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LINEAR MODELING OF THE DYNAMICS OF AN ELECTRODYNAMIC PISTON COMPRESSOR

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ABSTRACT

The force acting on the reciprocating piston during a compression cycle shows typically non-linear dynamic behavior in electrodynamic piston compressors. In order to increase the efficiency in design, in this paper, the compression cycle is modeled as a linear spring and a hysteretic damper by 4 different methods. Hysteretic damping coefficient is obtained in all of the 4 methods based on the dissipation energy equivalency corresponding to the area of pressure-volume diagram. Regarding the stiffness, the simplest method is to use the slope of two extreme points in the diagram. Another simple method is to derive the stiffness coefficient by applying piecewise equivalency of the potential energy. The other two methods are to apply describing function approach to the compression cycle alone and a single degree of freedom system comprising the reciprocating compressor cycle. Characteristics of the 4 linearized dynamic models are compared with the full nonlinear model, which are obtained by numerical integration for various parameters.

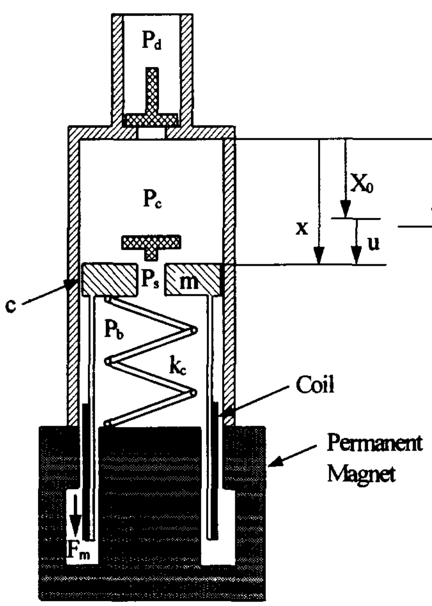
1. INTRODUCTION

Electrodynamic piston compressor is a small hermetic reciprocating compressor that is driven by a linear oscillating motor. Compressors of this kind have been studied through the years owing to its potential in cost reduction and efficiency increase[1-7]. Their basic structure is very simple; the piston is directly connected to the oscillating part of the linear oscillating motor and the cylinder is rigidly attached to the stationary part. Dynamics of the piston is, however, complicated, i.e., it is closely linked with electrodynamics and thermodynamics, because motion of the piston is not kinematically restricted.

The electrodynamic piston compressor which is the object in this study is schematically presented in Fig. 1. This is a simplified model commonly known as a Doelz compressor. Cylinder is assumed not to vibrate, and the dynamics of valve is neglected. Heat loss and valve loss are neglected. Therefore, polytropic processes of compression cycle are assumed to be isentropic processes, and suction and discharge processes to be isobaric.

In the mathematical modeling of this compressor, the pressure force acting on the piston due to compression cycle can be dealt with in two ways. One way is with an equivalent stiffness and damping[1,2] and the other way is with pressure difference between the front and back surfaces of piston and thermodynamic equations[3,6,7]. The latter is more accurate, but the mathematical model is nonlinear and, hence, the steady-state response can be obtained only by numerical integration. This makes it difficult and tedious to analyze the effects of system parameters. In the former, the equivalent stiffness and damping are assumed constant at a given design stroke and mean position of the piston, so that the mathematical model is linear and therefore can be solved analytically.

In this paper, 4 different methods are applied to model the pressure force with an equivalent stiffness and damping. To verify the usability of linear modeling in design, two system parameters of the compressor which has a capacity of a heat lift of 150 Watts from -23°C to 32°C are determined using a linearized model, and the results of the 4 different linear models and nonlinear model of this compressor are compared.



m = effective moving mass

c = viscous damping coefficient between piston and cylinder wall

k_c = stiffness of tuning spring

P_d = discharge pressure

P_s = suction pressure

P_c = pressure in compression chamber

P_b = back pressure

F_m = thrust of linear oscillating motor

X_p = static equilibrium position of piston in quiescence

X_0 = mean position of piston motion

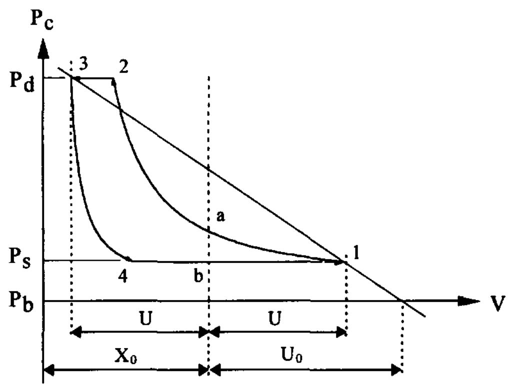
x = displacement of piston from cylinder head

u = displacement of piston from mean position of piston motion

Fig. 1 Schematic drawing of the electrodynamic piston compressor

2. NONLINEAR MATHEMATICAL MODEL

A typical pressure-volume diagram of an ideal compression cycle is shown in Fig. 2.



U = amplitude of piston

U_0 = offset of equivalent spring

Fig. 2 Ideal pressure-volume diagram of compression cycle

If the cross-sectional area of the cylinder is constant through the whole stroke of the piston, the pressure in compression chamber can be written by

$$P_c(t) = P_r\left(\frac{X_r}{x(t)}\right)^n \quad \text{if } P_c < P_s \text{ then } P_c = P_s \text{, and if } P_c > P_d \text{ then } P_c = P_d \tag{1}$$

where P_r and X_r respectively are P_d and $(X_0\text{-}U)$ for the positive stroke, and P_s and (X_0+U) for the negative stroke, and n the specific heat ratio. The pressure force F_g acting on the piston is given by

$$F_g(t) = A_p\left(P_c(t) - P_b\right) \tag{2}$$

where A_p is cross-sectional area of the piston.

Because the pressure force contains static force component, the governing equation of motion of the piston must satisfy static force equilibrium as well as dynamic force equilibrium as follows:

$$m\frac{d^2x(t)}{dt^2} + c\frac{dx(t)}{dt} + k_c\left(x(t) - X_p\right) - F_g(t) = F_m(t) \tag{3}$$

The thrust of a linear oscillating motor $F_m(t)$ is given by $\alpha i(t)$, where α is a constant linking the thrust and current i(t) in the motor. Effective moving mass m includes the piston mass with driving coil and a portion of the tuning spring mass. The viscous damping coefficient c represents the effect of the oil friction between the piston and cylinder wall[2,7].

The voltage equation of electrodynamic linear oscillating motor can be written as

$$\alpha\frac{dx(t)}{dt} + L_e\frac{di(t)}{dt} + R_e i(t) = v(t) \tag{4}$$

where L_e and R_e are the effective inductance and resistance respectively[2], and v(t) the supply voltage.

These equations are solved numerically by the 4th order Runge-Kutta method.

3. LINEAR MATHEMATICAL MODEL

In this compressor, the mechanical spring is used to tune the resonance frequency, and if the spring is not so soft then motion of the piston is nearly harmonic[2,6]. Thus, assuming that the piston does harmonic motion of $u(t)=U\cos\omega t$ across the mean position X_0, displacement of the piston relative to the cylinder head is represented by

$$x(t) = X_0 + u(t) \tag{5}$$

and the pressure force is modeled as below using the equivalent hysteretic damping coefficient h_{eq} and spring constant k_{eq},

$$F_g(t) = -\frac{h_{eq}}{\omega}\frac{du(t)}{dt} - k_{eq}\left(u(t) - U_0\right) \tag{6}$$

where U_0 is an offset of the equivalent spring from the mean position X_0.

The pressure force in equation (6) is divided into the static and dynamic components, so that the governing equations can also be divided into the static equilibrium and dynamic equilibrium equations as follows:

For the static equilibrium,

$$k_c\left(X_0 - X_p\right) - k_{eq}U_0 = 0 \tag{7}$$

For the dynamic equilibrium,

$$m\frac{d^2u(t)}{dt^2} + \left(c + \frac{h_{eq}}{\omega}\right)\frac{du(t)}{dt} + \left(k_c + k_{eq}\right)u(t) = \alpha i(t) \tag{8}$$

$$\alpha\frac{du(t)}{dt} + L_e\frac{di(t)}{dt} + R_e i(t) = v(t) \tag{9}$$

These equations can be solved analytically in frequency domain using complex notation if the equivalent stiffness and damping are defined.

The equivalent damping is defined as a hysteretic damping based on dissipation energy equivalency corresponding to the area of pressure-volume diagram. This area, which means work done per cycle by compressor, can be defined as follows:

$$W = \frac{2n}{n-1}P_sA_p\left(1 - r_p^{\frac{n-1}{n}}\right)\left(1 - \left(r_p^{\frac{1}{n}} - 1\right)\left(\frac{X_0 - U}{2U}\right)\right)U \tag{10}$$

where r_p is the ratio of discharge to suction pressure. Therefore, the equivalent hysteretic damping coefficients can be written by

$$h_{eq} = \frac{W}{\pi U^2} \tag{11}$$

The equivalent stiffness is defined by 4 different methods. The simplest method is, following Cadman[1], to use **the slope of two extreme points** 1 and 3 in Fig. 2. According to this method, the equivalent spring constant and offset can be represented as follows:

$$k_{eq} = \frac{A_p\left(P_d - P_s\right)}{2U} \tag{12}$$

$$U_0 = U + \frac{A_p\left(P_s - P_b\right)}{k_{eq}} \tag{13}$$

Another simple method is to apply **piecewise equivalency of the potential energy**. In a linear spring-damper system, the same amount of dissipation energy is always dissipated every quarter cycle. The potential energy is, however, alternately stored and released in the same amount over each quarter cycle, or vice versa, if we consider the half cycle of oscillation from the mean position. Therefore the difference between works done on this system over two successive quarter cycles from the mean position becomes the double of variation of the potential energy over one quarter cycle. Using these relations, the equivalent spring constant and offset can be written by

$$k_{eq} = \frac{\left(\Delta W_{a-3} - \Delta W_{3-b}\right) + \left(\Delta W_{b-1} - \Delta W_{1-a}\right)}{2U^2} \tag{14}$$

$$U_0 = \frac{\left(\Delta W_{a-3} - \Delta W_{3-b}\right) - \left(\Delta W_{b-1} - \Delta W_{1-a}\right)}{4k_{eq}U} \tag{15}$$

where $\Delta W_{i\text{-}j}$ is the variation of energy through the path from point i to point j in Fig. 2.
The third method is to **apply the describing function approach[8] to the compression cycle only**. With the displacement of piston as input and the pressure force as output, the describing function is obtained by numerical integration as follows:

$$N(U,\omega) = \frac{1}{U}\left(a_1 + jb_1\right) \tag{16}$$

where U is the amplitude of the piston displacement. Then equivalent spring constant and hysteretic damping coefficient can be obtained by comparing this equation with the equation (6) as follows:

$$k_{eq} = -\frac{a_1}{U} \tag{17}$$

$$h_{eq} = \frac{b_1}{U} \tag{18}$$

With simple manipulation, we can see that the equation (18) is identical with the equation (11). In the describing function method, the nonlinearity is often assumed odd, that is, the dc-component neglected. In this case, however, it can not be neglected because the dc-component given by the static equilibrium is dependent inherently on the parameters of the compression cycle as shown in equation (6) and (7). Using the dc-component, the offset of equivalent spring is obtained as follows:

$$U_0 = \frac{a_0}{2k_{eq}} \quad , \text{where } a_0 = \frac{1}{\pi}\int_0^{2\pi} F_g(t)d(\omega t) \tag{19}$$

The fourth method is to **apply the describing function approach to a single degree of freedom system (SDFS)** the motion of which is governed by

$$m\frac{d^2x(t)}{dt^2} + c\frac{dx(t)}{dt} + k_c\left(x(t) - X_p\right) - F_g(t) = F\cos\omega t \tag{20}$$

First, appropriate values of m, c and k_c are assumed in the acceptable design range and, then, X_p is determined in order that the min. and max. values of x may be coincident with the design

values using iterative numerical integration.
To obtain the describing function, the magnitude F is controlled, in such a way that the min. and max. values of x may be kept constant, because the shape of F_g vs. x depends on these. The undamped natural frequency ω_n is determined by selecting a frequency point where the real part of the describing function becomes zero, when F_g is modeled by equation (6). Then, the equivalent spring constant is obtained by

$$k_{eq} = \omega_n^2 m - k_c \tag{21}$$

and the offset of equivalent spring is obtained using equation (7) as follows:

$$U_0 = \frac{k_c\left(X_0 - X_p\right)}{k_{eq}} \tag{22}$$

4. APPLICATION AND THE RESULTS

Among the compression cycles, which have the capacity of a heat lift of 150Watts from -23°C to 32°C when the driving frequency is 60Hz, one is described as follows:

$P_d = 1468.6$ kPa, $P_s = 115.43$ kPa, $P_b = 115.43$ kPa, $n = 1.118$
$A_p = 100\pi$ mm^2, $X_0 = 11.0355$ mm, $U = 10.0355$ mm

The equivalent hysteretic damping coefficient of this compression cycle was obtained as 3795.08 N/m, and the other parameters were obtained by each method as the following table.

Method	k_{eq} [kN/m]	U_0 [mm]
the slope of two extreme points	21.157	10.0565
piecewise equivalency of the potential energy	10.456	5.7465
describing function of compression cycle	15.155	6.1215
describing function of SDFS	14.455	4.1595

When constructing the single degree of freedom system, the mass was assumed as 1.0 kg, the stiffness as 131.91 kN/m and viscous damping coefficient as 6.0 N·sec/m.
The effective mass and tuning spring were determined using the linearized model so that this compressor could produce the designed output at 220Vrms 60Hz and resonate at 57 Hz for the given electrodynamic linear oscillating motor. These were determined using linear model whose equivalent coefficients were obtained by applying describing function method to SDFS, and they are

$m = 1.546$ kg, $k_c = 160.41$ kN/m, $X_p = 10.66$ mm

The characteristics of this compressor were obtained using the nonlinear mathematical model. When solving the nonlinear model, the mean position was changed according to the driving frequency. Therefore, the input voltage was controlled to maintain that the top clearance, which is the distance between cylinder head and top dead center, was constant. In order to compare the results with those of linear mathematical models, the half of peak-to-peak value was assumed as amplitude. The frequency response characteristics are presented in the following figures with the deviations of the results of each linear model from those of nonlinear model.

As shown in Fig. 3, simulation results based on the nonlinear model confirm that this compressor has resonance at 57 Hz as initially intended. When the driving frequency was 60Hz, the input voltage was 218.2Vrms and input current was 3.8A for 1mm of top clearance. That is, deviation of the voltage from the design value was -0.8% and deviation of the stroke was 0.4% and therefore, to meet the exact design target, the X_p must be increased about 0.4%.

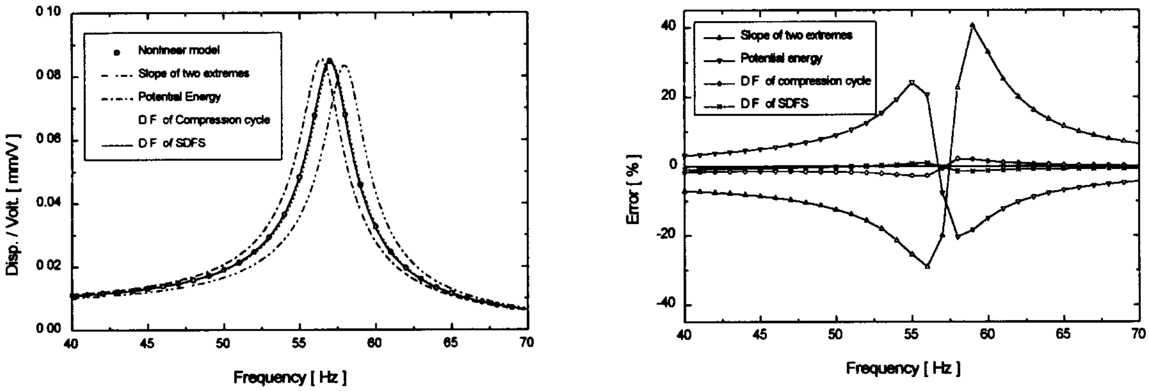


Fig.3 Frequency response characteristics of piston movement/supplied voltage and errors of linearized models with respect to the fully nonlinear model.

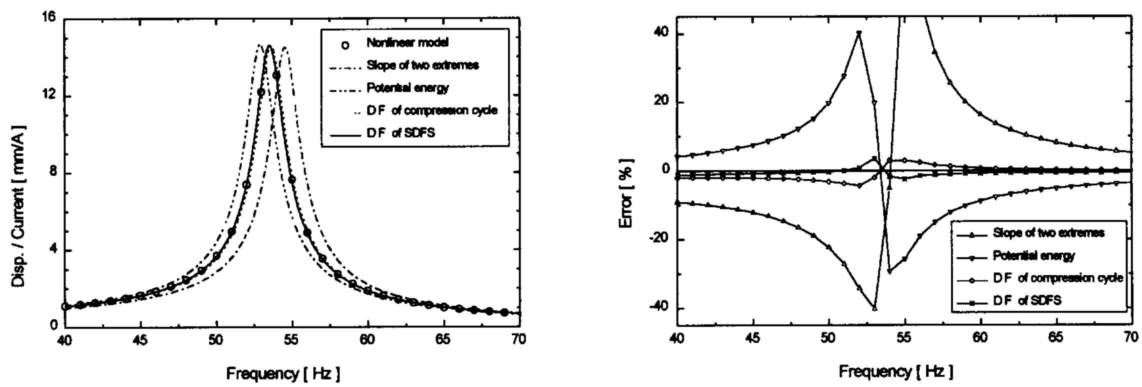


Fig.4 Frequency response characteristics of piston movement/supplied current and errors of linearized models with respect to the fully nonlinear model.

The resonance frequency of the linear model by the slope of two extreme points is higher than that of the nonlinear model as shown in Fig. 3, which means that the equivalent spring estimated by this method is stronger than the true one by about 44%. In case of the piecewise equivalency of the potential energy, the equivalent spring was estimated to be softer than the true one by about 28%.

As shown in Fig. 3 and 4, discrepancies between two describing function approaches for the linearization are very small, and both of them agree well with the nonlinear model within a few percents of error. The equivalent spring was estimated to be slightly stronger than the true one by the describing function approach to compression cycle only, and slightly softer by the other describing function approach. In case of error in frequency response characteristics, the root-mean-squared error of the latter approach was about 0.6~0.7% smaller than that of the former approach, but in case of computing time, it took about 1200 times longer. Although it is better approximation theoretically to apply the describing function approach to single degree of freedom model, it takes too much time for the iterative numerical integration as in the nonlinear model. Therefore, in a point of effectiveness the former is preferred.

5. CONCLUSIONS

Accuracy and effectiveness of four different linearized approaches for the modeling of an electrodynamic piston compressor were studied. The results of the describing function approaches agreed quite well with those of the nonlinear mathematical model. It is preferred to apply the describing function approach to compression cycle only in the aspect of computational effectiveness. Although it was not presented in this paper, the amplitude dependent nonlinear characteristics, like jump phenomena, also can be investigated by this approach.

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