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Invited Paper

HIGH FREQUENCY RAY ACOUSTICS MODELS FOR DUCT SILENCERS

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Hybrid modal/ray acoustics models for high frequency multimode sound propagation through finite-length dissipative duct silencers are being investigated and in this paper, the very simplest of these - involving no mean fluid flow, two dimensional sound propagation, a locally reacting duct liner and no area change in the silencer - is described. A mode matching scheme is also outlined. Numerical predictions of silencer attenuation from the hybrid model and the mode matching model are compared to experimental data taken from a two-dimensional silencer apparatus, and favourable agreement is noted.

1. INTRODUCTION

In predictive models for the acoustic attenuation of dissipative duct silencers both with and without mean fluid flow, it has often been assumed (see *e.g.* Cummings and Sormaz [1]) that the axial attenuation rate of the least attenuated acoustic mode will give a reasonable - possibly conservative - estimate of the silencer performance. Other, more complete, models (*e.g.* Mechel [2,3], Sormaz, Cummings and Nilsson [4], for finite length silencers without flow and Sormaz [5] for such silencers with flow) involve mode matching at the silencer's terminations, and allow for a specified multimode incident sound field. Finite element analysis has also sometimes been applied to finite length silencers. Either mode matching or fully numerical schemes can require considerable computational effort, and there may be some difficulty in identifying and tracking modes in the former case. At high frequencies, where there are many incident modes, the problem may become intractable and one is led to wonder whether a simpler approach, such as ray acoustics, can produce results of acceptable accuracy.

Ray acoustics models for duct acoustics have been investigated by several workers, such as Tester [6,7], Kempton *et al.* [8,9] and Boyd *et al.* [10]. Tester [6] compared ray acoustics and modal predictions of the sound field from a two dimensional line source in a duct with zero flow and locally reacting walls, and concluded that the ray acoustics models were "surprisingly" accurate. Kempton *et al.* [9] compared mode theory and ray theory predictions of the insertion loss of a fan jet engine inlet silencer to measured data and noted good agreement under certain conditions, though he concluded that further research on interference and diffraction effects in the ray theory was needed. Boyd *et al.* later presented ray theory predictions in which these effects were taken into account, for a two-dimensional duct with zero flow, and noted generally good agreement with other results.

In this paper, a very simple mode/ray formulation for sound attenuation in a finite length dissipative silencer containing zero flow is described, and the results are compared to those from a mode-matching treatment and measured data.

2. A HYBRID MODE/RAY MODEL FOR A DISSIPATIVE SILENCER

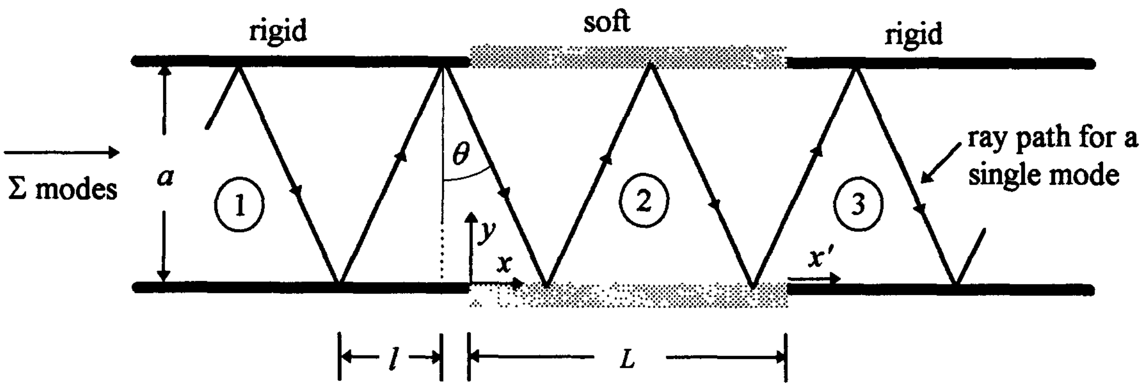

Figure 1. Finite length duct silencer with ray path.

In Figure 1 is shown a section of two-dimensional duct with a length L of soft, locally reacting, walls (the "silencer", region 2) and rigid inlet and outlet sections (regions 1 and 3 respectively). The duct is - for convenience - assumed to be anechoically terminated at both ends. A multimode sound field of radian frequency ω is incident from the left. Each propagating mode consists of the sum of two plane waves (treated here as rays), and one ray path - for a single mode - is shown. The other ray path for this mode consists of a vertical mirror image of that shown. The incident sound field is given as a summation of modes by

$$p(x,y;t) = e^{i\omega t} \sum_{m=0}^{\infty} P_m e^{-ik_m x} \cos(m\pi y / a), \qquad (1)$$

where m is the mode order and k_m the axial wavenumber, $\sqrt{[k^2 - (m\pi / a)^2]}$, k being the acoustic wavenumber, ω / c, and c the sound speed. The component rays of each mode propagate at angle $\theta = \cos^{-1}(m\pi / ka)$ to the y axis. Let us neglect any diffraction effects at the silencer inlet and outlet, and assume that purely specular reflection (at angle θ) of the rays occurs within both the rigid and soft duct sections. This assumption will be progressively better as the frequency increases. If the *dimensionless* normal impedance of the soft walls (referred to the characteristic impedance of air and equal for both walls) is $z_w = r_w + ix_w$, we may easily show that the sound power reflection coefficient of a ray from the duct walls is

$$\tau_\theta = [(r_w \cos\theta - 1)^2 + (x_w \cos\theta)^2] / [(r_w \cos\theta + 1)^2 + (x_w \cos\theta)^2]. \quad (2)$$

For a particular mode, the average number of reflections of its plane wave components from the duct walls within the length L is $n = L / a \tan\theta$. Then the space-averaged mean-squared sound pressure of the mode at the silencer outlet is given in terms of that at the inlet by

$$\langle \overline{p_m^2} \rangle_{out} = \tau_\theta^n \langle \overline{p_m^2} \rangle_{in}. \quad (3)$$

We note that, for the fundamental mode ($m = 0$), $n = 0$ and so this mode should proceed unattenuated. While this is not, of course, the case in practice, the attenuation rate of the fundamental mode is usually significantly lower, in the multimode region, than that of most of the higher modes. Furthermore, when many modes propagate, only a small proportion of the incident sound power is likely to be contained in the fundamental mode. This means that the error in predicted multimode attenuation brought about by the underestimate of fundamental mode attenuation will be small.

It is of interest to be able to predict both the multimode sound pressure level insertion loss *IL* (equal - in this model where there are no reflected waves - to the dB difference between the *y*-space-averaged mean-squared pressures at the silencer inlet and outlet) and the sound power transmission loss *TL*, defined in the usual way. Both quantities will, of course, depend on the relative amplitudes of the modes in the incident sound field. Perhaps the simplest assumption is that $\langle \overline{p_m^2} \rangle_{in}$ is independent of m. The *y*-space-averaged mean-squared pressure in the multimode inlet sound field is given simply as the sum of the space-averaged modal mean-squared pressures, over the number of propagating modes $N = \text{Int}(ka/\pi) + 1$ (Int(x) signifying truncation to the value of the largest integer less than or equal to x), since cross-terms between modes integrate to zero in the spatial averaging. For this assumption about the incident sound field, we have the result

$$IL = -10\log\left\{\frac{1}{N+1}\sum_{m=0}^{N}\{[(r_w - ka/m\pi)^2 + x_w^2]/[(r_w + ka/m\pi)^2 + x_w^2]\}^{L/a\sqrt{(ka/m\pi)^2-1}}\right\}. \quad (4)$$

For the comparison between experiment and prediction that will later be presented, however, we require the incident sound field generated by a point volume source on one of the duct walls, since this is readily predictable and fairly easily measurable.. In this case,

$$\langle \overline{p_m^2} \rangle_{in} = \langle \overline{p_0^2} \rangle (k/k_m)^2 / \Lambda_m, \text{ where } \Lambda_m = 1, m = 0; = 1/2, m \neq 0 \quad (5)$$

and $\langle \overline{p_0^2} \rangle$ is the mean-squared sound pressure in the fundamental mode. A slightly more complicated formula thaneq. (4) results,

$$IL = -10\log\left\{\frac{\sum_{m=0}^{N}\left(\frac{k}{k_m}\right)^2 \frac{1}{\Lambda_m}\left[\frac{(r_w - ka/m\pi)^2 + x_w^2}{(r_w + ka/m\pi)^2 + x_w^2}\right]^{L/a\sqrt{(ka/m\pi)^2-1}}}{\sum_{m=0}^{N}\left(\frac{k}{k_m}\right)^2 \frac{1}{\Lambda_m}}\right\}. \quad (6)$$

The transmission loss may be found by summing the modal sound powers in the incident and transmitted sound fields, and the result is

$$TL = -10\log\left\{\frac{\sum_{m=0}^{N}\left(\frac{k}{k_m}\right)\frac{1}{\Lambda_m}\left[\frac{(r_w - ka/m\pi)^2 + x_w^2}{(r_w + ka/m\pi)^2 + x_w^2}\right]^{L/a\sqrt{(ka/m\pi)^2-1}}}{\sum_{m=0}^{N}\left(\frac{k}{k_m}\right)\frac{1}{\Lambda_m}}\right\}. \quad (7)$$

Equivalent expressions have been found for three dimensional ducts with differing materials on opposite pairs of walls, but these are not given here for the sake of brevity. Equs. (6) and (7) are very simple in the light of the complexity of the incident sound field and the silencer geometry (albeit in only two dimensions), and it remains to be seen how well they compare to a more complete analysis and to experimental data.

3. A MODE-MATCHING FORMULATION

Here, we write expressions for the sound fields in regions 1,2 and 3 respectively as

$$p_1 = e^{i\omega t}\sum_{m=0}^{\infty}\Psi_m(y)(P_{1i}^m e^{-i\alpha_m x} + P_{1r}^m e^{i\alpha_m x}), \quad (8a)$$

$$p_2 = e^{i\omega t}\sum_{m=0}^{\infty}\Phi_m(y)(P_{2i}^m e^{-i\beta_m x} + P_{2r}^m e^{i\beta_m x}), \quad (8b)$$

$$p_3 = e^{i\omega t}\sum_{m=0}^{\infty}\Psi_m(y)P_{3i}^m e^{-i\alpha_m x'}, \quad (8c)$$

where $\Psi_m(y) = \cos(m\pi y/a)$ and $\Phi_m(y) = \cos(\gamma_m y) + A_m \sin(\gamma_m y)$, γ_m being the transverse wavenumber in the silencer section and A_m a constant. Subscripts *i* and *r* denote incident and reflected waves respectively. The duct is (as before) assumed to be anechoically terminated at both ends, so that not only are there no reflected modes in the silencer outlet section, but the reflected modes in the inlet section do not return to the silencer. We find γ_m from solutions to the eigenequations

$\tan(\gamma_m a/2) - iz_w\gamma_m/k = 0$ (odd modes); $\cot(\gamma_m a/2) + iz_w\gamma_m/k = 0$ (even modes). (9a,b)

These equations were solved by the use of Muller's method. Mode-matching was achieved by carrying out a least-squares match of sound pressure and axial particle velocity at $x = 0$ and $x = L$. In the case of sound pressure, the square of the sound pressure jump for a finite sum of $M+1$ modes at (for example) $x = 0$ was integrated across the duct width and then minimised *w.r.t.* P_{2i}^n. The jump in axial particle velocity at $x = 0$ was treated in the same way. The same process was carried out at $x = L$, but now the minimisation was *w.r.t.* P_{2r}^n. Taking $n = 0,1,\ldots.M$ at $x = 0$ gave rise to a set of $2M+2$ linear equations in the reflected and transmitted modal coefficients, assuming all incident modal coefficients in region 1 and reflected modal coefficients in region 2 were specified. The incident coefficients were readily found for a point source located on one of the duct walls a given distance from the inlet to the silencer. Initially, all P_{2r}^m values were equated to zero, and the set of equations was solved. The P_{2i}^m values were used for the set of incident modes at $x = L$, and the appropriate set of equations for the reflected and transmitted modal coefficients (again $2M+2$) was solved. The P_{2r}^m values were then used in the equations at the inlet to find new P_{2i}^m values, and the process was repeated until the modal coefficients ceased to change significantly. This iterative process reduced the number of equations to be solved simultaneously by a factor of two and, since only a small number of iterations was required, considerably reduced the computation time. A sufficiently large value of M was taken, so that the solution had converged adequately.

Once the modal coefficients had been determined, it was a straightforward matter to compute both the *IL* and *TL* of the silencer. The *IL* was found from the coefficients of the propagating modes incident on, and transmitted by, the silencer. In the mode-matching model, modes are reflected from the silencer inlet and outlet, in contrast to the ray model, though as far as the *IL* is concerned, their role is only in determining the transmitted modal coefficients.

4. EXPERIMENTS

To verify the predictions of the ray model by comparison to experimental data, a two-dimensional duct test apparatus was fabricated. A plan view of this is shown in Figure 2. The

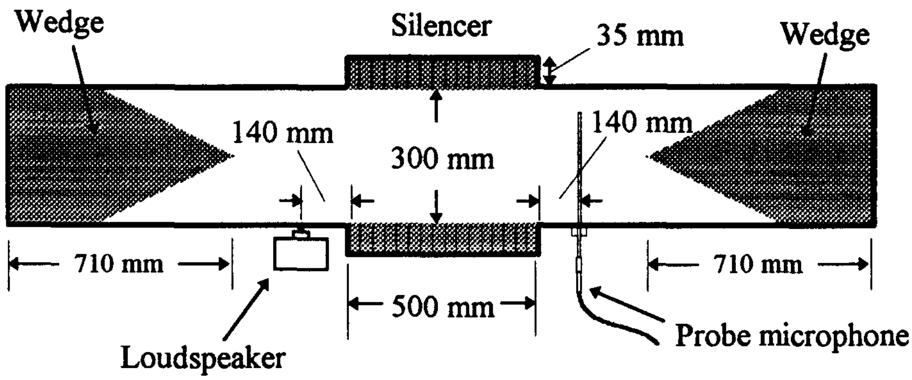

Figure 2. Experimental duct apparatus.

height of the duct was 25 mm, enabling two-dimensional mode propagation to occur up to about 6.9 kHz. Because the source loudspeaker - a medium-sized pressure driver - was connected to the duct by a 10 mm diameter hole located halfway along the 25 mm duct wall (this arrangement being intended to simulate the aforementioned point source), only even modes in this dimension should, in theory, have been excited and consequently the frequency range for two-dimensional modes was from 0 to 13.8 kHz. Non-idealities in construction of the duct would, in reality, impose a rather lower limit and experimental data were taken only up to 7.5 kHz. The soft walls in the silencer section consisted of a 35 mm thickness of partially reticulated polyurethane foam with a steady flow resistivity of 4390 SI rayl/m. The bulk acoustic properties of the foam - required to determine the normal surface impedance of the liner - were found by means of an impedance tube, from tests on two differing thicknesses of absorbent. The foam was cut into sections 25 mm wide, and these were separated by thin aluminium baffles placed transverse to the duct axis, to prevent wave propagation along the liner and render it essentially point reacting. These baffles would cease to be completely effective much above about 7 kHz, although no really sudden departure from point reacting behaviour would be anticipated at rather higher frequencies.

Random noise was fed from an amplifier to the speaker and the sound field was sampled by means of a probe microphone at 20 mm intervals. Single frequency sound pressure level data (in the form of a transfer function, referred to the voltage input to the speaker) were obtained from an FFT analyser and spatially averaged mean squared pressure figures were found at each frequency. The silencer section could be removed, thereby permitting measurement of the sound pressure level with no silencer present. The *IL* of the silencer could thus be found directly by means of this duct apparatus.

5. COMPARISON BETWEEN *IL* PREDICTIONS AND MEASUREMENTS

The normal surface impedance of the silencer liner was predicted from the bulk acoustic properties of the foam, as $z_w = z_a \coth \Gamma \ell$, z_a and Γ being the dimensionless characteristic

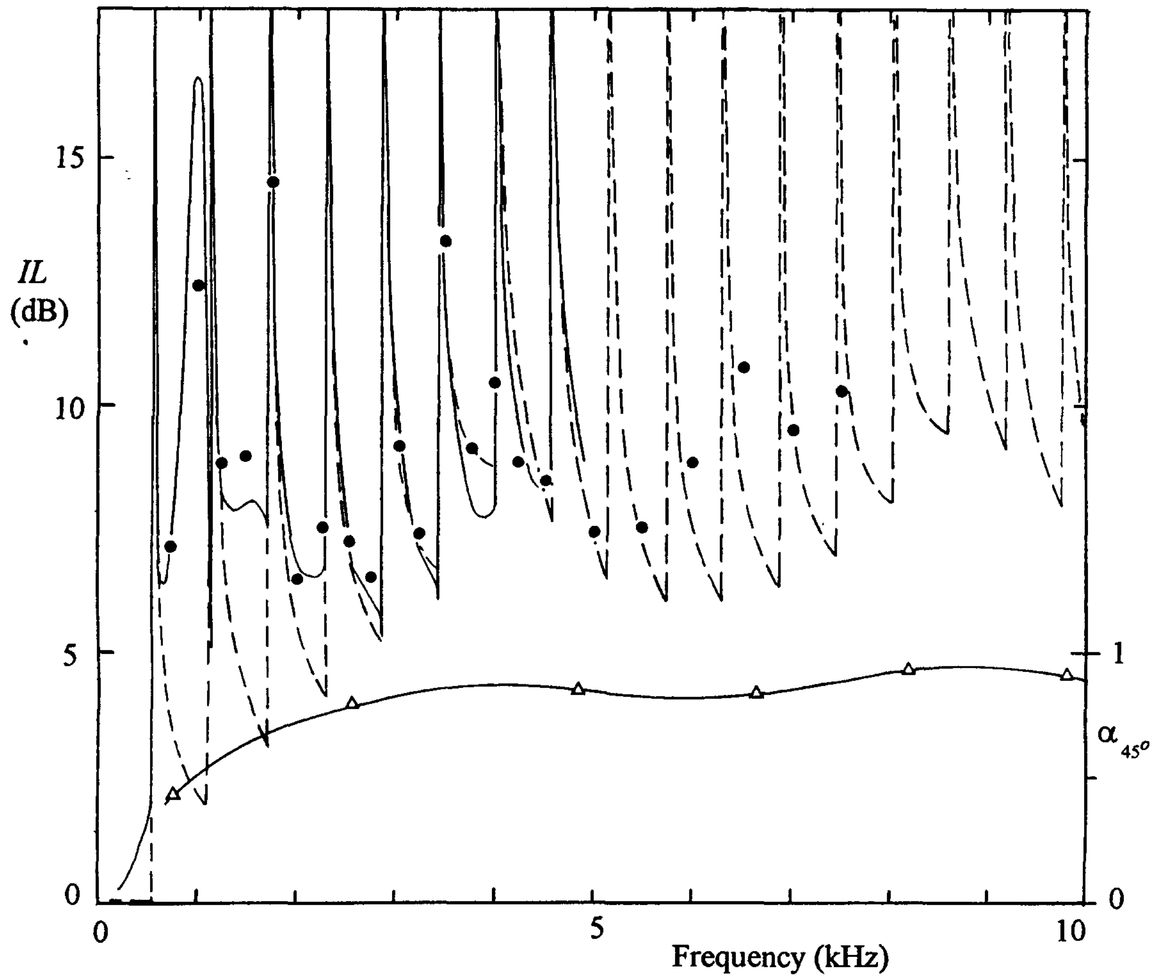

Figure 3. Predicted and measured *IL* of the silencer: ———, mode-matching predictions; — — —, ray theory; ●, measurements. Δ———Δ, α_{45^o} of liner.

impedance and the dimensional propagation coefficient respectively, and ℓ the liner thickness. Predictions were made of the *IL* of the silencer from 100 Hz to 10 kHz by the ray theory, and from 200 Hz to 4.9 kHz by the mode-matching theory. This upper frequency limit was imposed by numerical difficulties in the mode-matching, that had not been resolved at the time of writing. Comparison is made between the two *IL* predictions and measured data in Figure 3. Also shown is the predicted sound power absorption coefficient ($\alpha_\theta = 1 - \tau_\theta$) of the liner for plane waves incident at 45°.

We can see that the *IL* predicted by both methods exhibits a series of (theoretically infinite) peaks at the cut-on frequencies of higher modes in the inlet and outlet ducts. The minimum *IL* values, between these peaks, are all less than about 10 dB. The mode-matching predictions are in excellent agreement with the measured data up to 4.5 kHz (at which frequency 8 modes propagate). Up to 2.5 kHz, the ray theory does no predict the minimum *IL* values very accurately, but this would not be expected in view of the various approximations upon which it relies. Above this frequency, with five or modes propagating, its agreement with the mode-matching theory is good. Above 5 kHz, the ray theory is in satisfactory agreement with the measured data. We note undulations in the envelope of the minimum values of the *IL*, more obviously in the ray theory predictions. These can be qualitatively explained by examining the curve of α_{45^o}, which tends to follow the same pattern. An angle of 45° is, perhaps, representative of an average figure for all propagating modes. We can see from eq. (6) that a high wall absorption coefficient implies a high *IL* figure, at least on the basis of the ray model. The numerical predictions from the mode-matching model appear to follow this trend too, in the range 2.5-5 kHz.

6. DISCUSSION AND CONCLUSIONS

The very simple multimode ray model that has been described has given gratifyingly accurate predictions of the *IL* as compared to the more complete mode-matching model and measured data, in a particularly difficult comparison involving a point sound source. In more practical situations, one might assume equal modal power in the incident sound field, and here the agreement might be even better, although it would be difficult to carry out experiments compatible with this assumption. The loss of predictive accuracy in the ray model would seem to be more than offset by the great ease with which it may be employed. The ray model involves a much more "physical"- though less precise - description of the sound field in a silencer than that embodied in the modal formulation (or, for that matter, in a fully numerical treatment such as a finite element analysis).

As mentioned earlier, the two dimensional ray treatment has been extended to three dimensions (though the results are not given here), and the resulting formulae are still very straightforward. The three dimensional model has not yet been experimentally verified. Mean gas flow effects can, in principle, readily be included in the ray model, and in order to apply ray models to practical devices such as splitter silencers, geometrical effects such as area blockage also need to be included. More complicated devices such as offset banks of splitters might also be amenable to ray treatments. Particularly if the sound fields in the inlet and outlet ducts can be represented as modal summations, the considerable flexibility of ray models may well permit the formulation of hybrid treatments such as that described here.

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