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ATTENUATION OF ACOUSTIC WAVES BY A THIN CYLINDRICAL BUBBLY LAYER

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ABSTRACT

Using a bubbly liquid is an economical and effective technique in controlling underwater sound signals. When an acoustic wave passes through a bubbly liquid, some of the energy is reflected and some is lost due to dissipation. As a result, the transmitted signal would be attenuated. In this paper, a harmonic signal enclosed by a cylindrical bubbly layer is studied in the presence of different boundaries. By using the classical theory for dilute bubbly liquid and numerical integration, the transmitted waves through a thin layer are determined. Furthermore, experiment results of a cylindrical bubbly layer with finite air volume fraction around 10% are obtained and compared with those calculated from the dilute bubbly liquid theory. It is found that there is a remarkable difference between the results. Discussion is given for the explanation of the phenomenon and the design of bubbly layer for underwater sound control.

INTRODUCTION

Effect of air bubbles on underwater sound transmission has been studied for a few decades since bubbly layers play an important role in the underwater environmental and military engineering. Due to the nonlinearity and randomness of the mixture, physics of a bubbly liquid is difficult to be studied. However, linear theory for dilute bubbly liquid can be obtained. In this paper, bubbles are assumed to be small and spherical with equilibrium radius much shorter than the wavelength of the incident signal. As a result, the pressure inside a bubble can be treated as uniform and the bubble oscillates in simple harmonic motion. By applying Calflisch et al.'s theory (1985), the effective sound speed for a bubbly liquid is found. Due to the strong dispersive effect of a bubbly liquid, the effective sound speed is highly dependent on the incoming frequency. In order to apply the technique to real engineering problems, a harmonic point source enclosed by a cylindrical bubbly layer is considered. By using Taylor's expansion and numerical integration, transmitted pressure fields through a thin cylindrical bubbly layer are obtained in an infinite domain, near a free surface and near a flat sea floor. In addition, a bubbly liquid with finite air volume fraction around 0.1 is studied experimentally. Compared with theoretical results, the transmission coefficients measured from experiments are much higher than those predicted by the theory. It seems that the classical theory cannot be applied to the bubbly liquid with a finite air volume fraction.

EFFECTIVE WAVE NUMBER IN A DILUTE BUBBLY LIQUID

If bubbles are evenly distributed in the liquid and the radii of the bubbles are small compared with the wavelength of the signal, the medium can be considered as a homogeneous one with an effective sound speed. When a small spherical bubble with equilibrium radius R_o is driven by an acoustic wave with angular frequency ω, the instantaneous radius R' obeys the linearized Rayleigh Equation

$$\ddot{R}'+\omega_o^2 R' = -\frac{p}{\rho_o R_o^2}, \tag{1}$$

where p is the pressure, ρ_o the mean density, $\omega_o = \sqrt{3p_o/\rho_o R_o^2}$ the bubble resonant frequency and a time dependent factor $e^{-i\omega t}$ is omitted. In this case, the bubble behaves as a monopole. Applying Caflisch et al.'s theory (1985), the effective wave number for a dilute bubbly liquid is found to be

$$k_b = \sqrt{k^2 + \frac{3\beta_o\omega^2}{R_o^2\omega_o^2\left[1-\frac{\omega^2}{\omega_o^2}\right]}}, \tag{2}$$

where β_o is the air volume fraction of the liquid and k the wave number in pure liquid. The above expression neglects the effect of dissipation due to damping and viscosity. In order to take into account the damping effect of bubbles, the Rayleigh Equation must be modified by adding several damping terms into the equation. However, in this paper, only the simple expression k_b in Eq. (2) is considered.

For acoustic signals with frequencies below the bubble resonance frequency, the effective speed of sound is lower than that in pure liquid because of the increase in compressibility of the medium. When the incident frequency moves beyond the resonance frequency, the wave number becomes a positive purely imaginary number which means that there is no propagating mode but damping in the medium. Hsieh (1988) called it the "Dead Zone". As the frequency further increases to a critical value ω_a, which is called the anti-resonance frequency, a propagating mode appears again. The value of ω_a depends on the bubble resonance frequency as well as the air volume fraction. However, this theory is not applicable to high frequency signals because the signal wavelength is comparable to the bubble radii.

PROPAGATION OF LOW FREQUENCY SIGNAL THROUGH A THIN CYLINDRICAL BUBBLY LAYER

Consider a cylindrical bubbly layer with inner radius r_1 and thickness s with a point source placed along the symmetry axis in an infinite domain (Fig.1). The signal frequency is assumed to be smaller than the bubble resonance frequency so that k_b is positive and real. Denote the inner and outer liquid regions by L_1 and L_2 respectively and the bubbly region by B. The governing equations for harmonic signals with a time factor $e^{-i\omega t}$ in L_j (j=1,2) are the reduced wave equations

$$\nabla^2\phi_1 + k^2\phi_1 = -\frac{-\delta(r)\delta(z)}{r} \tag{3}$$

and

$$\nabla^2\phi_2 + k^2\phi_2 = 0 \tag{4}$$

respectively, where k is the wave number in pure liquid at the equilibrium state and ϕ is the velocity potential. The reduced wave equation for region B is

$$\nabla^2\phi_b + k_b^2\phi_b = 0. \tag{5}$$

On the surfaces of the bubbly liquid, the continuity of pressure and normal velocity implies

$$\phi_b = \phi_j \tag{6}$$

and

$$\frac{\partial\phi_b}{\partial r} = \frac{\partial\phi_j}{\partial r} \tag{7}$$

on r_1 and r_2 where $r_2= r_1+s$. By taking Fourier transform on z, the transmitted velocity potential is found to be

$$\phi_2^{FF} = \frac{-8}{\pi^3 r_1 r_2}\int_{-\infty}^{\infty}\left[\frac{H_o^{(1)}(k_r r)}{\Delta}\mathrm{Exp}[ik_z z]\right]dk_z, \tag{8}$$

where

$$\begin{aligned}\Delta = \; & \left[k_{br}H_o^{(1)}(k_r r_2)H_1^{(2)}(k_{br}r_2) - k_r H_1^{(1)}(k_r r_2)H_o^{(2)}(k_{br}r_2)\right] \\ & \qquad x\left[k_r H_o^{(1)}(k_{br}r_1)H_1^{(2)}(k_r r_1) - k_{br}H_1^{(1)}(k_{br}r_1)H_o^{(2)}(k_r r_1)\right] \\ & + \left[k_{br}H_o^{(1)}(k_r r_2)H_1^{(1)}(k_{br}r_2) - k_r H_1^{(1)}(k_r r_2)H_o^{(1)}(k_{br}r_2)\right] \\ & \qquad x\left[k_{br}H_1^{(2)}(k_{br}r_1)H_o^{(2)}(k_r r_1) - k_r H_o^{(2)}(k_{br}r_1)H_1^{(2)}(k_r r_1)\right],\end{aligned} \tag{9}$$

$$k_r = \sqrt{k^2 - k_z^2}, \tag{10}$$

and

$$k_{br} = \sqrt{k_b^2 - k_z^2}. \tag{11}$$

$H_o^{(1)}$ and $H_o^{(2)}$ are the zeroth order Hankel function of the first and second kind respectively.

If the thickness of the bubbly layer is small compared with the incident wavelength (ks <<1) and the inner radius of the cylindrical layer ($s<<r_1$), we can approximate Δ by expanding the Hankel functions with arguments $k_r r_2$ and $k_{br} r_2$ in Taylor series as

$$H(k_r r_2) = H(k_r r_1) + (k_r s)H'(k_r r_1) + O\left[(k_r s)^2\right] \tag{12}$$

and

$$H(k_{br} r_2) = H(k_{br} r_1) + (k_{br} s)H'(k_{br} r_1) + O\left[(k_{br} s)^2\right]. \tag{13}$$

Substituting the above into Eq. (8) and using some Hakel function relations, we obtain

$$\phi_2^{FF} = \frac{i}{8\pi}\int_{-\infty}^{\infty}\left[\frac{H_o^{(1)}(k_r r)}{1 - is(\pi r_1/4)\left(k_b^2 - k^2\right)H_o^{(1)}(k_r r_1)H_o^{(2)}(k_r r_1)}\mathrm{Exp}\left[ik_z z\right]\right]dk_z. \tag{14}$$

Analytical evaluation of the integral is impossible. In order to evaluate the integral numerically, we have to prove that the value within the interval is finite. A simple test is being used. The integral is evaluated numerically within a small interval around the singularity. The interval is divided into 512, 1024, 2048 and 5096 divisions respectively. The result shows that, omitting round-up errors, the values obtained from the four "resolutions" are nearly equal with accuracy up to four significant figures. It is seen that the integral in Eq. (14) has a finite value and is evaluated numerically by Simpson's rule.

Based on similar methods, the transmitted potentials of a point source through a thin cylindrical bubbly layer near a free surface and near a flat sea floor are evaluated numerically. Some of the results are shown from Fig. 2 to Fig. 7.

EXPERIMENTAL STUDY

Experiments are implemented in a small water tank with dimensions 1.8m x 1.2m x 0.5m. A sketch of the experimental apparatus is shown in Fig. 8. To make the bubbly layer, a flexible porous material is used as a bubble maker and is curved into a circle with inner radius of 12cm. Dry compressed air is injected into the bubble maker through a regulator and a flowmeter from an air compressor. A cylindrical bubbly layer of thickness 1cm is produced and the bubble radius lies between 1mm to 2mm. A hydrophone (B&K 8104) is placed along the symmetry axis of the layer as a sound source. On the other hand, the transmitted signal is picked up by a smaller hydrophone (B&K 8103) outside the bubbly layer. The input and output of the signal are controlled by a signal analyzer unit (B&K 2035A) with a 25kHz input module (B&K 3019) and a generator module (B&K 3106) installed. From Fig. 8, it is seen that a sound signal is generated from the analyzer and is amplified by a power amplifier (B&K 2713) before emitted from the hydrophone. The received signal is transmitted back to the analyzer. The data are then read by a PC through an IEEE 488 interface for further analysis. The PC is also connected to two XYZ tables for controlling the movements of the hydrophones.

In order to study the free field behavior of the system, reflected waves from the tank walls must be eliminated. This is done by capturing the free field time interval i.e. the time before receiving the reflected waves. For sinusoidal waves, their amplitudes are measured. To obtain an accurate result, the wavelength and frequency of an acoustic signal should be much smaller than the tank dimensions and the sampling rate of the analyzer respectively. The free field time interval (known as the reverberation time) in the water tank is about 0.35ms (approximately 8.7 cycles for a 25kHz signal). A train of sinusoidal waves is emitted and experiments show that the wave will reach steady state within a cycle. At each point, 100 measures are taken in which the mean and the standard deviation are calculated.

To study the behavior of the system with a water surface, one must include the reflected wave from the free surface. Measurement is started from the instant after the incidence of the wave reflected from the surface to the moment before the arrival of the other reflected waves. The source is placed near the surface in order to increase the time interval. Similar approach can be applied to study the case with a solid bottom. Results for a 25kHz point source through a cylindrical bubbly layer with air volume fraction around 0.1 are shown from Fig. 9 to Fig. 14. The graph of the average transmission coefficients against different incident frequencies is plotted in Fig. 15.

DISCUSSION

The reason for choosing a thin bubbly layer as an underwater sound attenuator is that, if the thickness of a bubbly layer is larger than the incident wavelength, there is a layer resonance when the incident frequency coincides with one of the eigenfrequencies of the bubbly layer (Yeung & Chwang 1996). As a result, the transmission coefficient will be boosted up.

As seen from Fig. 2 to Fig. 7, the distortion of the transmitted pressure fields by the thin cylindrical bubbly layer is very small. This can be verified experimentally from Fig. 9 to Fig. 14. However, the layer does reduce the sound levels of the signal as illustrated from the figures. The resonant frequency of bubbles set for numerical calculation is 5kHz. If the incident frequency is above that resonant frequency, the bubbly layer reaches the "Dead Zone" with its range dependent on the air volume fraction as indicated in Eq. (2). The transmitted pressure calculated within the "Dead Zone" approaches zero which means that waves cannot propagate through the "Dead Zone" and nearly all of the energy is reflected back. This theory, supported by previous experimental results (Commander & Prosperetti 1988), may be valid for very dilute bubbly liquid in which the "Dead Zone" is very narrow.

For finite air volume fraction, experimental results indicate that dilute bubbly liquid theory cannot be applied. The transmission coefficients obtained in the experiments are much larger than those predicted by the theory. It seems that the "Dead Zone" phenomenon does not appear in the system. At finite air volume fractions, interaction between bubbles in the liquid cannot be neglected. Including the randomness and nonlinear effects of the system, bubbles may not be sensitive to the incoming waves. Furthermore, in reality, bubbles are not stationary but in random motions. As a result, pressures recorded at each point fluctuate. It is seen that the distributions are normal with their mean values obeying the inverse square law along the line. Due to the influence of a bubbly layer, pressure fluctuations are at maximum when the point is near the layer. The standard deviation decreases as the observing point moves away from the layer.

CONCLUSIONS

By using the classical theory for a dilute bubbly liquid, the transmission of an acoustic wave through a thin cylindrical bubbly layer is obtained. At frequencies higher than the bubble resonant frequency, due to the "Dead Zone" effect, the transmission coefficients are very low. The "Dead Zone" range depends on the air volume fraction of the liquid. However, for a bubbly liquid with finite air volume fraction, theoretical results cannot be applied and the transmission coefficients obtained experimentally are much higher than those predicted by the theory.

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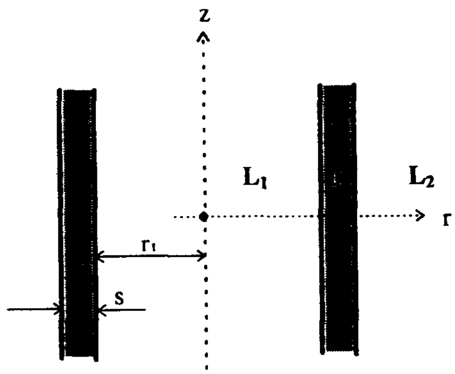

Fig. 1 A point source enclosed by a cylindrical bubbly layer

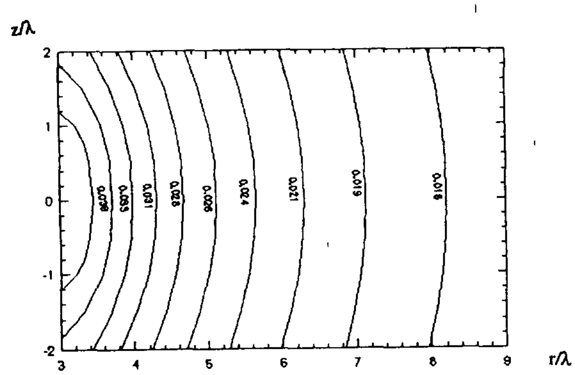

Fig. 2 A 2500Hz point source located at (0,0) in free field without a bubbly layer

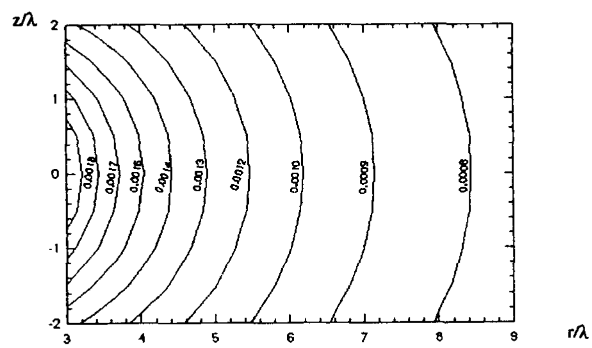

Fig. 3 A 2500Hz point source located at (0,0) in free field with a bubbly layer (β_o=0.001, f_o=3000Hz, r_1=1.8, s=0.2)

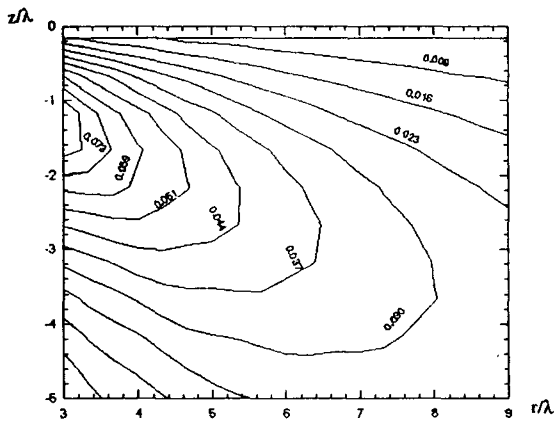

Fig. 4 A 2500Hz point source located at (0,-0.6) near a free surface without a bubbly layer

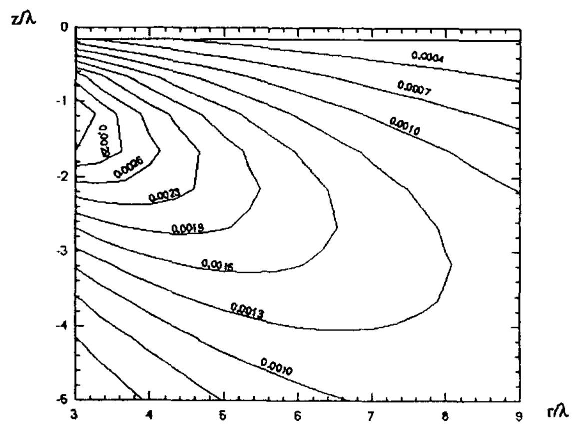

Fig. 5 A 2500Hz point source located at (0,-0 6) near a free surface with a bubbly layer (β_o=0.001, f_o=3000Hz, r_1=1.8, s=0.2)

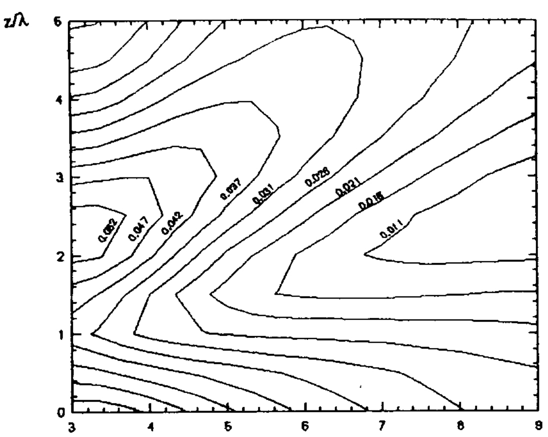

Fig. 6 A 2500Hz point source located at (0,0.6)
near a solid bottom without a bubbly layer

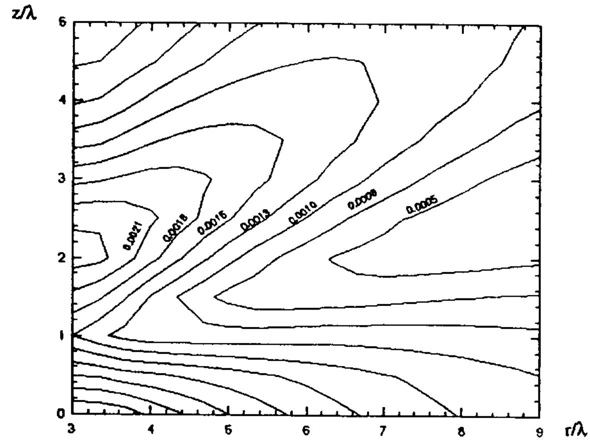

Fig. 7 A 2500Hz point source located at (0,0.6)
near a solid bottom with a bubbly layer
(β_o=0.001, f_o=3000Hz, r_l=1.8, s=0.2)

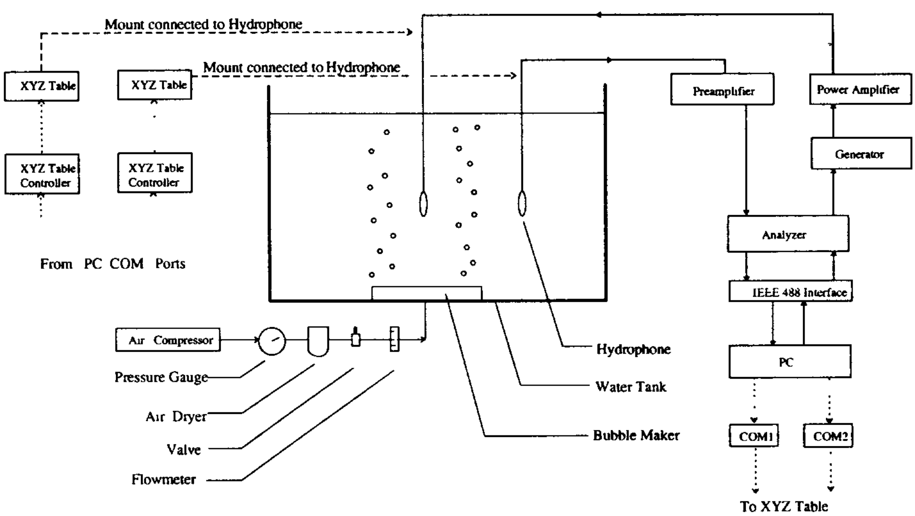

Fig. 8 Experimental setup

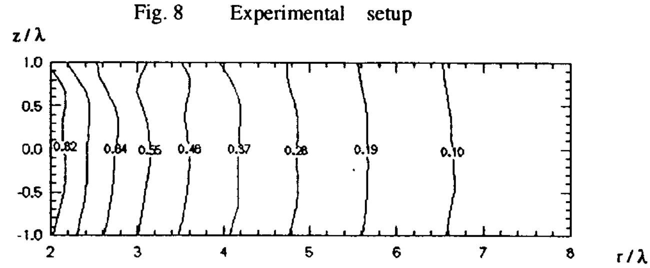

Fig. 9 Experimental result of a 25kHz point source located at (0,0)
in free field without a bubbly layer

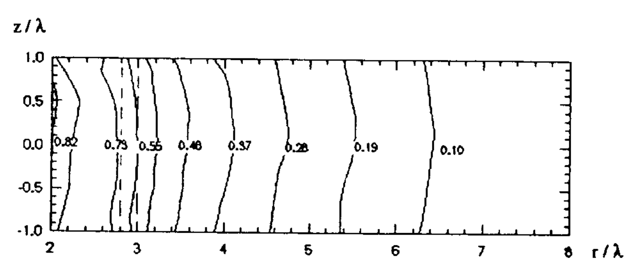

Fig. 10 Experimental result of a 25kHz point source located at (0,0)
in free field with a bubbly layer
(β_o~0.1, f_o~3000Hz, bubbly layer located between the dotted lines)

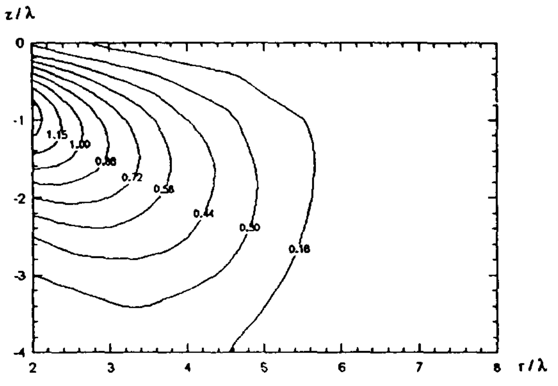

Fig. 11 Experimental result of a 25kHz point source located at (0,-0.6) near a free surface without a bubbly layer

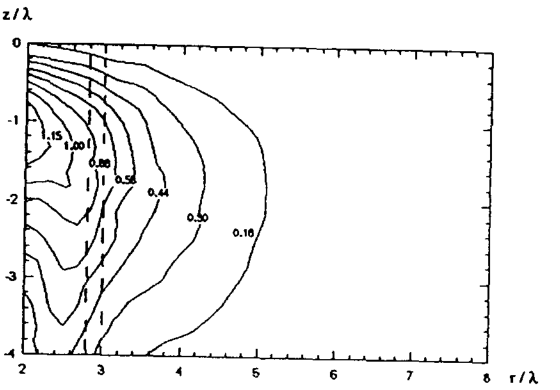

Fig. 12 Experimental result of a 25kHz point source located at (0,-0.6) near a free surface with a bubbly layer (β_0~0.1, f_0~3000Hz, bubbly layer located between the dotted lines)

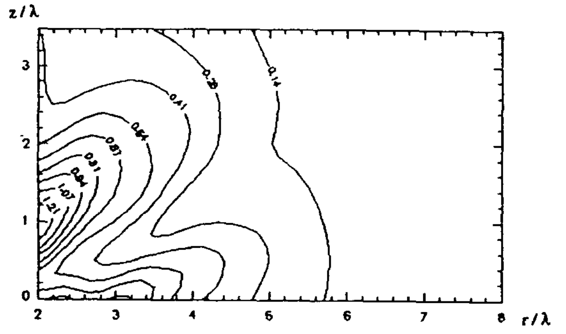

Fig. 13 Experimental result of a 25kHz point source located at (0,0.6) near a solid bottom without a bubbly layer

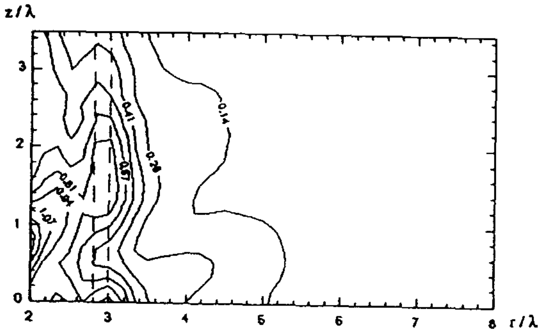

Fig. 14 Experimental result of a 25kHz point source located at (0,0.6) near a solid bottom with a bubbly layer (β_0~0.1, f_0~3000Hz, bubbly layer located between the dotted lines)

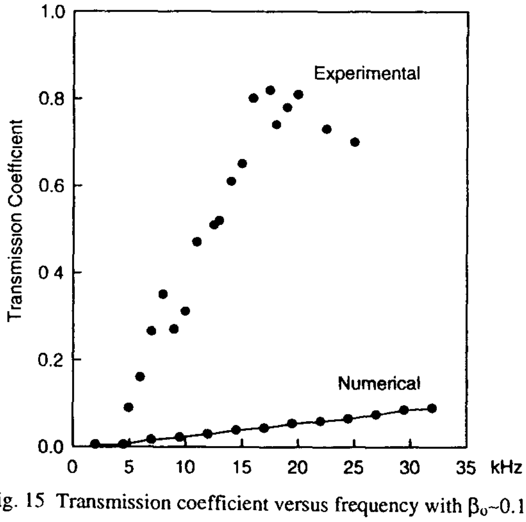

Fig. 15 Transmission coefficient versus frequency with β_0~0.1