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Invited Paper

IMPROVED METHOD FOR ESTIMATION OF COMPLEX MODULUS AND DAMPING

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ABSTRACT

This paper introduces an improved method for estimation of Complex Modulus as a continuous function of frequency. The classical method can normally be used up to a frequency which is a factor of 2 to 3 below the first resonant frequency for the testsetup. For the proposed improved method the frequency range is extended to well above the first resonant frequency for the test-setup. The method is verified by use of the well known 3 dB bandwidth method.

1. INTRODUCTION

One of the most fundamental relations governing the dynamic properties of materials is Hooke's law of elasticity. Hooke's law (see Fig. 1) states that there is a *linear relationship between stress (σ) and strain (ϵ)*. Stress is force per unit area, i.e. pressure, and it is measured in Newtons per square meter (N/m^2) also called Pascal (Pa). Strain is relative deformation, i.e. deformation divided by the original dimensions, and is therefore dimensionless. The ratio between stress and strain, E is called the modulus of elasticity or Young's modulus, and as for stress, it is measured in Pascal. Like mass density, the modulus of elasticity is a material constant. For steel it is approximately 200 GPa, and for aluminium it is approximately 70 GPa. Hooke's law describes many materials very accurately, especially metals which have relatively low damping coefficients. For many other materials, such as polymers and wood, the damping is so large that it cannot be ignored.

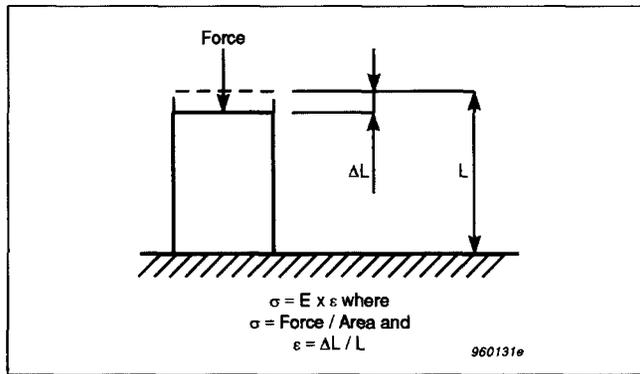


Fig. 1. Hooke's law

2. VISCOELASTIC MATERIALS

Viscoelastic models are often introduced when Hooke's law is deemed inaccurate. A few basic models are shown in the following:

Hooke: $\sigma = E\epsilon$

Newton: $\sigma = \eta \dot{\epsilon}$

Maxwell: $\dot{\sigma} / E + \sigma / \eta = \dot{\epsilon}$

Kelvin: $\sigma = \eta \dot{\epsilon} + E\epsilon$

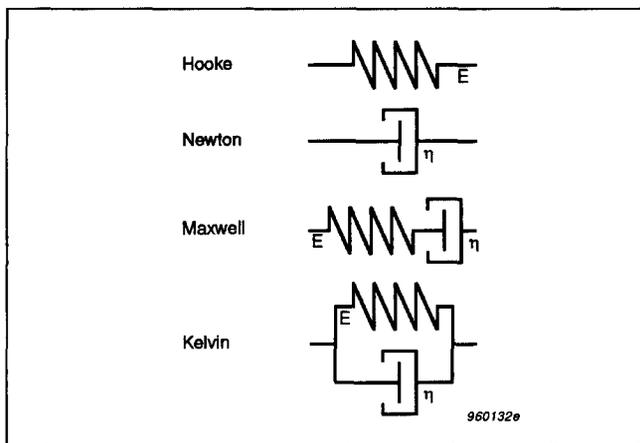


Fig.2. Spring and dashpot models of various viscoelastic materials

A dot above a symbol indicates time derivative. Hooke's law describes purely elastic materials, and Newton's law describes purely viscous materials. To make these equations more intuitively understandable, "spring and dashpot" models can be drawn (see Fig. 2).

Many more viscoelastic models can be constructed by adding more springs and/or dashpots, in series or in parallel, to the models above.

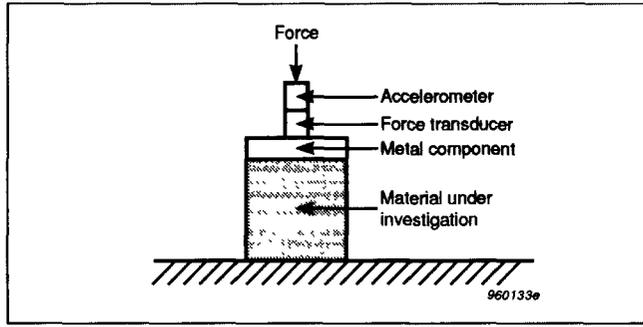


Fig.3. Simple set-up for measurement of complex modulus

The viscoelastic models introduce a time, and therefore also a frequency, dependency between stress and strain. If we look at the frequency domain representation of the relations, we get the following equations:

Hooke: $\sigma(j\omega) = E \times \varepsilon(j\omega)$

Newton: $\sigma(j\omega) = j\omega\eta \times \varepsilon(j\omega)$

Maxwell: $\sigma(j\omega) = \frac{j\omega\eta E}{j\omega\eta + E} \times \varepsilon(j\omega)$

Kelvin: $\sigma(j\omega) = (j\omega\eta + E) \times \varepsilon(j\omega)$

Where j is the imaginary number, and ω is the angular frequency in rad/s. The presence of j indicates that stress and strain are not usually in phase for viscoelastic materials. The advantage of the frequency domain representation is that we can return to the simple linear relationship used for Hooke's law. We just have to introduce the *complex modulus of elasticity*, E_c , which is a complex function of frequency.

$$E_c = E_c(\omega) = \frac{\sigma(j\omega)}{\varepsilon(j\omega)} \quad (1)$$

where E_c is defined as follows:

Hooke: $E_c = E$

Newton: $E_c = j\omega\eta$

Maxwell: $E_c = \frac{j\omega\eta E}{j\omega\eta + E}$

Kelvin: $E_c = j\omega\eta + E$

Note that for all viscoelastic models, the magnitude of E_c , i.e. the absolute Young's modulus, $|E_c|$, will either be constant or increase as a function of frequency. The imaginary part, $\text{Im}(E_c)$ of E_c divided by the real part, $\text{Re}(E_c)$ of E_c is referred to as the *Loss Factor* and is an indicator of the damping of the material. A number of other damping descriptors are described in Ref.[2].

3. MATERIAL PROPERTIES CONTRA SYSTEM PROPERTIES

When we want to measure the modulus of elasticity, the first method that comes to mind is a simple displacement versus force measurement (i.e. "mobility", compliance, receptance, flexibility or admittance measurement) on a block of the material of interest. Knowing the dimensions of the test specimen, the complex modulus can be calculated based on a simple frequency response function (FRF) measurement of compliance:

$$E_c = L/A \times 1 / FRF(\omega) \text{ where} \quad (2)$$

$$FRF(\omega) = \Delta L(\omega) / Force(\omega) \quad (3)$$

L is once again the length, and A is the cross-sectional area of the test specimen.

This is the so-called non-resonant method for the estimation of complex modulus (see Ref.[3]). See Ref.[1] for an introduction to frequency response measurements on simple SDOF systems. At low frequencies the method works really well, but at higher frequencies the mass of the test specimen and of the other components will begin to affect the measurement. Put in another way, at higher frequencies, the inertial forces cannot be ignored anymore. So as we get close to the first resonance frequency, the FRF is no longer only an expression of the material properties, but also of the system properties of the complete test specimen and test setup. Here is where the improved method comes in, see Ref. [5]. Since we know the geometry of the specimen, it is possible to find an analytic solution of how this simple system vibrates. If we assume that the metal components on top of the test specimen can be described as a lumped mass, the theoretical compliance of this simple system can be shown to be

$$Compliance(\omega) = \frac{\Delta L(\omega)}{Force(\omega)} = \quad (4)$$

$$\frac{\sin(\beta)}{\omega^2 (M_s / \beta \times \cos(\beta) - M \times \sin(\beta))} \quad \text{where}$$

$$\beta = \sqrt{\omega^2 L^2 \rho / E_c} \quad (5)$$

ρ is the mass density of the test material (kg/m^3), M_s is the mass of the test specimen ($A \times L \times \rho$), M is the lumped mass of the metal components on top of the specimen, and E_c is the complex modulus of elasticity of the specimen material.

The compliance ($\Delta L / Force$) is found from the frequency response measurement, and A , L , ρ , and M can be measured with great accuracy. This leaves only one unknown in equations (4) and (5), namely E_c , which can be found by a simple (Newton) iterative process.

The modified non-resonant method can be said to peel the system characteristics from the measurement data, leaving only the material properties.

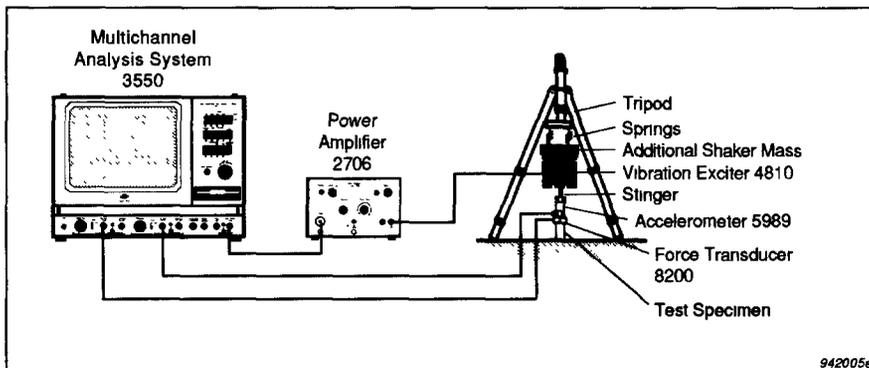


Fig.4. Test setup for the measurement of complex modulus

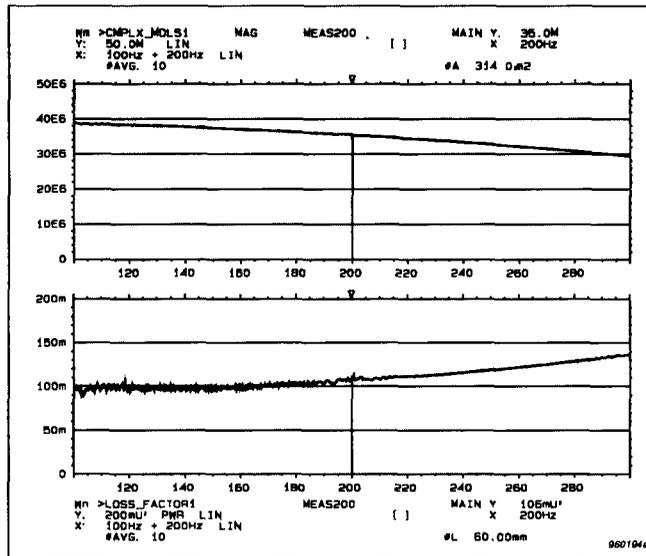


Fig.5. Magnitude of complex modulus and loss factor found using equation (2)

4. PRACTICAL MEASUREMENT

To perform the measurement, the test set-up shown in Fig. 4 was used.

The test specimen was made of rubber, it was cylindrical, with a diameter of 20 mm and a height of 60 mm. The following parameters were either measured or calculated.

- $A = 314 \text{ mm}^2$
- $L = 60 \text{ mm}$
- $\rho = 1058 \text{ kg/m}^3$
- $M_s = 19.9 \text{ g}$
- $M = 11 \text{ g}$

The first measurement (Fig. 5) was at low frequencies (100 to 300 Hz). The upper graph shows the User-Definable Display Function (UDDF) CMPLX_MDLS1 which is an inverted H1 frequency response function that has been scaled by the factor L/A , i.e. using equation (2). The lower graph shows the LOSS_FACTOR1, which is the ratio between the imaginary and real parts of the Complex Modulus.

Both curves look trustworthy, but as stated earlier, no viscoelastic model will allow for a decrease of the magnitude of the complex modulus of elasticity as a function of frequency.

If we look at Fig. 6, which shows a measurement at higher frequencies (100 to 900 Hz), the compliance frequency response function clearly indicates that there is a resonance at 552 Hz. So we can see that the shapes of CMPLX_MDLS1 and LOSSFACTOR1 are, at least, partly due to the fact that we are getting close to the first resonance frequency.

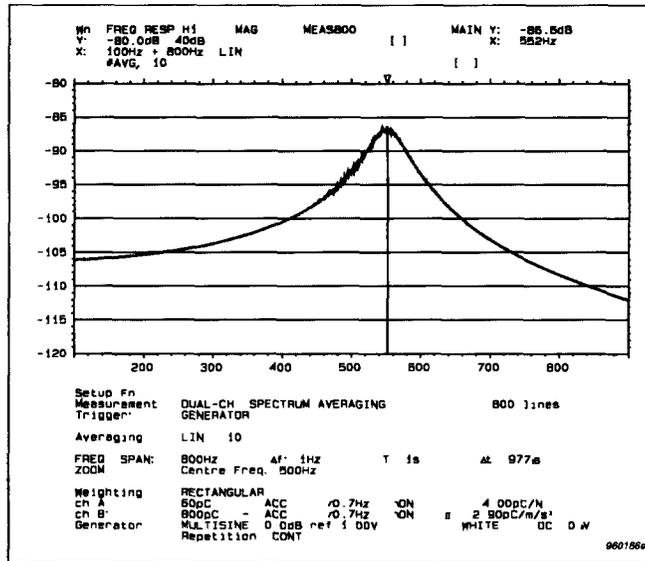


Fig.6. Frequency response function of compliance (100 to 900 Hz)

Fig.7 shows the two UDDFs *CMPLX_MDLS1* and *LOSS_FACTOR1*, and at this higher frequency range, it can clearly be seen that both are strongly affected by the resonance. The inertial forces disturb the measurement even at much lower frequencies than the resonance frequency, and the question is whether the displayed values can be trusted even at 100 Hz. What is needed is a method to remove the effect of the inertial forces, and this is precisely what the modified non-resonant method described earlier will do.

The Brüel&Kjær Multichannel Analysis System Type 3550 can perform the computations needed to compensate for the system characteristics of this simple system using equations (4) and (5). See Ref. [4]. The result of the calculations of *CMPLX_MDLS* and *LOSS_FACTOR*, which are described much more in details in Ref. [6], are shown in Fig. 8. The two curves look very different from the curves in Fig.7. The effect of the resonance cannot be seen anymore! The magnitude of the complex modulus of elasticity can be seen to be slightly increasing, and is about 40 MPa. The loss factor is very close to 0.1 in the frequency range from 100 to 900 Hz. In the 100 to 900 Hz frequency range the complex modulus of elasticity can be said to be approximately:

$$E_c = 40 \text{ MPa} + 4 \text{ MPa} \times j$$

At higher frequencies, this method is also very sensitive to errors in the analytical model; if the real system deviates too much from the model, then errors will be introduced. All system parameters (*A*, *L*, ρ *M_s*, *M*) must be measured with high accuracy.

Finally, we can check whether the loss factor is consistent with the loss factor found by the traditional resonance bandwidth method. This method states that the loss factor at a resonance is approximately equal to the half-power bandwidth divided by the resonance frequency. The *UDAI LF_AT_RES* will perform this calculation at the resonance which is selected by the main cursor. Fig.9 shows the frequency response function in the upper graph, and the loss factor found by the modified non-resonant method in the lower graph. In the upper cursor setup, the *UDAI LF_AT_RES* has been activated and is reading out 0.089, and at the same frequency the main cursor is reading out 0.095 in the lower graph.

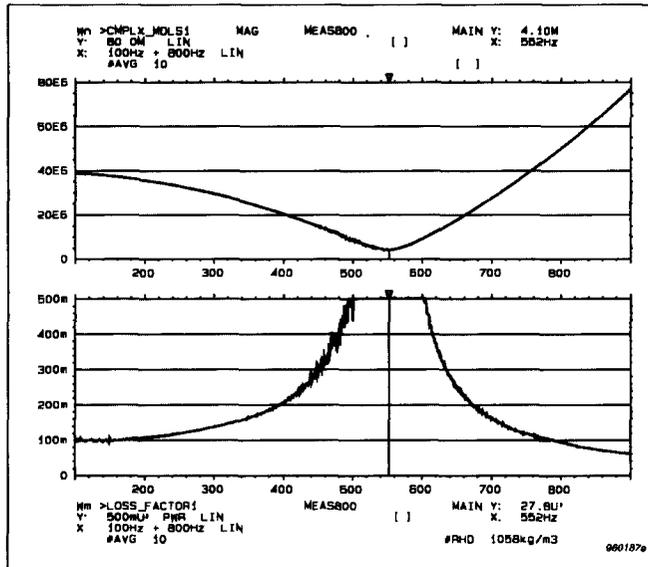


Fig. 7. Both complex modulus and loss factor are erroneous close to the resonance frequency

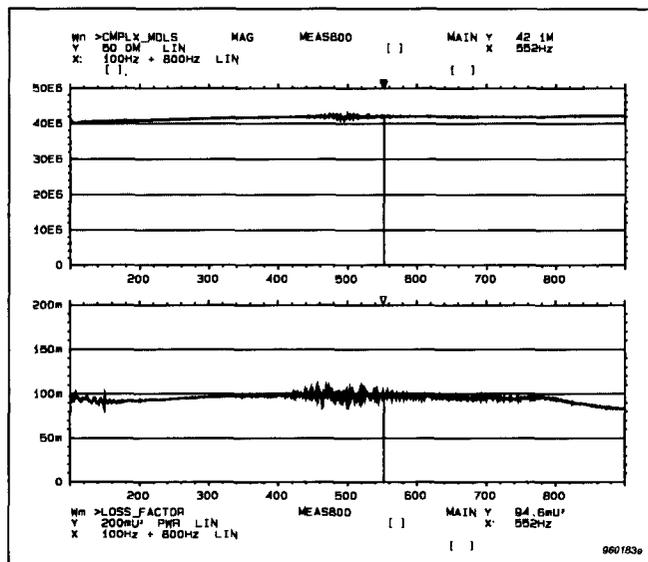


Fig. 8. Magnitude of complex modulus and loss factor found using equation (4) and (5)

5. CONCLUSION

In this paper it has been demonstrated that a better estimate of the complex modulus of elasticity can be calculated based on a simple non-resonant measurement. The advantages of this modified non-resonant method as opposed to the widely used resonant method, are twofold. First, it delivers not only damping, but also stiffness, and second, the information is available at all frequencies, and not only at the resonant frequencies.

It can be seen (see Ref. [6]) that the modified non-resonant method has some problems at higher frequencies. One reason for this is due to the fact that the model used is not 100% exact. Any small misalignment between the higher resonance frequencies in the model and the real ones

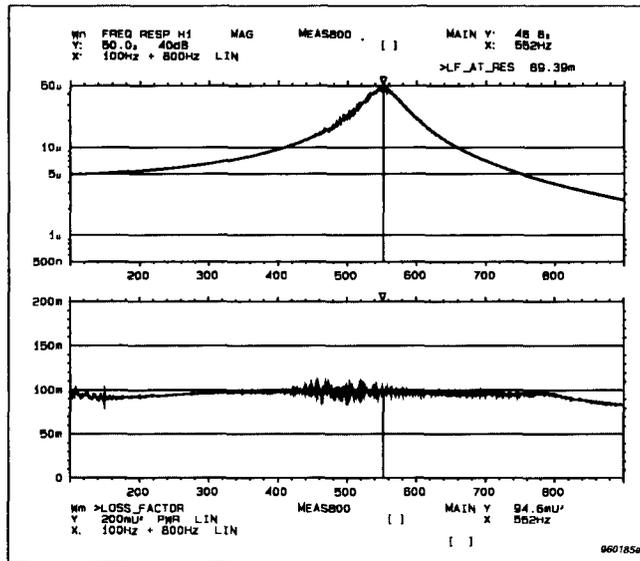


Fig. 9. Loss factor calculated by resonance bandwidth method (upper cursor setup), and loss factor calculated by modified non-resonant method (lower graph)

will influence the estimated complex modulus. It was also seen (see Ref. [6]) that the loss factor estimate got more and more noisy at higher frequencies. This is again understandable if we remember that the inertial forces dominate at higher frequencies, i.e. stiffness and damping have very little effect on the FRF. But all in all, the modified non-resonant method is a clear improvement over other methods.

6. REFERENCES

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