of the transformed modes over a wide range of frequencies, the model was extended to enable simpler implementation of the filters.

It was found that if the modal filters were constructed simply by a constant weighting and time delay applied to the error sensor outputs, rather than by a full frequency dependent implementation, then good control could be maintained with only a small reduction in effective bandwidth. The results of the experimental work showed that construction of an acoustic sensor to measure sound power radiation using a small number of filtered pressure sensors is practical.

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