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# USING FREQUENCY LIMITED PROPAGATION DATA TO ESTIMATE THE TORTUOSITY OF POROUS MATERIALS

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## ABSTRACT

The tortuosity or structure factor of a porous medium represents the increase in resistance to normal air flow due to the air following an indirect path relative to the macroscopic acoustic pressure gradient. Along with flow resistivity and porosity, tortuosity is one of the important parameters required in many of the theoretical models used to predict the acoustic properties of porous materials. A commonly used technique for measuring the tortuosity involves saturating the material with an electrically conducting fluid, however, this is inappropriate for unconsolidated granular media such as soils. Alternatively, the tortuosity can be estimated from the high-frequency asymptote of the measured phase speed. In practice, the maximum frequency at which propagation measurements can be made will be experimentally limited. This paper outlines a simple method by which the tortuosity of a medium can be readily estimated, with the aid of a simple theoretical calculation, from experimental data that have not reached the asymptotic value. The method assumes a prior knowledge of the flow resistivity and porosity of the medium. To validate the technique, estimates of the tortuosity of plastic foams and soils are compared with values obtained through a more time consuming curve-fitting approach.

## INTRODUCTION

Flow resistivity, $\sigma$, and porosity, $\Omega$, are two parameters required in many theoretical models used to predict the acoustic behaviour of porous materials. Another important parameter is the tortuosity, $q^2$ (sometimes denoted $\alpha_\infty$). This can be visualised by considering an ideal porous medium consisting of parallel, uniform tubes inclined at an angle $\theta$ to the surface normal. The factor $q$ is the ratio of pore length to sample length, $1/\cos\theta$. For complex media, the tortuosity can only be obtained experimentally, and can be considered as being due to an 'effective' inclination of the pores. It is a purely geometric quantity, independent of frequency and the properties of the fluid in the pores.

A number of techniques are available for measuring the tortuosity. An electrical conductivity method has been used to determine the tortuosity of foams[1] and glass-bead media[2]. However, this approach is inappropriate for unconsolidated soils as the structure would be changed by saturation with the conducting fluid. Probe microphone measurements,[3] made inside both glass beads and sand, allowed the propagation constant to be determined at frequencies below 2 kHz. The tortuosity was then estimated by fitting the data with a high flow resistivity / low frequency approximation of a rigid-frame model for uniform unconnected pores. This method requires a knowledge of the shape factor, *S*, of the pores, which may limit the reliability of the result.

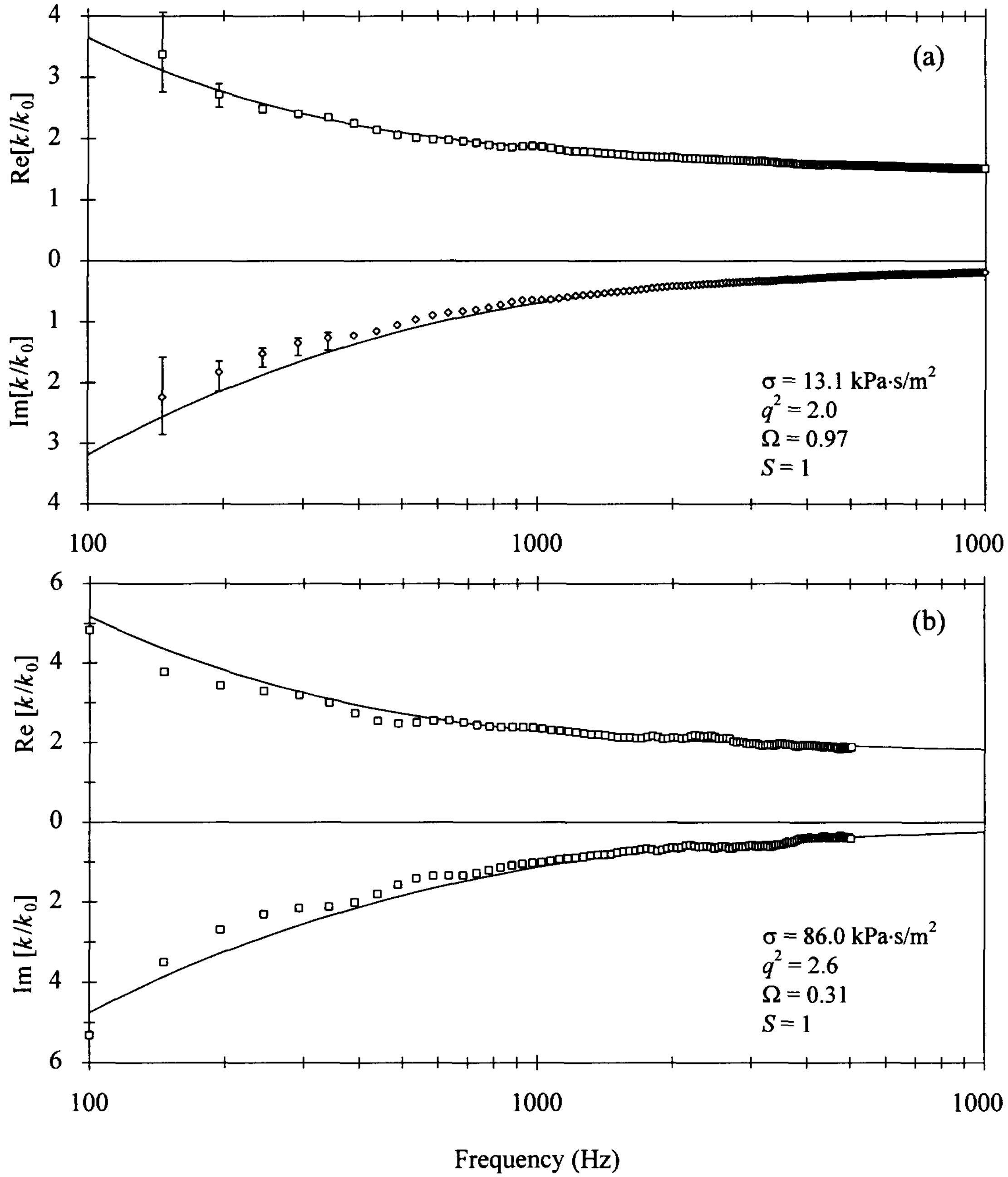


*Figure 1. Measured propagation constant $k/k_0$ compared with predictions of the Biot-Allard model for (a) high porosity foam, and (b) compacted soil with 14% (w/w) moisture.*

The normalised propagation constant is denoted $k/k_0$, where $k_0 = \omega/c_0$ is the wavenumber for propagation at speed $c_0$ in air. The real and imaginary parts of $k/k_0$ relate to the speed, $c$, and attenuation of sound in the medium, with $\mathrm{Re}[k/k_0] = c_0/c$. Attenborough[4] established that, in the 'classical' rigid-frame model, the high-frequency limit of $k/k_0$ is equal to $q$, and suggested[5] that the tortuosity of a medium can be estimated from this asymptotic value.

In practice, the maximum frequency at which propagation constant measurements can be made will be restricted, and the high-frequency limit of $k/k_0$ may not be reached. For example, the propagation speed in a high-porosity foam, determined by using an impulse technique,[6] has an upper frequency limit of 10 kHz, as indicated in Fig.1(a). For a compacted soil[7] with a moisture content about 14% by weight, the large surface reflection coefficient restricted the practical upper frequency limit for measurement of transmitted sound to 4 kHz, see Fig.1(b).

The greater the flow resistivity, the higher the frequency at which measurements are required to ensure that the asymptotic limit is reached. The problem is apparent from Fig.2, where the phase speed predicted by the Biot-Allard model[8] for $q^2 = 3$, $\Omega = 0.4$, and $S = 1$ is shown for a range of flow resistivities. All curves have the same asymptotic value, indicated by the dashed line. It is apparent that for a flow resistivity in the order of $10^6$ Pa·s/m$^2$, the speed at 4 kHz has only reached about 65% of the asymptotic value. Even for $\sigma = 10^4$ Pa·s/m$^2$ the speed is only 95% of the limit at 10 kHz. Attenborough[5] suggested using the parameter

$$\mu = S\left(8\rho_0\omega q^2 / \sigma\Omega\right)^{1/2} \quad \geq \quad 10 \qquad (1)$$

where $\rho_0$ is the density of air, as a criterion for deciding if, at a given frequency $\omega$ the speed is sufficiently close to the asymptotic value. The inset to Fig.2 shows the values of this parameter, at 10 kHz, for each of the flow resistivities. This data suggests that the criterion should be at least $\mu = 20$ for $\sigma = 10^4$ Pa·s/m$^2$.

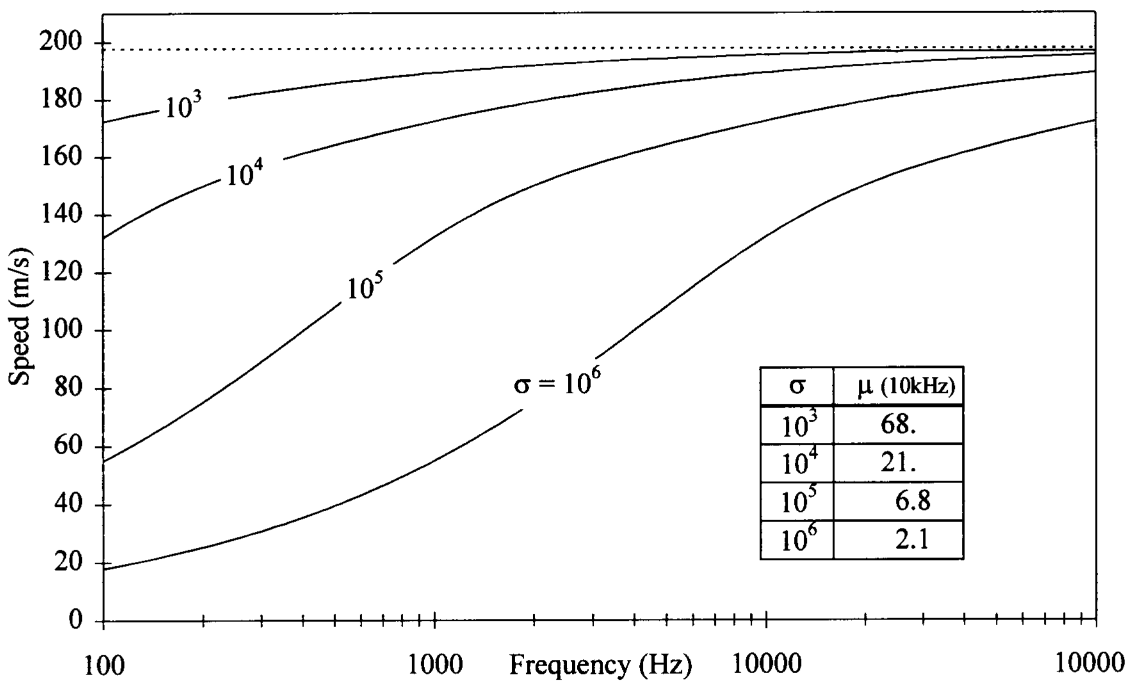


*Figure 2. Phase speed predicted by the Biot-Allard model for $q^2 = 3$, $\Omega = 0.4$, $S = 1$, and the indicated flow resistivities. The dashed line represents the asymptotic speed limit.*

Rather than obtaining a direct estimate of the tortuosity from the acoustic data, a series of calculations can be performed using a theoretical model to determine the tortuosity which, along with the other known parameters, provides the best fit to the experimental curves at the higher frequencies. This technique is particularly useful in soils, where the high frequency data are restricted. However, such a process is relatively time consuming. This procedure was used with the data shown in Figs. 1(a) and (b) to obtain values which can be compared with those deduced using the estimating technique to be described in the next section.

## ESTIMATING THE TORTUOSITY FROM LIMITED SPEED DATA

The following procedure outlines a simple method by which the tortuosity of a medium can be quickly and reliably estimated, with the aid of one theoretical calculation, from speed or wavenumber data that does not satisfy the high frequency criterion. The method assumes a prior knowledge of the flow resistivity and porosity of the medium.

It is useful to define a normalised speed, $c(\omega)/c_\infty$, as the ratio of the phase speed at frequency $\omega$ and the asymptotic speed as $\omega \to \infty$. At a given frequency, the ratio $c(\omega)/c_\infty$ increases with tortuosity, even though $c(\omega)$ decreases. From Attenborough's approximation,[4] the square root, $q$, of the tortuosity is related to $c_\infty$ by

$$q = \frac{k(\omega \to \infty)}{k_0} = \frac{c_0}{c_\infty}. \qquad (2)$$

The problem is to estimate $q$ from speed measurements that have not reached the asymptotic value. If a first approximation of $q$, designated $q(\omega) = c_0/c(\omega)$, is obtained at the highest available frequency, $\omega$, from the value $\mathrm{Re}[k(\omega)/k_0] = q(\omega)$, then $q$ is given by

$$q = q(\omega)\frac{c(\omega)}{c_\infty}. \qquad (3)$$

A theoretical calculation of the ratio $c(\omega)/c_\infty$ is then made with the known $\sigma$ and $\Omega$, using the approximation $q(\omega)$ in place of $q$. Equation (3) can then be used to calculate $q$ with the ratio $c(\omega)/c_\infty$ acting as a 'correction factor' for the approximation $q(\omega)$. A reasonable estimate can be made without changing $S$ from unity, which means both the Biot-Allard model[8] and the commonly used Attenborough[4,5] rigid-frame model give the same results. A simple polynomial approximation to the Biot-Allard model is also available,[9] significantly simplifying the calculation of the normalised speed.

## TEST OF THE PROCEDURE

As an example, from the foam data shown in Fig.1(a), $\mathrm{Re}[k/k_0]$ at 4 kHz gives $q(\omega) = 1.58$. Assuming $(1.58)^2$ for the first estimate of tortuosity and the independently measured values of $\sigma$ and $\Omega$ specified in Fig.1(a), the normalised speed shown by the lower dashed curve in Fig.3(a) is predicted. Reading off the normalised speed at 4 kHz gives the correction factor $q/q(\omega) = 0.87$, or $q = 0.87 \times 1.58 = 1.38$, which compares well with the value of 1.4 [ie. the $q^2 = 2.0$ indicated in Fig.1(a)] obtained by trial and error through curve fitting.

This method would still be valid if, for example, the highest frequency available was 2 kHz. At this frequency, the foam results give as a first estimate $q(\omega) = 1.69$, corresponding to the upper dashed curve in Fig.3(a). From this, the correction factor of 0.84 gives $q = 1.42$, which is also in good agreement with the value obtained by curve-fitting.

Perhaps it is more beneficial to verify that the method works for soils, due to the difficulty of measuring the tortuosity by other means. Two examples of soils in different physical states, and with greater flow resistivities than the foam, have been considered. The first makes use of the data given in Fig.1(b) for a compacted soil with 14% moisture and $\Omega = 0.31$, while the second uses sound speed measurements in a dry sandy soil[7] with 2% moisture and $\Omega = 0.40$. In both cases, the upper frequency limit of the data was 4 kHz. The normalised speed curves for the two media are shown as the solid curves in Figs. 3(a) and (b), respectively. In each case, the upper and lower solid curves correspond to the estimated $q(\omega)$ at 2 and 4 kHz. The results are compared along with the other examples in Table 1.

The normalised speed curves for the compacted soil give values of $q = 1.75$ and 1.66 from the 2 and 4 kHz estimates, while curve-fitting suggests a value of 1.6. The dry soil had an even larger flow resistivity of $\sigma = 2.4\times10^6$ Pa·s/m$^2$; three orders of magnitude greater than that of the foam. The initial estimates of $q(\omega) = 6.44$ and 5.80 result in $q = 4.42$ and 4.35, which are much closer to the value of 4.3 obtained by curve-fitting. However, as with the other examples, the lower frequency estimate is not in as good agreement with the curve-fitting result as that deduced from the 4 kHz data.

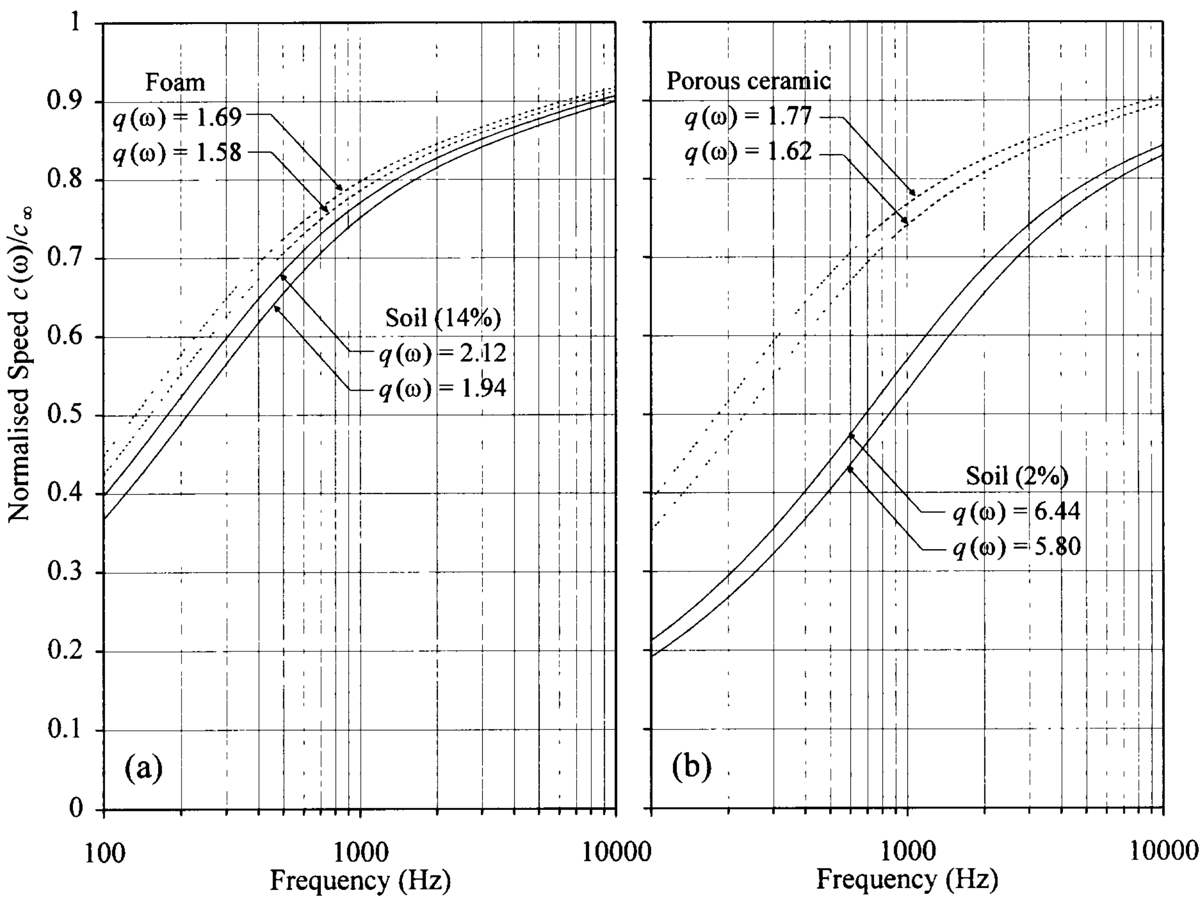


*Figure 3. (a) Normalised speed calculated for parameters corresponding to the foam and soil data of Figs. 1(a) and (b) respectively, each with two values of $q(\omega)$ as indicated. (b) Two further examples: a high flow resistivity soil[7] with 2% moisture and the porous ceramic material[8].*

| Medium | Acoustic foam | | Compacted soil (14%) | | Dry soil (2% water) | | Porous ceramic[8] | |
|---|---|---|---|---|---|---|---|---|
| σ (Pa·s/m$^2$) | $1.31\times10^3$ | | $8.6\times10^4$ | | $2.4\times10^6$ | | $4.45\times10^4$ | |
| Frequency (kHz) | 2 | 4 | 2 | 4 | 2 | 4 | 2 | 4 |
| First estimate $q(\omega)$ | 1.69 | 1.58 | 2.12 | 1.94 | 6.44 | 5.80 | 1.77 | 1.62 |
| Correction factor $\frac{q}{q(\omega)}$ | 0.84 | 0.87 | 0.83 | 0.86 | 0.69 | 0.75 | 0.82 | 0.85 |
| Better estimate of $q$ | 1.42 | 1.38 | 1.75 | 1.66 | 4.42 | 4.35 | 1.46 | 1.38 |
| By curve fitting | 1.4 | | 1.6 | | 4.3 | | 1.3 (measured) | |

*Table 1. Calculation of tortuosity from measured propagation constant data for S = 1.*

As a final example, it is useful to consider a material for which the tortuosity has been independently measured. Champoux and Stinson[8] obtained precise acoustic data between 50 Hz and 4 kHz, for a porous ceramic material and determined the tortuosity, non-acoustically, to be $q^2 = 1.7$. [Note that in their paper, the real and imaginary components of $k$ are interchanged compared to this work.] After converting their data to $k/k_0$, the values shown in Table 1 of $q = 1.46$ and 1.38 were obtained using the dashed curves in Fig.3(b) for first estimates at 2 and 4 kHz, respectively. Although these compare well with the measured value of $q = 1.3$, the agreement is not as good as might be expected. The reason for this is discussed in the next section.

## THE EFFECT OF SHAPE FACTOR ON THE ESTIMATION OF TORTUOSITY

In general, $S$ can only be determined by theoretical curve-fitting once $q$ is known. Consequently, the present technique for estimating $q$ requires that some assumption be made about the value of $S$. In the above examples, the normalised speed curves were calculated assuming $S = 1$. This is a reasonable approximation for the first three media considered, but is not necessarily appropriate for the porous ceramic material.

One advantage of the Champoux and Stinson[8] data is that they had independently measured the tortuosity, permitting $S = 1.37$ to be determined by curve-fitting using the Biot-Allard model. This data allows us to examine the effect of a non-unity $S$ on the estimation of $q$ by our technique.

Table 2 shows the results for the porous ceramic material based on $q(\omega)$ estimated at 4 kHz and calculated for three values of $S$. It can be seen that the value of $q$ decreases as $S$ is increased. The best estimate is obtained for the shape factor closest to the actual value for the material. However, in general we have no prior knowledge of the magnitude of $S$. This suggests that a compromise may be reached by calculating the normalised speed curves by assuming that $S = 1.5$. Indeed, if the compacted soil data from Table 1 is recalculated using $S = 1.5$, then the 4 kHz prediction changes from 1.66 to 1.55.

| Shape Factor | $S = 1.0$ | $S = 1.5$ | $S = 2.0$ |
|---|---|---|---|
| First estimate $q(\omega)$ | 1.62 | 1.62 | 1.62 |
| Correction factor $\frac{q}{q(\omega)}$ | 0.85 | 0.79 | 0.72 |
| Better estimate of $q$ | 1.38 | 1.28 | 1.17 |

*Table 2. Effect of S on the correction factor at 4 kHz for the porous ceramic material[8] with known values of $\Omega = 0.432$, $q = 1.3$ and $S = 1.37$.*

## CONCLUSION

Soils and other granular materials can be structurally altered when a sample is removed to measure the important acoustic parameters. Particularly for soils, the direct measurement of tortuosity presents difficulties as it normally entails saturating the medium with a liquid. Further, the high flow resistivity of soils limits the availability of high frequency data from which the tortuosity might be deduced. This paper has presented a method by which an estimate of the tortuosity may be obtained from frequency limited acoustic data. A good estimate may be obtained from measurements at 4 kHz, while a tolerable approximation can be achieved from data at 2 kHz or lower. It appears that using $S = 1.5$ may be a reasonable compromise when $S$ is unknown.

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