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SIMULATION EXAMINATIONS OF VEHICLE SUSPENSIONS WITH NONLINEAR DAMPING AND RIGIDITY CHARACTERISTICS.

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SUMMARY

Mathematical model of suspensions together with its geometrical dimensions has been presented in this paper. It is a 6 mass discreet dynamic system with 36 freedom grades. Its concentrated masses are connected by elastic and attenuating elements of non-linear characteristics. Apart from springs and silencers there are also wheel guiding elements (rockers, lateral control rods, etc.).The model is activated to vibrations by road unevenness. Input functions are of stochastic nature. Changes of many element parameters of the model were taken under consideration in simulating examinations (type of suspensions, mass distribution, stiffness and attenuation characteristics changes, etc.). The results were presented in a graphic form together with comparative analysis.

INTRODUCTION

Design engineering involves selecting such characteristics which secure perfection of engine functioning. The following quality features ensure this: stability, reliability, precision, low level of disturbances. They are closely related to the character of dynamic processes in engines. Therefore it is necessary at the design stage to gather information about the influence of construction parameters on these processes. Constructional diagnostic results allow to proceede smoothly from designing stage to production stage.

MODELS OF VEHICLE SUSPENSIONS

Simulation examinations of mathematical models are of great importance in testing dynamic mechanical systems. Basic task of vibration analysis is to introduce and to solve equations which define motion of examined mechanical system. Complexity of most the phenomena does not make the equations too accurate. Some simplifications must be introduced in modelling process - from mechanical system to physical model, mathematical model, algorithm and finally to computer programme. This programme is to solve discrete dynamic systems presented by differential equations:

$$\frac{dx_i}{dt} = f_i (t ; x_1,...,x_n; y_1,...,y_m; z_1,...,z_k; \lambda_1,..., \lambda_L) \qquad (1)$$

with initial conditions for $t = t_O$:

$$x_i(t_O) = x_{Oi} \qquad (i = 1,...,n) \qquad (2)$$

System (1) and (2) can be presented in vector form:

$$\dot{x} = f (t, x, y, z, \lambda), \qquad x(t_O) = x_O \qquad (3)$$

where $x^T = (x_1, ..., x_n)$ is a variable vector of state (components of this vector are time functions; they can also be random functions), $x_O^T = (x_{10},x_{20},...,x_{nO})$ - is a vector of initial conditions (components of this vector can be determined or they can be variables), $f^T = (f_1,f_2,...,f_n)$ - is a nonlinear vertical function, $y^T = (y_1,y_2,...,y_m)$ - is a vector of constructional variables (components of this vector can be constant functions), $z^T = (z_1,z_2,...,z_k)$ - is a vector of external interactions which affect the system (they can be kinematically or dynamically forced with several determined z_i components or random functions of time which are correlated or non-correlated), $\lambda^T = (\lambda_1,\lambda_2,...,\lambda_L)$ - is a vector of system parameters (these are the additional values describing dynamic system and they do not change during the process of designing; real or random quantities can be vectors components). Admissible system is defined by state equations, inequality and function bounds for optimization of nonlinear dynamic systems:

$$\Phi = \{(x,y) : \dot{x} = f(t, x, z, \lambda); x(t_O) = x_O; g(x, y, \lambda, z) = 0; \varphi(x, y, z, \lambda) >0\} \qquad (4)$$

where $g = (g_1,g_2,...,g_g)$ and $\varphi = (\varphi_1,\varphi_2,...,\varphi_p)$ are additionally some nonlinear vertical functions.

In diagnostics at the designing stage, analysing dynamic phenomena in suspensions seems to be quite essential since any changes of suspension characteristics influence:

— comfort of driving,
— steerability and driving safety,
— truckload protection,
— protection of all the elements of vehicle (long life),
— road surface protection.

Vibrations are stimulated in vehicles by engine and power transmission system, as well as by road surface, its profile and driving speed. Vibrations of power transmission systems are caused by unbalanced masses whilst engine vibrations mainly by unbalanced gas forces and inertia moment. Suspension vibrations caused by rough road surface, changing rolling friction cannot be eliminated. Modelling of vehicles is therefore difficult because of their complexity. This is also due to element distortion, hence each examined system has unlimited number of freedom degrees.

Correctness of mathematical model (that is making it as close as possible to the actual conditions) is not easy to achieve since it is a system of equations of physical model motion. Position of the model in space is defined by coordinates in a given reference system.

Displacement system (translation coordinates) and angular displacement (rotational coordinates) are most frequently used. Limiting the number of freedom degrees results in obtaining more discrete model. Such limitations in case of vehicles are quite appropriate because bodywork distortions and axle distortions are smaller than those in suspension springs and tyres. Assuming rigidity of these elements, a system of combined masses elastically supported with damping can be formed.

Such analytical model is only applied when it is assumed that a vehicle is of symmetric construction in relation to longitudinal axles. Vehicles bodywork does not vibrate transversly at pure vertical forcing when left wheel and right wheel are exposed to identical forcing. Therefore in linear models, vertical displacements and longitudinal tilts are not associated with side tilts and they do not interact. Further simplifications can be obtained in m_2 bodywork mass and J_{2y} moment of inertia can be replaced by m_{2p}, m_{2t} concentrated masses which are placed above axles and m_{sp} mass is placed in the central past of bodywork. When coupling mass is $m_{sp} = 0$ then m_{2p} and m_{2t} mass displacements are independent. In that way there are only two vibrating systems instead of system with five degrees of freedom; each of them has three degrees of freedom which makes all the calculations easier. Such coupling is quite common in vehicles. In four-wheel vehicles the situation changes when wheel suspensions are considered see (Fig 1.).

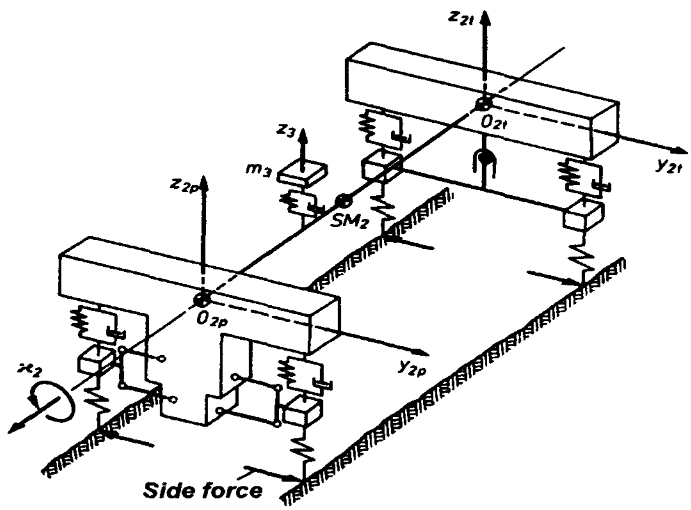

Fig 1. Analytical model of a vehicle where suspension has been considered.

Analytical model assumes that a vehicle is symmetrical to longitudinal axles. Apart from springs and silencers the model has wheel suspension elements related to bodywork which are very important in transferring forces while driving. These forces are transferred to bodywork not only by springs and silencers but also by joints. Vertical motion of a wheel corresponds to side displacement of wheel contact point with road surface. Due to such wheel displacements, side forces occure and they influence vibrations of a vehicle. There are also changes in wheel camber angles which cause additional side forces.

Bodywork mass on front axle is m_{2p} and on rear axle is m_{2t}, coupling mass is $m_{sp} = 0$, m_{2p} mass displacements are marked z_{2p} and y_{2p}, m_{2t} mass displacements are marked in a similar way. Lateral tilts of bodywork are marked κ_2 as previously, whereas ϕ_2 longitudinal tilts are due to z_{2p}, z_{2t} displacements and ψ_2 rotations are due to y_{2p} and y_{2t} displacements.

Equations of motion are arranged for front and back partial system. O_{2p} is a reference point for determining displacements of the front part of bodywork and O_{2t} point referes to the rear one. Side forces acting on wheels are presented as vectors.

The model is described by two dissociated systems of equations in the form of matrix - one is for vertical vibrations and longitudinal tilts:

$$M_Z\ddot{z} + K_Z\dot{z} + C_Z z = R_z h_\Sigma \tag{5}$$

with vector of unknowns:

$$z^T = (z_3, z_{2p}, z_{2t}, z_{1p}, z_{1t}, F_{y\Delta p}, F_{y\Delta t}) \tag{6}$$

and the other one for lateral displacements and tilts:

$$M_\chi\ddot{\chi} + K_\chi\dot{\chi} + C_\chi\chi = R_\chi h_\Delta \tag{7}$$

with vector of unknowns:

$$\chi^T = (\chi_2, \chi_{1p}, \chi_{1t}, \chi_{2p}, \chi_{2t}, F_{y\Sigma p}, F_{y\Sigma t}) \tag{8}$$

where:

z_{1i} — ficticious vertical displacements,
χ_{1i} — fictitious lateral tilts,
h_Σ — symmetrical forcing resulting from inequality sums in left and right trace,
C — rigidity matrix,
K — damping matrix,
M — inertia matrix,
R — forcing interaction matrix.

Linearity of a system is assumed because the presentation of nonlinear systems is more difficult since the principle of superposition cannot be applied. Identification of the results gets more difficult with the increase of model freedom degrees. Therefore it is essential to check linearity of the system and to limit its freedom degrees considering only the most essential ones. One of the methods which are applied is to check how superposition principle functions. To do this, forcing by harmonic signals is performed at input.

$$u_1(t) = A\sin\omega_1 t \tag{9}$$
$$u_2(t) = A\sin\omega_2 t$$

Signals should be fed the first time in form of $u_1 + u_2$ sum and the second time as u_1 and u_2 respectively. Then amplitudes corresponding to forcing are measured at output. If there are different frequencies, it is an indication that the system is nonlinear.

Vibrations in suspensions are accompanied by:

— hydraulic damping in shock absorbers,
— constructional damping in springs, splined connections, etc.,
— internal friction, particularly in rubber elements.

Evaluation of energy dissipation efficiency is based on:

— logarithmic decrement of damping,
— specific damping coefficient,
— phase delay angle.

The above mentioned indicators are quite effective in recognising linearity of a system. Logarithmic decrement of damping is constant for linear systems. The shape of hysteresis loop indicates nonlinearity of a system. Defining coefficient of specific damping evaluates nonlinearity of a system and so do the phase examinations of phase trajectory. There are no general methods of evaluating parameters of all kinds of nonlinearity of nonlinear systems. Identification of structurally nonlinear models is difficult and the choice of structure considerably influences the quality of a model. Approximate solutions are most frequently applied for nonlinear systems therefore quasi-linear systems, methods based on linearization of models, etc., are used. First the parameters of a system which has been made linear are identified and then parameters of nonlinear system are exchanged on the basis of relations between model parameters and elasticity and damping coefficients.

SIMULATION EXAMINATIONS

Model of suspension presented in Fig 1. was considered in the examinations. Elements of the model can be:
— displaced vertically,
— tilted longitudinally,
— lilted laterally,
— rotated round vertical axis.

Runge-Kutty numerical method of fourth order was applied to solve the system of equations in matrix form. Nonlinear characteristics of damping and elasticity relevant to the actual construction elements were made linear.

The following types of suspensions were analysed:
a) independent front suspension with double asymmetric lateral rockers and dependent rear suspension - driving axle,
b) independent front suspension (single lateral rockers) and rear independent suspension (single helical rockers),
c) independent front suspension (lateral rocker and Mc Pherson column).

Simulations were carried out for different damping characteristics, rigidity, mass distribution, load at harmonic and stochastic forcing. Function of reinforcement was determined for:
— vertical accelerations of the front part of bodywork,
— vertical accelerations of the rear part of bodywork,
— dynamic loads of front wheels,
— angular accelerations of longitudinal tilts in bodywork,
— difference of side forces effecting front wheels,
— difference of side forces effecting rear wheels,
— lateral accelerations of the front part of bodywork,
— lateral accelerations of the back part of bodywork,
— sums of side forces effecting front wheels,
— sums of side forces effecting rear wheels,
— angular accelerations of bodywork lateral tilts.

Examples of experiment results have been presented in pictures.

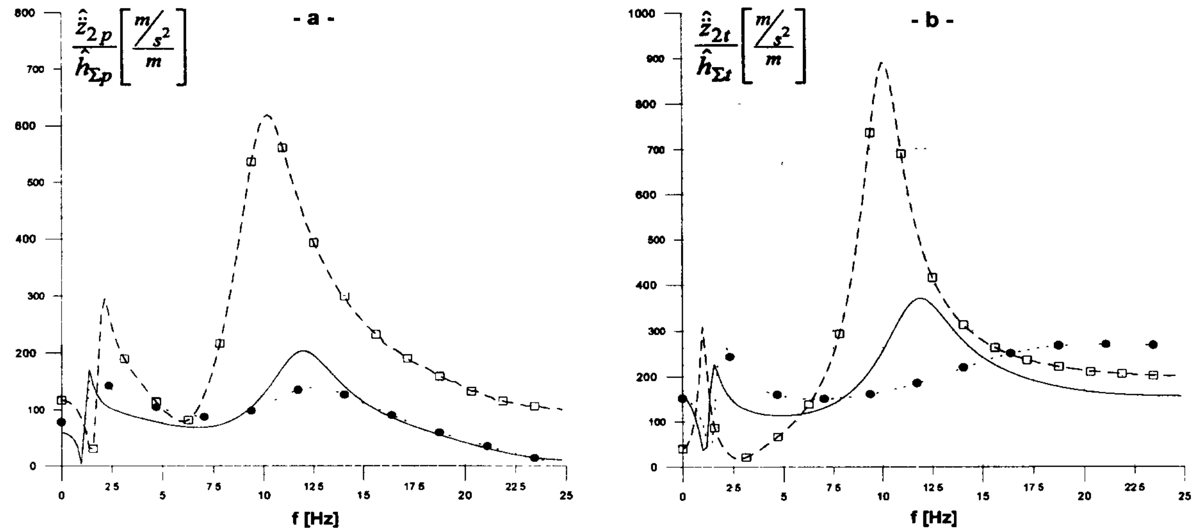

Fig 2. Reinforcement function for vertical accelerations in: a) the front part of bodywork, b) the rear part of bodywork.

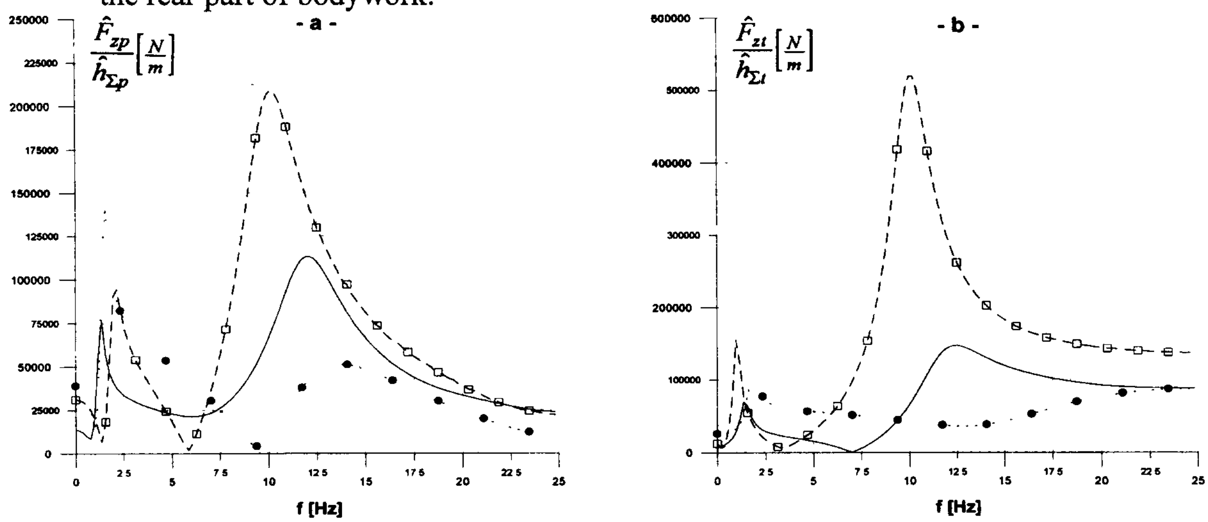

Fig 3.Reinforcement function for dynamic loads in: a) front wheels, b) rear wheels.

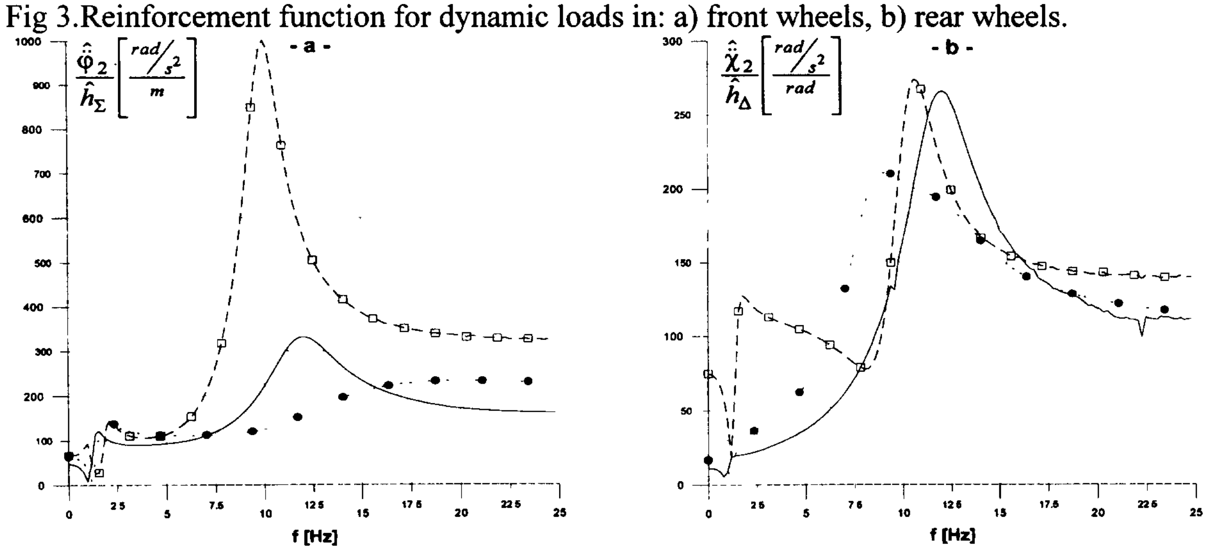

Fig 4. Reinforcement function for angular accelerations of: a) longitudinal tilts in bodywork, b) lateral tilts in bodywork.

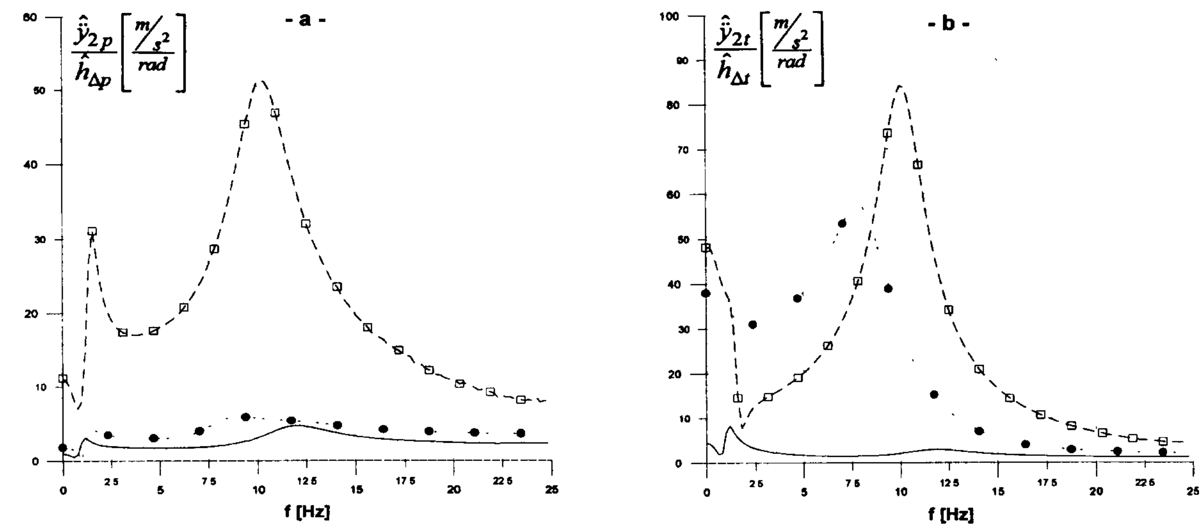

Fig 5. Reinforcement function for lateral accelerations in: a) the front part of bodywork, b) the rear part of bodywork.

Fig 2÷5: front — single lateral rockers, rear — single helical rockers,

front — double asymmetrical lateral rockers rear — rigid axle (driving axle),

front Mc Pherson, rear Mc Pherson,

f [Hz] – forcing frequency.

CONCLUSIONS

Having completed simulation examinations of three different types of linearized suspensions with nonlinear rigidity and damping characteristics the following conclusions can be drawn:

a) growth of suspension rigidity causes increase of both bodywork free vibration frequency and axle free vibration frequency. In the case of reinforcement function for angular accelerations of longitudinal tilts there is growth of rigidity accompanied by decrease of amplitudes within the whole frequency range. Differences in experimental results are relatively small for Mc Pherson type of suspension and for suspension with double lateral rockers comparing to those of single lateral rockers or single helical rockers. The same applies to vibrations and lateral tilts. Increase of amplitudes can be observed for rigid axles and it goes as far as free vibration frequency of the axle.

b) changes of damping influence the value of amplitudes but they do not influence free vibration frequencies of body work and axles. Decrease of damping causes growth of vertical acceleration amplitudes in the area of bodywork and axle resonance as well as in the second resonance area of limitation. Considerable decrease of amplitudes is noticed between them. Dynamic load of wheels increases and amplitudes of longitudinal tilts decrease. Amplitudes of bodywork lateral tilts are also smaller. However differences occur in suspensions with single rockers at free vibration frequencies of bodywork where there is growth of amplitude and for Mc Pherson type of suspension where there are angular lateral tilts with maximum value.

c) changes of wheel loads for $m_{2p}+m_{2t}$ = constant have the following influence:

- with loaded rear part of bodywork and unloaded front part of bodywork angular accelerations increase for single lateral rockers, they do not change for Mc Pherson type of suspension and they increase for single rockers and rigid axles,
- with loaded front and unloaded rear part of vehicle it is the opposite,
- with uniform loading of front and rear there is a decrease of vibration amplitudes within the whole range of frequencies.

d) type of kinematic vibration forcing considerably influences vibrations. In the case of stochastic forcing amplitudes grow for almost all magnitudes comparing to those of sinusoidal forcing.

In the above presented cases results can be easily distinguished which helps to choose the best solutions in suspensions for a particular type of vehicle. Simulation examinations which are part of constructional diagnostics make the designing stage shorter and they can diagnose prototype as well. They are also more economical than the traditional examinations at work-stand.

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