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SURFACE MOBILITY FOR A RECTANGULAR CONTACT REGION UNDER A UNIFORM VELOCITY DISTRIBUTION

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ABSTRACT In the study of structure-borne sound and vibration isolation, mobility is used to reflect the characteristics of power transmission of a supporting structure. When the contact area between an exciting machine and its supporting structure is larger than the governing wave length, surface mobility is applied. Surface mobility is influenced by many factors such as the pattern and frequency of excitation and the dimensions and shape of the contact area. By means of analyses of these factors, the effects of power transmision through the contact area can be obtained under different conditions. In this paper, the effective point mobility and surface mobility over rectangular contact areas, for the assumption of a uniform conphase velocity distribution, are studied. An infinite homogeneous thin plate is chosen as a supporting structure. The resulting force distribution and effective point mobility within the contact area are calculated. Using effective point mobility, surface mobility is also calculated. By this method, not only is the total power transmission calculated from an excitor to the thin platelike support, but also the detailed pattern of power transmission within the contact areas can be predicted.

1. INTRODUCTION

In the study of vibration isolation between an exciting machine and a supporting structure, mobility is used to reflect the characteristics of the supporting structure and hence the transmission of vibrations and structure-borne sound into the supporting structure. Point mobility can readily measured and, in some instances, easily calculated, however in most practical cases contact occurs over a significant surface area. Work reported here is part of an on-going study into the effects of large area contact betweeen a machine and its supporting structure. Hammer and Petersson [1] developed the concept of surface mobility and applied it to a strip-like contact area. Subsequent work by the current authors [2-5] has applied the concept of surface mobility to circular and rectangular contact regions. These studies have shown that there are significant differences between point mobility and various cases of surface mobility. Hence the development of methods to predict and measure surface mobility is an important aspect of the control of structure-borne sound and vibration.

In the current study, as in previous ones, theoretical calculations of surface mobility have been developed assuming that the supporting structure is an infinite thin plate subject to bending waves. The surface mobility over circular contact areas was studied by Norwood, Williamson and Zhao [2,3] for a variety of contact conditions and assumptions. Rectangular contact regions subjected to uniform conphase force distributions have been studied previously using the complex power [4] and the effective point mobility [5] approaches to surface mobility. The current study uses the effective point mobility concept to study rectangular contact regions subjected to a uniform conphase velocity distribution.

Different contact conditions between an exciting machine and its supporting plate will lead to different force and velocity distributions over the contact region. When the contact region is formed between a very soft compliant isolator and a stiff supporting plate, the force distribution may approximately be uniform and conphase. Alternatively for a relatively stiff isolator and a more flexible supporting plate, a uniform and conphase velocity distribution will be a closer approximation to the true contact conditions. In the study reported below surface mobility is explored assuming a uniform conphase velocity distribution over rectangular contact areas for an infinite plate. This complements previous studies of surface mobility over rectangular contact areas where uniform force distributions were assumed [4,5].

2. BASIC THEORY

The supporting plate is assumed to be an infinite, homogeneous thin plate in order to simplify the analysis. Energy loss is also neglected in the calculations. The thin plate assumption implies that the thickness of the plate is only a fraction of the governing wavelength. Thus the bending wave equation for homogeneous thin plate is valid [6]. This equation is rewritten as

$$\Delta\Delta v - k^4 v = \frac{j\omega}{B}\sigma(x,y) \tag{1}$$

where v represents the spatial transverse velocity of the plate; k is the bending wave number; B the flexural stiffness of the plate; $\sigma(x,y)$ the force per unit area at point (x,y); Δ the two dimensional Laplace operator. The general solution of equation (1) for arbitrary given force distributions is a combination of cylindrical Hankel functions of the second kind [6] as

$$v(x,y) = \iint_S M_0 \sigma(x_0,y_0)\Pi(kr)dx_0 dy_0 \tag{2}$$

where M_0 is the ordinary point mobility; r is the distance between the excitation point (x_0,y_0) and the observation point (x,y); S is the excitation region; and $\Pi(kr)=H_0^{(2)}(kr)-H_0^{(2)}(-jkr)$ is the propagation function defined as the difference between two Hankel functions.

For a rectangular contact area firmly attached to an infinite, homogeneous thin plate, according to the assumption of a prescribed uniform conphase velocity distribution, a mixed boundary condition must be handled. Thus equation (2) becomes an integral equation of Fredholm type of the first kind in terms of unknown force distribution. To solve this integral equation problem, a force distribution which should meet the uniform velocity boundary conditions must be found. Polynomials of Chebychev type and trigonometric type may be employed to approximate the force distribution. Such approaches, however, are not very flexible for plates of arbitrary shape. Therefore, a discretised model is used in this study to calculate the force distribution.

3. DISCRETISED MODEL

The aim of this study is to explore rectangular contact areas, hence a discretised model was set up as shown in Figure 1.

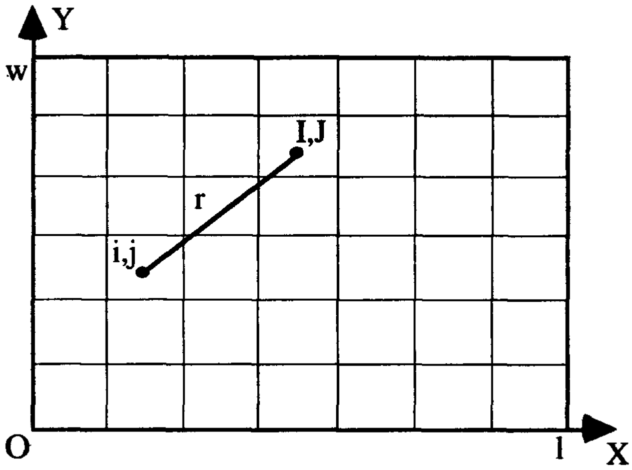

Figure 1 The Discretised Model of a Rectangular Contact Area

The force distribution is approximated by assuming a constant force distribution in each subregion. The centre of the subregion represents the point at which the force and the velocity act. l is the length of the rectangular contact area; w the width; n the number of row of the subregion and m the number of the column. Thus, the length and width of each subregion are dl=l/m and dw=w/n respectively. The force F over subregion (I,J) is equal to

$$F(I,J)=\int_{-dw/2}^{dw/2}\int_{-dl/2}^{dl/2}\sigma(I,J)dxdy \tag{3}$$

The response to this force at subregion (i,j) can be approximated by assuming that a point force, F(I,J), acts at the centre of subregion (I,J). Hence v(i,j|I,J), the velocity at (i,j) caused by the force at (I,J) is given by [6]:

$$v(i,j|I,J)=M_0F(I,J)\Pi(kr) \tag{4}$$

Therefore the velocity at subregion (i,j) caused by all forces over the contact region can be obtained

$$v(i,j)=\sum_{I=1}^{n}\sum_{J=1}^{m}M_0F(I,J)\Pi(kr) \tag{5}$$

where r is the distance between the two central points of subregion (i,j) and (I,J).

The velocities of all subregions thus can be shown as below

$$v(1,1)=\sum_{I=1}^{n}\sum_{J=1}^{m}M_0F(I,J)\Pi(kr)$$

$$v(1,2)=\sum_{I=1}^{n}\sum_{J=1}^{m}M_0F(I,J)\Pi(kr)$$

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$$v(n,m)=\sum_{I=1}^{n}\sum_{J=1}^{m}M_0F(I,J)\Pi(kr)$$

which can be written in a matrix form of

$$\{\bar{v}\}=[T]\{\bar{F}\} \tag{6}$$

where $\{\overline{v}\}$ consists of v(i,j), i=1,n, j=1,m; $\{\overline{F}\}$ consists of F(I,J), i=1,n, j=1,m. Both of $\{\overline{v}\}$ and $\{\overline{F}\}$ are column vectors which have nxm elements; [T] is a square transfer matrix with nxm rows.

To fulfil the assumption of a uniform velocity distribution, the velocity over the whole contact area should be a uniform and in phase. Without loss of generality a unit velocity is assumed, hence

$$\begin{aligned} \mathrm{Re}[v(i,j)] &= 1 \\ \mathrm{Im}[v(i,j)] &= 0 \end{aligned} \qquad \text{for } i=1,n \text{ and } j=1,m \tag{7}$$

Thus the force distribution over the contact area, $\{\overline{F}\}$, can be calculated by equation (6).

4. EFFECTIVE POINT MOBILITY AND SURFACE MOBILITY

Using the effective mobility definition of surface mobility [1-3], the effective point mobility at subregion (i,j) can be expressed by

$$M^e(i,j) = \frac{v(i,j)}{F(i,j)} \tag{8}$$

As known from condition of equation (7) that v(i,j)=1, effective point mobility can be obtained as

$$M^e(i,j) = \frac{1}{F(i,j)} \tag{9}$$

The total complex power over the whole excited contact area can be written as

$$Q = \frac{1}{2}F^*V = \frac{1}{2}|F|^2 M^s \tag{10}$$

where F is the total force acting over the whole contact area and M^s is the surface mobility over that area. The total complex power can also be expressed in terms of the forces acting over the subregions and the effective point mobilities at these areas as

$$Q = \frac{1}{2}\sum_{j=1}^{m}\sum_{i=1}^{n} F(i,j)^* v(i,j) = \frac{1}{2}\sum_{j=1}^{m}\sum_{i=1}^{n} |F(i,j)|^2 M^e(i,j) \tag{11}$$

Combining equation (10) and (11), the surface mobility can be obtained, based on the effective point mobility, as

$$M^s = \frac{1}{|F|^2}\sum_{j=1}^{m}\sum_{i=1}^{n} |F(i,j)|^2 M^e(i,j) \tag{12}$$

5. CALCULATION AND DISCUSSION

The theory derived in the previous section has been applied for uniform velocity excitation over a rectangular contact area which aspect ratio of width w to length l is 1:2. Results for the cases for various Helmholtz number, kw, are presented and discussed below.

5.1. FORCE DISTRIBUTION

Using equation (6), the forces over the rectangular contact areas are calculated by dividing the contact region into subregions of m by n. The results are shown in Figures 2 and 3.

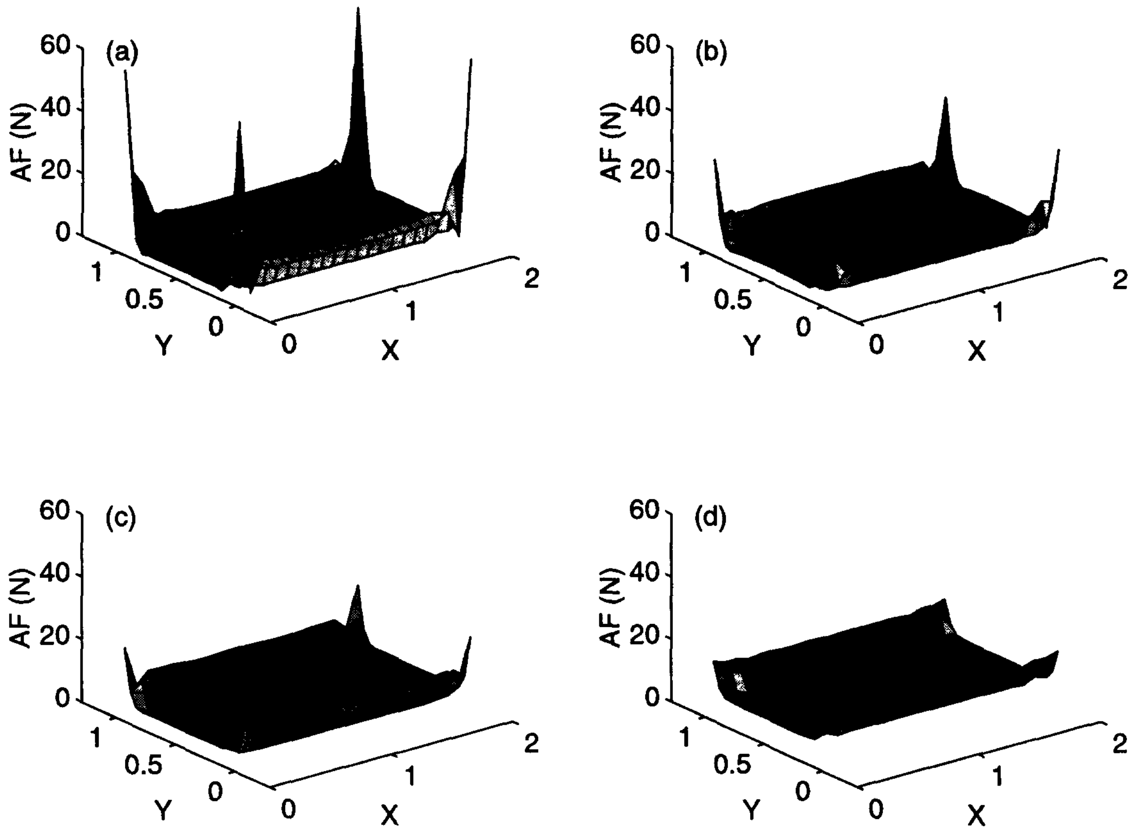

Figure 2 The amplitude of the force (AF) over a rectangular contact area (X=normalised length, Y=normalised width) for various values of kw: (a) kw=1; (b) kw=5; (c) kw=10; (d) kw=20.

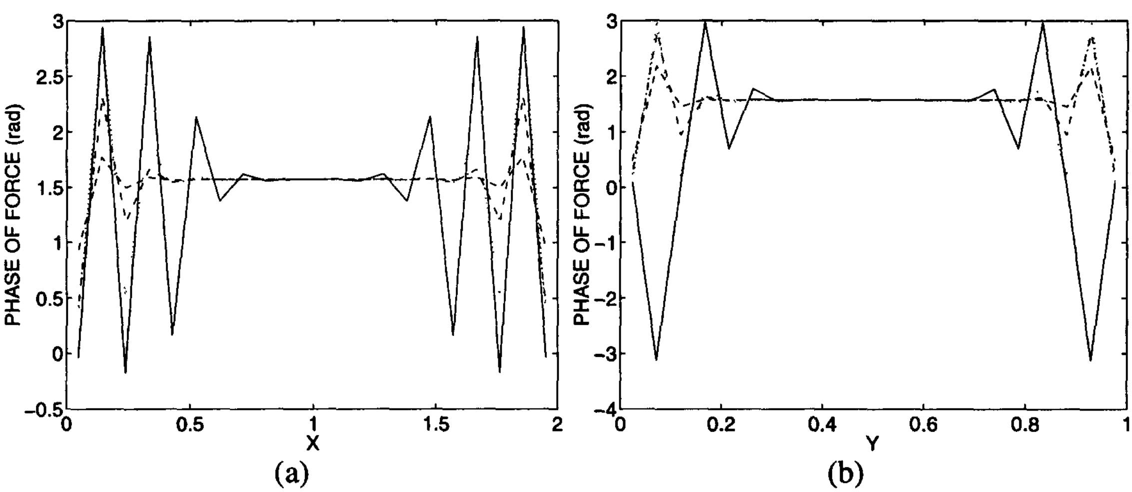

Figure 3 The phase of the force of a rectangular contact area along a mid-line in the contact region: solid line-kw=1; dotted line-kw=5; dashdot line-kw=10; dashed line-kw=20. (a) Along X direction at Y=0; (b) Along Y direction at X=0.

Figure 2 shows the amplitude of the force distributed over a rectangular contact area at different values of kw. Since $k=2\pi/\lambda$, kw is proportional to the ratio of the contact width, w, to the bending wave length, λ. From Figure 2 it can be seen that the values of the force acting along the edges of the contact area are much higher than that over the central part of the contact area. The forces at the corners are the largest. The force distributions over the rectangular contact area are also symmetrical. Most of the force is distributed along the edges. Due to the effect of the aspect ratios of the rectangular contact area, it can be seen easily that the force acting along the long edges are much larger than that acting along the short edges. As kw increases, the values of the forces over the central part of the contact area become larger and larger, but the maxima of the forces decrease.

Figure 3 shows the phases of the force along a central cross section in different directions. In Figure 3, the changes of the phases of the force are illustrated for different values of kw as well. As kw increases, the changes of the phases trend to be smooth in the central part of the contact area.It also can be seen that phase oscillates around $\pi/2$. The forces at the edges of the contact region in Figure (a) and (b) of Figure 3 have zero phase. Moving inwords from an edge of the contact region, the phase rises rapidly from 0 to π, 180° out of phase, then oscilates about $\pi/2$, 90° out of face. In the central region a steady phase of $\pi/2$ predominates.

5.2. EFFECTIVE POINT MOBILITY OVER THE CONTACT AREA

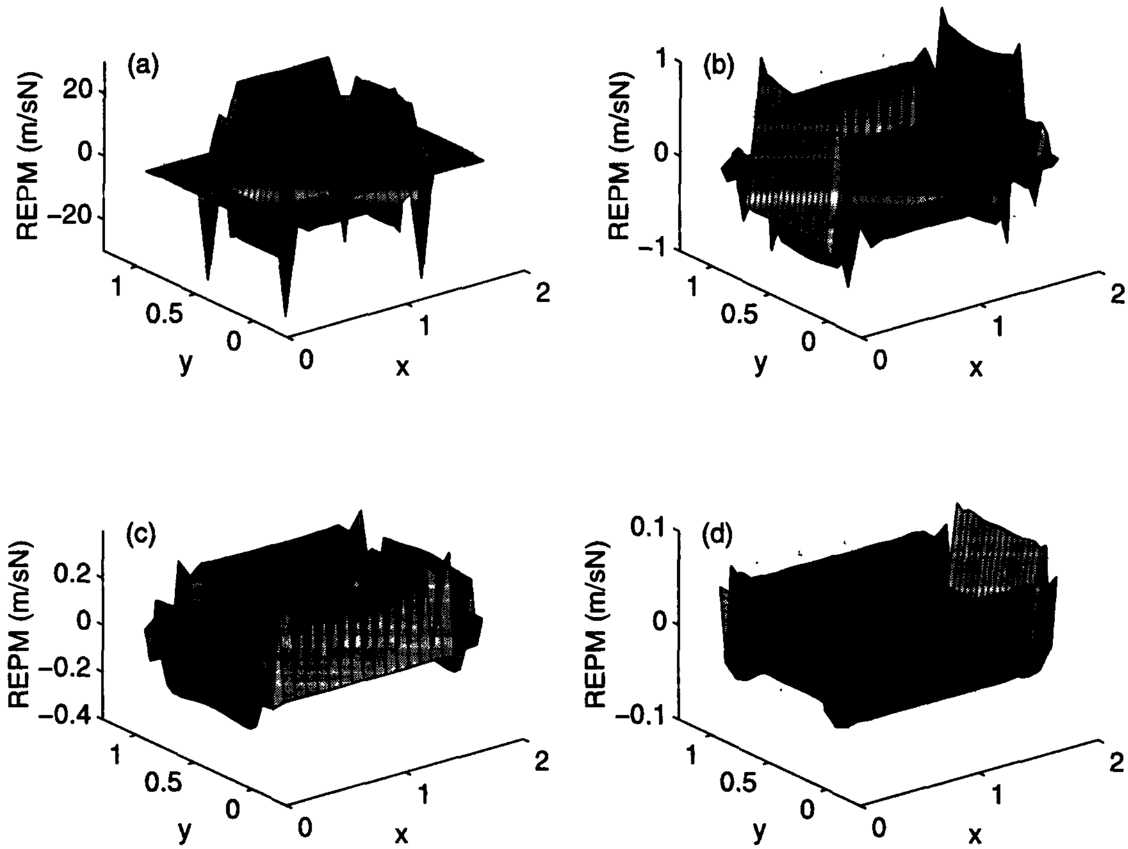

Figure 4 The real part of the effective point mobility (REPM) over a rectangular contact area (X=normalised length, Y=normalised width) for various values of kw: (a) kw=1; (b) kw=5; (c) kw=10; (d) kw=20.

For the case under study, effective point mobility has been calculated using equation (9) under the assumption of uniform conphase velocity distribution. Note that the real part of the effective point mobility reflects the amount of energy transferred at a point. Hence Figures 4 shows both the distribution of power transmission and force over the contact areas.

Figures 4 shows the distribution of the real part of the effective point mobility over a rectangular contact area in which the aspect ratio is l/w=2 for different values of kw. From Figure 4 it can be seen that the distributions of the effective point mobility are symmetrical about the central axes of the contact area, but take different values in the x and y directions. This means that the cases of power transmission are different in the two directions. When kw is small, the power transmitted is more in the x direction than in the y direction. As kw becomes larger and larger, the distribution of the mobilities trends to be more evenly distributed. The region around the edge has very small values of the effective point mobility. However large values, that is regions of significant power transfer, occur nearer the centre. As kw increases, the main region of transmission expands outward and the values become smaller and smaller. This implies that the main power of transmission region of the supporting plate is a ringlike region which moves outward with increasing kw. In addition, the Figures also show that the value can be positive or negative which means that there exist some points through which power is transmitted into the supporting plate (positive value) and the other points where power is transmitted back from the supporting plate to exciting machine (negetive value). The values of effective point mobility decrease rapidly with increasing kw, i.e. as the wavelength becomes short, the power input reduces.

5.3. SURFACE MOBILITY

Surface mobility based on a uniform conphase velocity distribution can be obtained by equation (12). Figure 5 gives the result of calculation of real part of surface mobility over a rectangular contact area normalised to the ordinary point mobility. This results is similar to that calculated under the assumption of uniform conphase force distribution except that in the uniform force case, periodic dips in surface mobility occur [4]. It also can be seen that, as kw increases, the ability to transmit power into the contact area of the supporting plate decreases markedly.

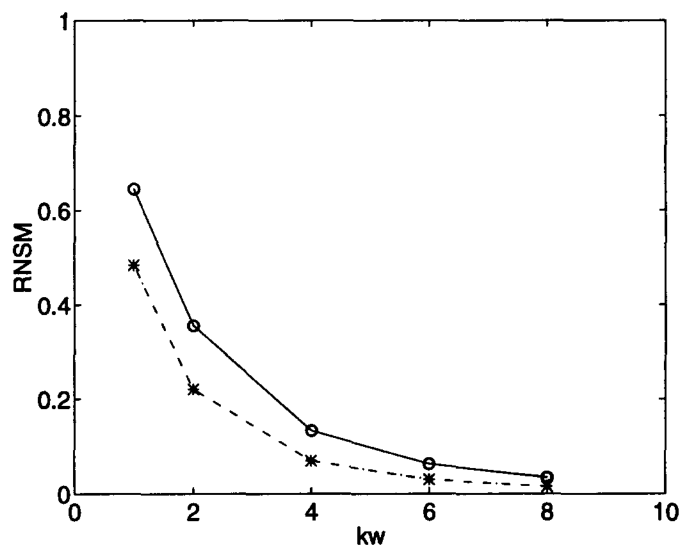

Figure 5 The real part of normalised surface mobility (RNSM): *-l/w=2; o-l/w=1.

6. CONCLUSION

A method of calculating the force distribution and surface mobility for a uniform conphase velocity over a two-dimensional rectangular contact area on an infinite thin plate has been developed. Using this method, the force distribution, effective mobility and surface mobility have been calculated and investigated by a discretised model.

The results of these calculations show that the force is concentrated on the edges of the contact area. When the aspect ratio of the contact area is not equal to 1, the long edges bear most of the force. The forces distributed over the contact area are out of phase which means there exist both compressive and tensile forces acting in different parts of the contact area at the same time.

The real part of effective point mobility is distributed in a ringlike manner. The values of the real parts of effective point mobility in the central region and at the edges are very small. That is most of power transmission occurs within the ringlike region. Power can be transmitted from the exciting machine into the supporting plate and back from the supporting plate to the exciting machine which depends on wether the value of the effective point mobility at a point is positive or negative. As kw increases, the ringlike part expands outward and the values of the effective point mobility decrease. The aspect ratio of the contact area also influences the distribution of effective point mobility leading to larger values along the longer sides.

The real part of surface mobility for the case of uniform conphase velocity was obtained. The results of calculation show that the real part of surface mobility decreases rapidly as kw increases which implies power transmitted into the supporting plate decreases as the wavelength decreases.

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