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Invited Paper

A PROBABILISTIC EVALUATION METHOD FOR VARIOUS TYPE SOUND INSULATION SYSTMES BASED ON KULLBACK'S INFORMATION CRITERION AND MIXED TYPE NON-STATIONARY SYSTEM MODEL

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ABATRACT In general, it is difficult to find some large scale model only from a bottom up way viewpoint for complicated sound insultion systems like non-parallel double wall or sound-bridge type double wall. Furthermore, in the acual environment, the input flluctuates non-stationarily and the output is observed under contamination of the background noise. In this paper, for the above complicated systems with non-stationary random input, a new evaluation method is proposed by newly introducing a multiplicative-additive system model on an intensiy scale. Owing to non-Gaussian property of input, output signals and background noise, the usual identification method such as least-squares error method is not appropriate. So, a new identification method based on Kullback's information criterion is proposed to deal with the non-Gaussian property. Next, the method predicting the response output probability distribution for arbitrary random input without contamination of the background noise is proposed. Here, according to original non-negative property of intensity quantity, a statistical type Laguerre series expansion is first employed as the output probability distribution form. Its expansion coefficients can be predicted by employing the above identified model. Finally, the proposed method is experimentally confirmed too by applying it to the actual sound insulation systems.

1. INTRODUCTION

For the evaluation of the environmental acoustic system, it is important to predict thewhole probability distribution closely related to the well-known evaluation indexes L_{eq} and L_x(x=5,50,95) of the response output to an arbitrary random input such as a road traffic noise. Usually, in an actual environmental acoustic system, the input fluctates randomly in non-Gaussian distribution form and even the characteristic of the acoustic system changes according to the change of the

internal factors
within a long time interval. Furthermore, the output is contaminated by the background noise. Under such complicated situation, it is too difficult to derive the model based on the internal mechanism according to physical laws and establish the evaluation method of the non-stationary output without the background noise only from a bottom up viewpoint. For practical use, some model from a top down viewpoint is inevitably required. As such a model, by employing the multiplicative model to the change of the internal factors and the additive model to the contamination of the background noise as an external factor, a mixed type model on an intensity scale can be taken.

In this paper, for the sound insulation system with a complicated sound insulation wall such as non-parallel double wall and sound-bridge type double wall, by introducing the mixed model of the multiplicative-additive type with only two system parameters, the stochastic evaluation of the response output to an arbitrary random input is considered especially in a unified forn. First, by assuming that the statistical moments of the background noise can be known in advance, the identification method is proposed by newly introducing the Kullback's information criterion matched to non-Gauusian fluctuation of the observed output. Next, once after deriving the method to determine the statistical moments of the random parameter reflecting the change of the internal factors, the prediction method is proposed to evaluate the probability distribution of the response output to an arbitrary random input. Then, by noting that the response output on an intensity scale takes only the non-negative value, the probabability distribution is in advance assumed to be expressed in a statistical type Laguerre series expansion form.

Finally, the proposed method is experimentally confirmed by applying it to the specific actual sound insulation systems.

2. A STOCHASTIC EVALUATION METHOD FOR THE RESPONSE OUTPUT OF SOUND INSULATION SYSTEM

2.1 INTRODUCTION OF MIXED TYPE MODEL FOR SOUND INSLATION SYSTEM

First, let us introduce a very simplified macro model of the sound insulation system in the actual enviroment. It is assumed that x,y,v and z denote the input, the output , the background noise and the output contaminated by the background noise of stationary type on an intensity scale, respectively. Here, it is noted that the statistiscal moments of v can be known in advance by observing the output when the input is not added. Since the output fluctuation reflecting the change of the internal factors is not caused without the existence of input, it is reasonable to express the system by the following the multiplicative model:

$$y=(a+\varepsilon)x+b \quad , \tag{1}$$

where ε is the random parameter reflecting the change of the internal factors. Since the output is contaminated by the background noise, the sound insulation system can be practically and macroscopically represented by the mixed model of multiplicative-additive type:

$$z = (a + \varepsilon) x + b + v. \tag{2}$$

2.2 IDENTIFICATION BASED ON KULLBACK'S INFORMATION CRITERION

2.2.1 KULLBACK'S INFORMATION CRITERION

By assuming that $P_0(y)$ and $P(y;\mathbf{a})$ denote the true probability density function and the probability density function with unknown estimation parameter vector $\mathbf{a}$, respectively, Kullback's information criterion is defined as follows:

$$I = \int P_0(y) \log \frac{P_0(y)}{P(y;\mathbf{a})} dy = \int P_0(y) \log P_0(y)\, dy - \int P_0(y) \log P(y;\mathbf{a})\, dy \ . \tag{3}$$

From the well-known property of the Kullback's information, this satisfies that $I \geq 0$ and here an equal sign is valid only when $P_0(y)=P(y;\mathbf{a})$. Since the Kullback's information criterion can be regarded as the distance between two probability distributions $P_0(y)$ and $P(y;\mathbf{a})$, the estimation problem of $\mathbf{a}$ becomes the problem minimizing I with respect to $\mathbf{a}$. Here, in the utmost right hand of Eq.(3), the first term is constant. So, the estimation problem is reduced to minimizing the second term with respect to $\mathbf{a}$. The extremal condition of this term yields

$$\frac{\partial}{\partial \mathbf{a}} \int P_0(y) \log P(y;\mathbf{a})\, dy = \int P_0(y) \frac{\partial / \partial \mathbf{a}\, P(y;\mathbf{a})}{P(y;\mathbf{a})} dy = 0 \ . \tag{4}$$

2.2.2 RECURSIVE ESTIMATION ALGORITHM

Let us denote the probability distribution of actually observed z and the probability distribution of z described by Eq.(2) by $P_0(z)$, $P(z;a,b)$, respectively. Upon replacing $P_0(y)$, $P(y;\mathbf{a})$ to $P_0(z)$, $P(z;a,b)$ in Eq. (4), we can directly obtain the following necessary condition:

$$\left\langle \frac{\partial/\partial a\, P_0(z;a,b)}{P_0(z;a,b)} \right\rangle_z = \left\langle \frac{\partial/\partial b\, P_0(z;a,b)}{P_0(z;a,b)} \right\rangle_z = 0, \tag{5}$$

where $\langle\ \rangle_z$ denotes an average operation with respect to z.

In order to derive the recursive estimation algorithm based on Eq.(5), it is necessary to prepare the framework of the probability distribution form $P(z;a,b)$ of z described by Eq.(2). This is derived as follows. First, by employing the joint probability density function $P(x,v,\varepsilon)$ of x,v,ε and Eq.(2), the probability measure-preserving transformation leads to the joint probability density function $P(x,v,z)$ of x,v,z :

$$P(x,v,z) = P(x,v,\varepsilon) \left| \partial\varepsilon/\partial z \right| \Big|_{\varepsilon \to \frac{z-b-v}{x} - a} \ . \tag{6}$$

Since x, v and ε are statistically independent, this is rewritten as follows:

$$P(x,v,z) = P(x) P(v) P_\varepsilon(\varepsilon) \left| \partial\varepsilon/\partial z \right| \Big|_{\varepsilon \to \frac{z-b-v}{x} - a}, \tag{7}$$

where $P(x)$, $P(v)$ and $P_\varepsilon(\varepsilon)$ denote the marginal density functions of x, v and ε, respectively. By integrating Eq.(7) with respect to x,v, P(z;a,b) can be expressed by

$$P(z;a,b)=\left\langle \frac{1}{x} P_\varepsilon\left(\frac{z-b-v}{x}-a\right)\right\rangle_{x,v} \quad , \tag{8}$$

where $\langle\,\rangle_{x,v}$ denotes an average operation about x and v. Since the random parameter ε randomly fluctuates symmetrically around zero, it is supposed that $P_\varepsilon(\varepsilon)$ can be approximated by the Gaussian distribution with mean 0 and variance σ_ε^2. After all, P(z;a,b) is represented as

$$P(z;a,b)=\left\langle \frac{1}{\sqrt{2\pi}\,\sigma_\varepsilon}\frac{1}{x} e^{-\frac{1}{2\sigma_\varepsilon^2}\left(\frac{z-b-v}{x}-a\right)^2}\right\rangle_{x,v} \quad . \tag{9}$$

Substituting Eq.(9) into Eq.(5) yields the following simultaneous equations:

$$\left\langle \frac{\partial/\partial a\, P(z;a,b)}{P(z;a,b)}\right\rangle_z=\left\langle\left\langle \frac{1}{\sqrt{2\pi}\,\sigma_\varepsilon}\frac{1}{\sigma_\varepsilon^2 x}\left(\frac{z-b-v}{x}-a\right) e^{-\frac{1}{2\sigma_\varepsilon^2}\left(\frac{z-b-v}{x}-a\right)^2}\right\rangle_{x,v} \Big/ \left\langle \frac{1}{\sqrt{2\pi}\sigma_\varepsilon}\frac{1}{x} e^{-\frac{1}{2\sigma_\varepsilon^2}\left(\frac{z-b-v}{x}-a\right)^2}\right\rangle_{x,v}\right\rangle_z=0 \tag{10}$$

and

$$\left\langle \frac{\partial/\partial a\, P(z;a,b)}{P(z;a,b)}\right\rangle_z=\left\langle\left\langle \frac{1}{\sqrt{2\pi}\,\sigma_\varepsilon}\frac{1}{\sigma_\varepsilon^2 x^2}\left(\frac{z-b-v}{x}-a\right) e^{-\frac{1}{2\sigma_\varepsilon^2}\left(\frac{z-b-v}{x}-a\right)^2}\right\rangle_{x,v} \Big/ \left\langle \frac{1}{\sqrt{2\pi}\sigma_\varepsilon}\frac{1}{x} e^{-\frac{1}{2\sigma_\varepsilon^2}\left(\frac{z-b-v}{x}-a\right)^2}\right\rangle_{x,v}\right\rangle_z=0 \quad . \tag{11}$$

Since these simultaneous equations are nonlinear regression functions, it is difficult to solve Eqs.(10) and (11) directly. So, by applying the well-known Robbins-Monro's stochastic approximation method to these, the following recursive estimation algorithm is obtained:

$$\begin{bmatrix}\hat{a}_{k+1}\\ \hat{b}_{k+1}\end{bmatrix}=\begin{bmatrix}\hat{a}_{k}\\ \hat{b}_{k}\end{bmatrix}+\Gamma_k\begin{bmatrix}\frac{\partial}{\partial a}P(z_k;\hat{a}_k,\hat{b}_k)\Big/P(z_k;\hat{a}_k,\hat{b}_k)\\ \frac{\partial}{\partial b}P(z_k;\hat{a}_k,\hat{b}_k)\Big/P(z_k;\hat{a}_k,\hat{b}_k)\end{bmatrix}, \tag{12}$$

where $\Gamma_k=\mathrm{diag}(\Gamma_{1k},\Gamma_{2k})$ is the gain matrix satisfying the Robbins-Monro's convergency condition.

2.3 PREDICTION OF THE RESPONSE OUTPUT PROBABILITY DISTRIBUTION

Let us consider the prediction of the probability distribution of the response output to an arbitrary random input without the contamination of the background nois. However, there still remains the problem on how to estimate the statistical information of ε. So, before considering the objective final problem, let us estimate the statistical moments of the random parameter ε. By employing the statistical independency of x, v and ε, Eq. (2) gives

$$\langle z^n \rangle = \sum_{i=0}^{n} \sum_{j=0}^{n-i} \frac{n!}{i!\,j!\,(n-i-j)!} \cdot \langle \varepsilon^i \rangle \langle v^j \rangle \langle x^i (a\,x+b)^{n-i-j} \rangle \tag{13}$$

This can be rewritten as

$$\langle \varepsilon^n \rangle = \frac{1}{\langle x^n \rangle} \left[\langle z^n \rangle - \sum_{i=0}^{n-1} \sum_{j=0}^{n-i} \frac{n!}{i!\,j!\,(n-i-j)!} \cdot \langle \varepsilon^i \rangle \langle v^j \rangle \langle x^i (a\,x+b)^{n-i-j} \rangle \right]. \tag{14}$$

Therefore, according to Eq.(14), by employing the same data of the input and observed output used in the above identification procedure, the statistical moments of ε can be estimated recursively.

After this preparation, the probability distribution of the response output can be predicted as follows. Since the output on an intensity scale takes only non-negative values, its probability density function can be expressed in a statistical type Laguerre series expansion form:

$$P(y) = P_\Gamma(y; m_y, s_y) \sum_{n=0}^{\infty} A_n L_n^{(m_y-1)}\left(\frac{y}{s_y}\right) \tag{15}$$

with

$$m_y = \frac{\langle y \rangle^2}{\langle (y - \langle y \rangle)^2 \rangle}, \quad s_y = \frac{\langle (y - \langle y \rangle)^2 \rangle}{\langle y \rangle} \tag{16}$$

and

$$A_n = \frac{\Gamma(m_y)\, n!}{\Gamma(m_y + n)} \left\langle L_n^{(m_y-1)}\left(\frac{y}{s_y}\right) \right\rangle, \tag{17}$$

where $P_\Gamma(\ )$ and $L_n^{(\alpha)}(\ )$ are the gamma distribution and the associated Laguerre polynomial defined respectively as

$$P_\Gamma(\zeta; m, s) = \frac{1}{\Gamma(m)\, s} \left(\frac{\zeta}{s}\right)^{m-1} e^{-\frac{\zeta}{s}} \tag{18}$$

and

$$L_n^{(\alpha)}(x) = \sum_{j=0}^{n} \binom{n+\alpha}{n-j} \frac{(-x)^j}{j!}. \tag{19}$$

Here, by employing Eq.(19), Eq.(17) can be expressed in terms of n-th moment of y as follows:

$$A_n = \frac{\Gamma(m_y)n!}{\Gamma(m_y+n)}\sum_{r=0}^{\infty}(-1)^r\binom{n+m_y-1}{n-r}\frac{1}{r!}\frac{\langle y^r\rangle}{s_y^{\,r}} \quad . \tag{20}$$

From Eqs.(16) and (20), the present problem is reduced to the prediction of the moments of y. Upon employing Eq.(1) under the assumption of the statistical independency of x and ε, the moments of y can be predicted by

$$\langle y^r\rangle = \sum_{i=0}^{r}\binom{r}{i}\langle\varepsilon^i\rangle\left\langle x^i\,(a\,x+b)^{r-i}\right\rangle . \tag{21}$$

3. APPLICATION TO ACTUAL SOUND INSULATION SYSTEMS

Our experiments has been made to identify the sound insulation systems and predict the probability distribution of the resposnse output. Figure 1 shows the experimental setup. Between a 50.3 m^3 transmission room and a 24.6m^3 reception room, each of a single wall, a non-paralell double wall and a sound-bridge type double wall consisting of the aluminuium panel has been attached. The panel is 1.2mm thick, 840mm wide and 1740mm long. In these experiments, by supplying the road traffic noise to the loudspeaker in the transmission room, the sounds in both rooms have been measured through two sound level meters and recorded in a data recorder. Later, they were sampled simultaneously at every one second and 1000 pairs of data have been obtained. The sound measured in the reception room and the white noise generated by a noise generator as a background noise have been composed by the digital computer. The reason why this has been done is that the probability distribution of the output without contamination of the background noise has been required to compare it to the theoretical distribution predicted by the proposed method. Then, in order to make the influence of the background noise notable intentionally, the average power of the background noise has been adjusted 3 dB lower than the one of the output.

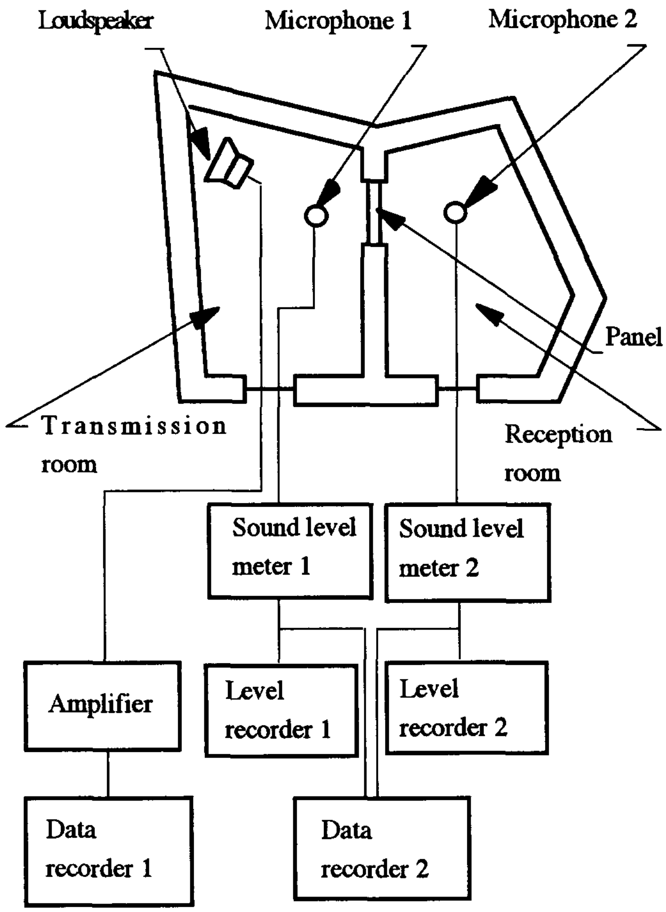

Fig. 1 Experimental setup.

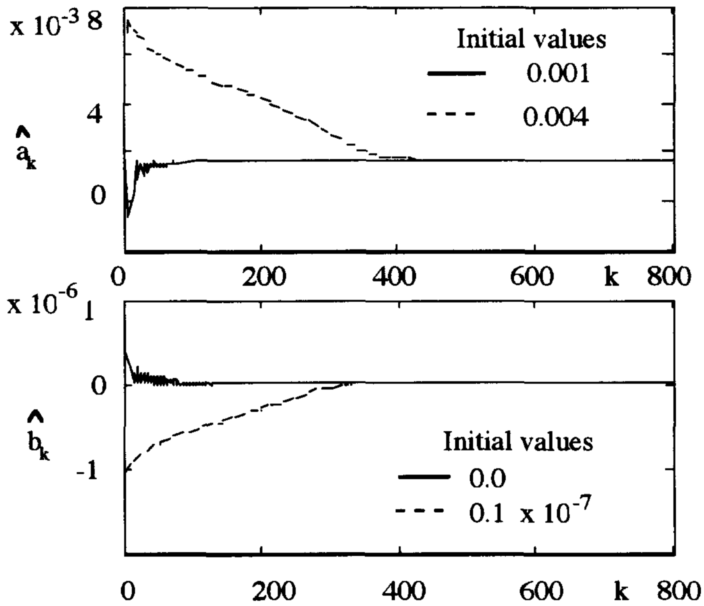

Fig. 2 The estimated values of a, b for a sound insulation system with non-parallel doubel wall.

First, for each sound insulation system, by employing the first 800 pairs of data, two parameters a, b of the mixed model have been estimated. Figure 2 shows the convergence process of the parameters a,b for a sound insulation system with non-parallel double wall. For each of a, b, each of the estimated values has been converged to almost the same value regardless of two different initial values, respectively.

Next, once after estimating the statistical moments of ε, the probability distribution of the response output to the remaining 200 sound input data has been predicted according to the method stated in section 2.3 and compared to the experimental probability distribution.

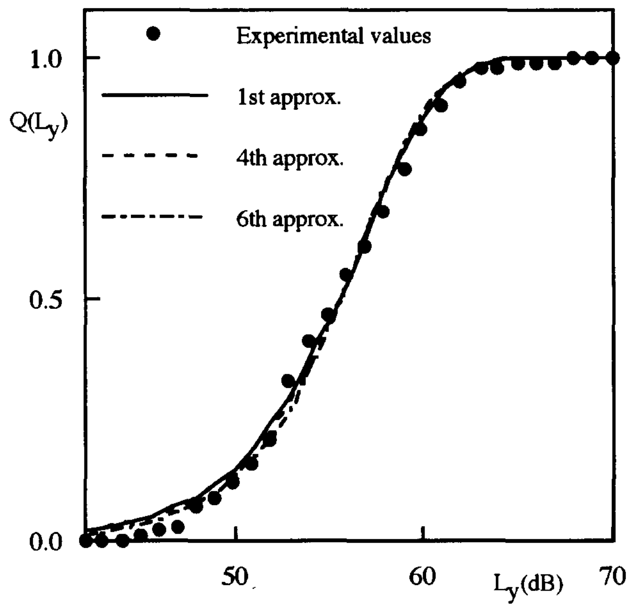

Fig. 3 A comparison between theoretically predicted curves and experimentally sampled values for a sound insulation system wit non-parallel double wall.

Figures 3 and 4 show the results for a sound insulation sytem with non-parallel double wall and a sound insulation sytem with sound-bridge type double wall, respectively. In each figure, the n-th approximation curve of the theoretical probability distribution has been calculated by truncating Eq. (15) except first n terms. In both figures, we can see that even first approximation curves agree well with experimental values. Furthermore, even though increasing the degree of the approximation of the theoretical curve, it can be recognized that the theoretical curve explains the experiment stably without divergence.

4. CONCLUSION

It is originally difficult to evaluate the complicated sound insulation systems such as non-parallel double wall and sound-bridge type double wall, by applying only the well-known acoustic theory. In this paper, for the evaluation of these complicated systems, some new method predicting the probability distribution of the response output has been proposed. By considering the change of internal factors and the existence of the background noise, the mixed model of multiplicative-additive model on an intensity scale has been first introduced. Then, the estimation algorithm of model parameters has been derived by employing the Kullback's information criterion mached to the arbitrary non-Gaussian type fluctuations. For the evaluation of the response output, the method predicting the expansion coefficients of the probability distribution of the response output expressed by a statistical type Laguerre series expansion has been proposed. Finally, the effectiveness of the proposed method has been confirmed experimentally by applying it to the specific actual sound insulation systems.

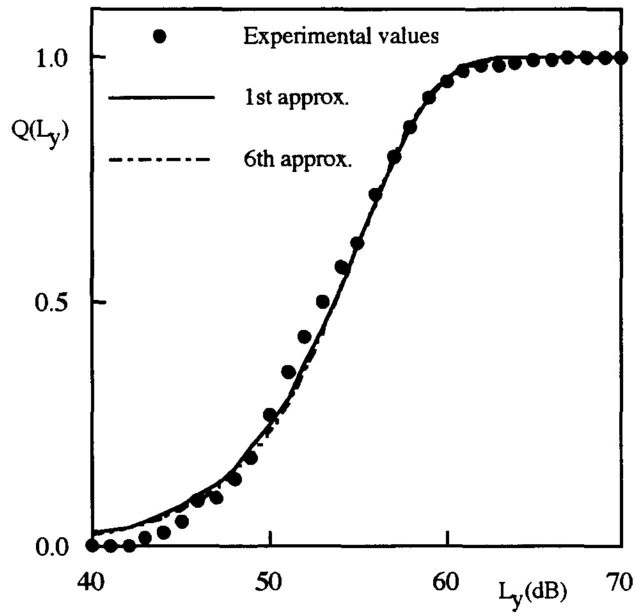

Fig. 4 A comparison between theoretically predicted curves and experimentally sampled values for a sound insulation system with sound-bridge type double wall .

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