FIFTH INTERNATIONAL CONGRESS ON SOUND AND VIBRATION

DECEMBER 15-18, 1997
ADELAIDE, SOUTH AUSTRALIA

Specialist Keynote Paper

STOCHASTIC SIGNAL INFORMATION PROCESSING FOR ACOUSTIC ENVIRONMENT—USE OF WIDE SENSE DIGITAL FILTER

Mitsuo OHTA* and Akira IKUTA**

* Faculty of Engineering, Kinki University
** Faculty of Human Life & Environmental Science,
Hiroshima Women's University

ABSTRACT: This paper gives a basic and essential idea of our works on the stochastic signal information processing for acoustic environment, that is based not only on the lower order linear correlation but also on various types of the higher order nonlinear correlations. The importance of employing these higher order nonlinear correlations is discussed through the concrete establishment of wide sense digital filters. These digital filters are fundamentally based on the hierarchical expansion expression of Bayes' theorem by using the orthogonal polynomials. The proposed digital filters can estimate any kinds of statistics of arbitrary functional type of a state variable including the lower order statistics connected with the material-side countermeasurements (from a bottom-up way viewpoint) and the higher order statistics connected with the human-side evaluation indices (from a top-down way viewpoint). The validity and the effectiveness of these filters are experimentally confirmed by applying them to the real acoustic environmental problems.

1. INTRODUCTION

Digital signal processing bounded only in the virtual world of computer has necessarily a very high degree of freedom (*The excess of analysis turns into foolishness*). That is, in the virtual world of computer, the concept of time is just one of the axes in the N dimensional space, and the operations from the present to the past would be possible (anti-causative). However, if we discuss only in the closed world of computer, there are some apprehensions that it artificially rarefies the dynamics on the time axis in the real world of actual engineering (*A skilled swimmer dies in the river*). We have to open anyhow our eyes to the outer real world. It goes without saying that the real universe is changing moment by moment not along with an artificially operation time axis but along with a real time axis unable to go backward (*The greatest enemy of freedom is self-indulgence*). Every existence is a historical existence, which was ever, is now, and will be, a Heraclitus' ever-living Fire following a natural law (*Freedom is a self-consciousness of inevitability. There is no freedom without law (Hegel); He himself is his greatest enemy*). Sometimes, the information processing along with this real time axis is called as a "*signal information processing*" in distinction from a mere "*symbolic information processing*". The former must obey a causative law, however on the contrary, the latter does not necessarily obey it, and in the "*symbolic information processing*" a variety of artificial operations are possible. In this article, we positively take the standpoint of "*signal information processing*" in order to clearly discriminate it from "*symbolic information processing*" .

The acoustic phenomenon in the actual sound environmental system involves a variety of compound problems. Not only the natural but also the social factors make them further complicated and diversified. Moreover, man's response to it (as individual and/or group) is not originally uniform, which seems to be the very manifestation of humanity. Therefore, we think the human-side noise evaluation standards should be made from the delicate and multifarious viewpoints, which admit every possible complexity, variety, and minuteness. In fact, many kinds of noise evaluation standards are proposed so far from all sorts of viewpoints. They are, for example, L_{eq} (Equivalent Sound Pressure Level), L_{dn} (Day-Night Average Sound Level), L_{NP} (Noise Pollution Level), L_x ((100-x) Percentage Point of the Sound Level Distribution), TNI (Traffic Noise Index), and so on [1,2].

Principally, an acoustic phenomenon is not a propagation of substance but a propagation of energy and has no D. C. component. So, the original sound pressure with no any carrier wave is full of changeability and its frequency range is so wide that it can have a variety of fluctuation patterns with non-stationary and non-Gaussian properties. These are the special characteristics of sound environmental system. Therefore, especially in the stochastic state estimation problem, we can not all-out rely upon only the usual orthodox methodology like Kalman filter [3,4], which is based solely on linearity, Gaussian property, least squares error criterion, first order correlation, and/or the lower order statistics of mean and variance, and so forth. It might be an origin of research to take the fine-grained and generalized analytical stance as much as possible (*A small leak will sink a great ship*). Concretely, we must first admit the complexity and the variety of the acoustic phenomenon without simplifying, from the phenomenon-oriented viewpoint rather than from the one-sided methodology-oriented viewpoint (*A stitch in time saves nine*), and then, must positively sit face to face against it.

In this article, we show through some actual experiments that how the various type higher order nonlinear corelations contribute to the acoustic signal information processing more effectively than the usual case of employing only the lower order linear correlation.

2. BASIC STANCE FOR SIGNAL INFORMATION PROCESSING

The prediction and/or estimation problems are essentially to pick up one most possible value among many other probable data within the allowable error limit. Therefore, ultimately, it must be probabilistic. The deterministic signal information processing of sound environmental data has in essence a limit of the estimation accuracy, because the sound data has originally a variety of stochastic fluctuation patterns in the presence of background noise.

The prediction and/or estimation problems of sound environmental systems have the following two big aspects. One is the bottom up causative aspect, i.e., which is the extrapolation from the past to the future by using the deterministic and/or the statistical informations on multifarious correlative relationships latent in the past data (*The best prophet is the past*). The other is the top down normative aspect, i.e., which is the selection of the optimum error criterion, or in other words, the establishment of a goal for decision making by man. Of course, it might be an origin of a signal information processing to combine organically these two aspects (on the same ring and in the same stream of real time, if it is possible). The basic doctrines of our works from two essential viewpoints in contrast, which may be of somewhat overstatements, are illustrated in Fig.1. Especially, in order to establish the signal information processing methodology along with a real time axis with the firm foothold on the physical phenomenon, it seems to be one of the basic attitudes to analyze the physical quantities with a handhold of physical laws (e.g., law of inertia in a physical time), and afterwards then to convert them to the human sensory quantities. From this point of view, in some evaluation of the environmental sound data, it is important to first pay attention to the physical quantities of universal type (e.g., power-scaled variable, energy or acoustic intensity) rather than to the human sensory quantities. The advantages of using the power-scaled variable is as follows: (i) it is a universal type quantity connected with every type of change in physical phenomenon other than sound or vibration, (ii) the simple additive law of energy is valid, and it is easy to make the "separation and/or integration" logic based on this additive law, (iii) it has a kind of averaging function in itself (e.g., an inertia or a memory effect) over the multi-points or over the multi-variables, and it sometomes enables the reduction of the multi-variate treatment into a single-variate one, (iv) it has an aspect that it is connected fairly with the subjective quantity as for the statistical evaluation indices of sound environmental data (e.g., L_{eq} or L_{NP}).

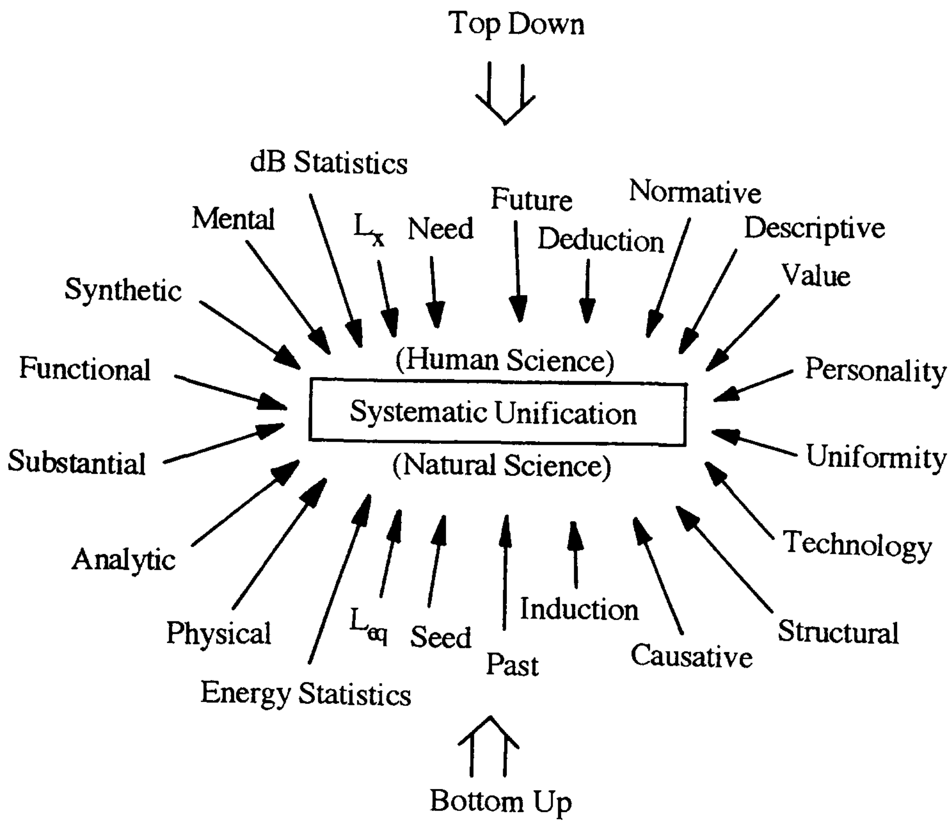

Fig.1 Concept of systematic unification.

However, this power-scaled variable (or an acoustic intensity) fluctuates only within the positive region. It is under the control of a basic binding condition that the variance approaches to zero when its mean turns to zero. Thus, in general, it shows the typical non-Gaussian and non-stationary characteristics. Furthermore, the appearance of nonlinear transformation closely connected with the human response or with the physical countermeasure is a problem (e.g., a transform from a sound pressure into an energy (or an intensity) as for L_{eq} or L_x, a logarithmic transform, a transform into arbitrary stochastic evaluation quantities, and so on). Dozens of noise evaluation quantities proposed so far are roughly classified into the following two categories. One is the category connected with an energy averaging (its typical example is L_{eq}), and the other is the category connected with the fluctuation pattern of noise or connected with the shape of its whole probability distribution form (its typical examples are L_x (x=1,5,10,50,90,95)). Many of noise evaluation quantities actually in use are the compromise, the combination, or the transform of the above. The methodology-oriented evaluation stance itself comes into a big problem at the very start of research (*A frog in the well doesn't know the open sea*). For example, the main objective of Kalman filter is an estimation of the conditional mean value, and the conditional covariance is only regarded artificially as the criterion of estimation error.

Generally speaking, not only the lower order statistics but also the higher order statistics must be considered in order to find the signal information processing method to be matched with both sides of the "matter-oriented countermeasure" (bottom-up) and the "human-oriented evaluation" (top-down) (*Look on both sides of the shield*). The lower order statistics are originally full of steadiness and reappearance supported by a large amount of data. That is, they are indeed useful for the evaluation of a hard physical system, and are certainly concerned with the "matter-oriented countermeasure". However, the auditory system is very sensitive to the end of the sound level probability distribution form. Accordingly, the estimation of the higher order statistics supported, on the contrary, by a small amount of data (e.g., L_5 or L_{10}) is essential for the evaluation of the environmental sound. For all of these reasons, it seems essentially to be best to build up the organically unified solid structure without any damaging the original whole image, i.e., the lower order statistics as a trunk of a tree, and the higher order statistics as green leaves on the gracefully shaped branches (*Rome was not built in a day*). For example, in the modeling of the physical phenomenon, the structure of the model is usually determined by a time series analysis based on the lower order linear correlation (*A word is not enough for the unwise*). Here, the higher order fluctuation is often taken, sometimes artificially, to be a noise component. The objective sense of value, which can treat the complexity of an object and the human response as they are, is to be thought much of taking the preference of the phenomenon rather than the artificiality. The pliability to use positively a various type of the higher order nonlinear correlation other than the usual linear correlation is first required to cope with every type of variability of the objective phenomenon and the delicate human-side evaluation index. Even if the dynamic model is employed, which continues to go forward and forward (*A hunter in pursuit of a deer sees no hills*), it is sometimes needed to come to a stop at each time-stage, and then deeply pay attention to the end of fluctuation (*Still waters run deep*). It is necessary to construct a whole solid image of a successive state estimation algorithm by introducing every kind of the delicate posture of the statistical analysis (series and parallel, force and love, production and natural environment, all of these need to be unified) (*When one closes his hand, it becomes a fist, and when he opens it, it becomes a* *palm*). Movement does not confront stillness but they are complementary to each other. After all, movement does not include stillness. On the basis of these two contrastive viewpoints, we should construct a solid structure to meet with a variety and a complexity of natural phenomenon together with a human-side evaluation. That is, we must go through the concrete affliction of finding out a signal information processing method where our attention is paid to the relevance between the bottom-up physical countermeasure and the top-down human-side evaluation, i.e., ultimately, technology and value.

3. IMPORTANCE OF THE UNIFIED HIERARCHICAL EXPANSION EXPRESSION

The introduction of a universal expression to describe every type of a probability distribution of a real stochastic existence is strongly required at the very start of analysis. This is because not only the mean and variance but also the median, the 90% range of distribution, a variety of noise evaluation indices proposed so far, and/or the synthetic characteristic values, must be predicted and/or estimated. In fact, we have often employed the infinite series expansion expressions for the realizations of the concrete signal information processing algorithms proposed in the following sections. This is not to discuss in complete separable form the lower order moments, but to discuss synthetically a whole image after constructing a hierarchical structure where the information of the higher order moments is incorporated as much as necessary over the information of the lower order moments (*Even the five-storied pagoda is erected from the first story*). The information of the lower order moments is located at the beginning terms of the series expansion expression, and that of the higher order moments is at the succeeding expansion terms.

The hierarchical structure and the systematic unification are introduced in this way from the very start of analysis. The information of the lower order and the higher order moments cannot be divided mechanically only from the practical business-like sense of value. If the lower order information is only used, the higher order logical sense may be lost. On the contrary, if the higher order information is only used, the physical foothold may be lost. We have to try to avoid a risk that it automatically falls into the self-righteous and one-sided research unconsciously (*A man bent on gain appreciates nothing else*).

The important problem is how we can reflect on the expansion coefficients of the series the advance information of to what extent their statistics may be reliable and useful for the long-term prediction. Therefore, even if the framework of the distribution is universal, the actual effectiveness of this expression is naturally limited. Our notice must be taken to this point especially for the sound environmental problem where the irregularity of objective phenomenon, indeterminacy of future prediction, and multiformity of the response of man come out particularly. For example, it is necessary to discuss the accuracy and the empirical reliability of the processing results based on finite sampled observation data. The followings must be further considered in more detailed: (i) how to determine an optimum number of the expansion terms, (ii) how to determine an optimum number of sampled observation data, (iii) the way of extracting the observation data from the population, and so on (*A journey of a thousand miles starts with but a single step*).

4. HIGHER ORDER SIGNAL INFORMATION PROCESSING FOR ACOUSTIC SYSTEMS

Our present works start from the Bayes' theorem, because it is the mathematical truth that can be applicable even for the complex human-side evaluation indices (e.g., L_x), and it is further applicable for the non-Gaussian and nonlinear nature of the real system on the countermeasure-side. If we start, like many other conventional works, from the stochastic differential equation of the n-th order with Brownian motion as an input, the higher order infinitesimal of Δt is to be very often neglected artificially from the very start of analysis. As a result, there is no place to take into consideration the higher order statistics that are essential for the delicate estimation of the environmental sound and vibration assessments. Another advantage of the introduction of the Bayes' theorem is that we do not need to make any artificial assumptions in advance on the whole probability distribution form, or do not need to introduce any specific type artificial evaluation indices without any physical meaning. Further, all the higher order nonlinear correlation between input and output are naturally reflected in this Bayes' estimation principle. Thus, in our works, new types of general digital filters that can estimate not only the lower order statistics (e.g., L_{eq}) but also the higher order statistics directly connected with the form of a whole probability distribution (e.g., L_x (x=1,5,10,50,90,95)) are derived based on this Bayes' theorem.

4.1 State Estimation Based on the Incomplete Observation Data with Amplitude Limitation

In the actual stochastic phenomenon, the observation process very often shows a complex fluctuation pattern apart from a standard Gaussian distribution. Furthermore, the state variables in the actual acoustic system is usual to fluctuate in a non-stationary form over a long time interval owing to a temporal change of system parameters and/or statistical properties of the random input, even if showing a stationary property in a short time interval. Especially, in the actual case when the observed signal of the random phenomenon is contaminated by an additional noise, it sometimes occures a loss or distortion of data owing to the existence of a definite dynamic range or a reliable range of measurement instruments [5].

In this Section, in order to reasonably removing this effect of observed data loss or distortion based on an amplitude limitation, a new trial on the stochastic signal information processing matched to this data loss or distortion is proposed from the fundamental viewpoint of study. More specifically, a state estimation method based on the incomplete observation data with loss or distortion is theoretically proposed through an establishment of wide sense digital filter under the actual situation with the additional noise of an arbitrary distribution type.

4.1.1 Acoustic System with Incomplete Observation of Amplitude Saturation

Let us consider an arbitrary acoustic system with state variable of arbitrary distribution type, and express the system equation as:

$$x_{k+1} = F_k(x_k, u_k), \tag{1}$$

where x_k denotes the state variable at a discrete time k, u_k is the random input with known statistics, and $F_k(\ )$ is a known nonlinear function of x_k and u_k at a discrete time k. Hereupon, x_k and u_k are statistically independent of each other. Furthermore, the observation equation is established by considering the amplitude saturation based on the dynamic range of measurement instrument, as follows:

$$y_k = E_k(x_k, v_k), \qquad z_k = g(y_k), \tag{2}$$

where $E_k(\ )$ is the known input-output relationship in the case when the measurement data are not affected by the saturation, and $g(\ )$ denotes the non-linear function describing the saturation characteristic given by

$$g(y) = \begin{cases} a & (y<a) \\ y & (a \leq y \leq b) \\ b & (y>b) \end{cases} . \tag{3}$$

Therefore, y_k is defined as the output signal of the acoustic system at a discrete time k before the signal passes through the saturation characteristic of Eq.(3), and z_k is the actually observed data at a discrete time k. Furthermore, v_k is the additional noise with known statistics and is independent of x_k.

4.1.2 Establishment of Wide Sense Digital Filter

In order to derive an algorithm of successively estimating the state variable x_k, let us consider the Bayes' theorem on the conditional probability density functions as the fundamental relationship.

$$P(x_k | Z_k) = P(x_k, z_k | Z_{k-1})/P(z_k | Z_{k-1}), \tag{4}$$

where Z_k $(=\{z_1, z_2, \ldots, z_k\})$ denotes a set of observations until a time k. Next, the conditional joint probability density function $P(x_k, z_k | Z_{k-1})$ of the state variable x_k and the observed value z_k at a time k should be expanded in a general form of the statistical orthogonal expansion series. Hereupon, the product of two marginal probability density functions for the state variable x_k and the observation z_k with a distortion of data due to the saturation characteristic $g(\ )$ is taken as the 1st term of the series expansion. These two fundamental probability density functions are denoted by $P_0(x_k | Z_{k-1})$ and $P_0(z_k | Z_{k-1})$, which can be artificially chosen as the probability density functions describing the dominant parts of the actual fluctuation, or as the well-known standard probability density functions like Gaussian or Gamma type distribution functions. Then, the orthogonal series type expansion expression of Bayes' theorem is obtained as [6,7]

$$P(x_k | Z_k) = \frac{P_0(x_k | Z_{k-1}) P_0(z_k | Z_{k-1}) \sum_{m=0}^{\infty} \sum_{n=0}^{\infty} F_{mn} \phi_m^{(1)}(x_k) \phi_n^{(2)}(z_k)}{P_0(z_k | Z_{k-1}) \sum_{n=0}^{\infty} F_{0n} \phi_n^{(2)}(z_k)}$$

$$= \frac{P_0(x_k | Z_{k-1}) \sum_{m=0}^{\infty} \sum_{n=0}^{\infty} F_{mn} \phi_m^{(1)}(x_k) \phi_n^{(2)}(z_k)}{\sum_{n=0}^{\infty} F_{0n} \phi_n^{(2)}(z_k)} \tag{5}$$

with

$$F_{mn} = \langle \phi_m^{(1)}(x_k) \phi_n^{(2)}(z_k) | Z_{k-1} \rangle, \tag{6}$$

where two functions $\phi_m^{(1)}(x_k)$ and $\phi_n^{(2)}(z_k)$ are the orthonormal polynomials of degrees m and n, with the weighting functions $P_0(x_k | Z_{k-1})$ and $P_0(z_k | Z_{k-1})$ respectively, and must satisfy the following orthonormal relationships:

$$\int \phi_m^{(1)}(x_k) \phi_n^{(1)}(x_k) P_0(x_k | Z_{k-1}) dx_k = \delta_{mn}, \tag{7}$$

$$\int \phi_m^{(2)}(z_k)\phi_n^{(2)}(z_k)P_0(z_k| Z_{k-1})dz_k = \delta_{mn}. \tag{8}$$

Based on the unified expression of Bayes' theorem in Eq.(5), the recurrence algorithm for estimating an arbitrary i-th order polynomial function $f_i(x_k)$ of the state variable x_k can be derived. Here, the function $f_i(x_k)$ can be expressed in a series expansion form by use of $\{\phi_m^{(1)}(x_k)\}$:

$$f_i(x_k) = \sum_{j=0}^{i} h_j\phi_j^{(1)}(x_k), \tag{9}$$

where h_j are appropriate constants in the orthonormal expansion of $f_i(x_k)$. Thus, the estimate for the function $f_i(x_k)$ can be easily derived in a universal form of the infinite series expansion as follows:

$$\begin{aligned}\widehat{f_i}(x_k) &= < f_i(x_k)| Z_k > \\ &= \int \sum_{j=0}^{i} h_j\phi_j^{(1)}(x_k) \frac{P_0(x_k| Z_{k-1}) \sum_{m=0}^{\infty}\sum_{n=0}^{\infty} F_{mn}\phi_m^{(1)}(x_k)\phi_n^{(2)}(z_k)}{\sum_{n=0}^{\infty} F_{0n}\phi_n^{(2)}(z_k)} dx_k \\ &= \frac{\sum_{m=0}^{i}\sum_{n=0}^{\infty} F_{mn}h_m\phi_n^{(2)}(z_k)}{\sum_{n=0}^{\infty} F_{0n}\phi_n^{(2)}(z_k)}.\end{aligned} \tag{10}$$

In the above equation, the orthonormal relationship of Eq.(7) is used.

4.1.3 Realization of Estimation Algorithm

In order to make the general theory for estimation algorithm more concrete, the well-known Gaussian distribution is adopted as the fundamental probability density function $P_0(x_k| Z_{k-1})$ for the state variable x_k, because this probability density function is the most standard one. Furthermore, the Beta distribution is adopted as $P_0(z_k| Z_{k-1})$ for the observation z_k with the amplitude restriction [a, b] of the fluctuation range due to the saturation characteristic (cf. Eq.(3)) of measurement instrument, as follows:

$$P_0(x_k| Z_{k-1}) = \frac{1}{\sqrt{2\pi\Gamma_k}}\exp\{-\frac{(x_k - x_k^*)^2}{2\Gamma_k}\}, \tag{11}$$

$$P_0(z_k| Z_{k-1}) = \frac{1}{B(\gamma_k^*,\ \alpha_k^*-\gamma_k^*+1)(b-a)}\left(\frac{z_k-a}{b-a}\right)^{\gamma_k^*-1}\left(1-\frac{z_k-a}{b-a}\right)^{\alpha_k^*-\gamma_k^*} \tag{12}$$

with

$$x_k^* = <x_k| Z_{k-1}>,\ \Gamma_k = <((x_k - x_k^*)^2| Z_{k-1}>,\ \alpha_k^* = \frac{(z_k^*-a)(b-z_k^*)}{\Omega_k^2}-2,\ \gamma_k^* = \frac{z_k^*-a}{b-a}\{\frac{(z_k^*-a)(b-z_k^*)}{\Omega_k^2}-1\},$$

$$z_k^* = <z_k| Z_{k-1}> = <g(E_k(x_k,\ v_k))| Z_{k-1}>,\ \Omega_k = <((z_k - z_k^*)^2| Z_{k-1}> = <(g(E_k(x_k,\ v_k)-z_k^*)^2| Z_{k-1}>. \tag{13}$$

Then, the orthonormal functions with two weighting probability density functions in Eqs.(7) and (8) can be given in the forms of Hermite polynomial and Jacobi polynomial [8]:

$$\phi_m^{(1)}(x_k) = \frac{1}{\sqrt{m!}} H_m\left(\frac{x_k - \overset{*}{x}_k}{\sqrt{\Gamma_k}}\right), \tag{14}$$

$$\phi_n^{(2)}(z_k) = \sqrt{\frac{\Gamma(\overset{*}{\alpha}_k - \overset{*}{\gamma}_k + 1)(\overset{*}{\alpha}_k + 2n)\Gamma(\overset{*}{\alpha}_k + n)\Gamma(\overset{*}{\gamma}_k + n)}{\Gamma(\overset{*}{\alpha}_k + 1)n!\Gamma(n + \overset{*}{\alpha}_k - \overset{*}{\gamma}_k + 1)\Gamma(\overset{*}{\gamma}_k)}}\, G_n\left(\overset{*}{\alpha}_k, \overset{*}{\gamma}_k; \frac{z_k - a}{b-a}\right). \tag{15}$$

Thus, the expansion coefficient F_{mn} can be concretely expressed in the form reflecting the effects of the observation mechanism causing the above amplitude saturation and the statistics of the additional noise.

$$F_{mn} = \frac{1}{\sqrt{m!}} \sqrt{\frac{\Gamma(\overset{*}{\alpha}_k - \overset{*}{\gamma}_k + 1)(\overset{*}{\alpha}_k + 2n)\Gamma(\overset{*}{\alpha}_k + n)\Gamma(\overset{*}{\gamma}_k + n)}{\Gamma(\overset{*}{\alpha}_k + 1)n!\Gamma(n + \overset{*}{\alpha}_k - \overset{*}{\gamma}_k + 1)\Gamma(\overset{*}{\gamma}_k)}}$$
$$< H_m\left(\frac{x_k - \overset{*}{x}_k}{\sqrt{\Gamma_k}}\right) G_n\left(\overset{*}{\alpha}_k, \overset{*}{\gamma}_k; \frac{g(E_k(x_k, v_k)) - a}{b-a}\right) | Z_{k-1} >. \tag{16}$$

Furthermore, by considering Eq.(1), the prediction step essential to perform the recurrence estimation can be given in a general form for an arbitrary polynomial function $h_r(x_{k+1})$ of the state variable as follows:

$$\overset{*}{h_r}(x_{k+1}) = <h_r(x_{k+1}) | Z_{k-1}> = <h_r(F_k(x_k, u_k)) | Z_{k-1}>. \tag{17}$$

By use of the estimates given by Eq.(10) and the statistics of u_k, the above prediction algorithm can be evaluated concretely.

4.2 State Estimation Based on Time Transitions of Moment and Probability Function

The random signal in the actual sound environment exhibits various non-Gaussian distribution forms, and is usually contaminated by the inevitable additional external noise (i.e., background noise) of arbitrary distribution type. Based on the observed noisy data mixed with the external noise, in order to estimate several evaluation quantities for a specific signal (e.g., L_x, L_{eq} and peak value etc.), it is fundamental to estimate the fluctuation wave form of only a specific signal at every instantaneous time.

On the other hand, in the actually observed time series data for the sound environment, there certainly exist nonlinear higher-order correlations, in addition to a well-known linear correlation of simple type. It is obvious that the introduction of various type higher order statistical properties of the state supported by the system and observation equations which are frames in the state transition provides the precise state estimation in some new type noise removal algorithm.

In this Section, a digital filter for estimating recursively the fluctuation wave form of only the specific signal based on the observed data contaminated by an arbitrary external noise of non-Gaussian type is considered. Concretely, state estimation methods corresponding to each model in two cases: i) adopting the time series regression model expressed in the first order moment form and ii) adopting the whole of conditional probability density function without smoothing the fluctuation form, as the system equation, are derived [9].

4.2.1 Time Series Regression Model Based on Expansion Representation of Distribution

Consider a stochastic process x(t). Let its instantaneous value at a discrete time k be x_k. All of linear and nonlinear correlation informations between x_k and the p-variable past value $\mathbf{x}_{k-1}$ $(= (x_{k-1}, x_{k-2}, \ldots, x_{k-p}))$ is contained in the conditional probability density function $P(x_k|\,\mathbf{x}_{k-1})$ of x_k for given $\mathbf{x}_{k-1}$. Especially, when x_k is to be predicted from $\mathbf{x}_{k-1}$, the expectation of x_k for given $\mathbf{x}_{k-1}$.

$$\widehat{x_k} = <x_k|\,\mathbf{x}_{k-1}> = \int x_k P(x_k|\,\mathbf{x}_{k-1})dx_k, \tag{18}$$

i.e., the regression function, can be used as the prediction x_k for x_k.

The prediction error ε_k is defined as

$$\varepsilon_k = x_k - \widehat{x_k}. \tag{19}$$

If the relation between x_k and $\mathbf{x}_{k-1}$ can well be represented by Eq.(18), ε_k is an accidental error and can be represented by a white noise model with mean 0. For this purpose, first, the joint probability density function $P(x_k, \mathbf{x}_{k-1})$ for time series x_k and $\mathbf{x}_{k-1}$ is expanded as follows. The standard distribution representations $P_0(x_k)$ and $P_0(\mathbf{x}_{k-1})$ are considered which can approximate the essential configurations of the fluctuations of x_k and $\mathbf{x}_{k-1}$, respectively. Using those distributions, the joint probability function is expanded in advance in the orthonormal form as follows:

$$P(x_k, \mathbf{x}_{k-1}) = P_0(x_k)P_0(\mathbf{x}_{k-1}) \sum_{m=0}^{\infty} \sum_{\mathbf{n}=\mathbf{0}}^{\infty} A_{m\mathbf{n}} \psi_m^{(1)}(x_k)\psi_{\mathbf{n}}^{(2)}(\mathbf{x}_{k-1}),$$

$$A_{m\mathbf{n}} = < \psi_m^{(1)}(x_k)\psi_{\mathbf{n}}^{(2)}(\mathbf{x}_{k-1}) > \tag{20}$$

with

$$\mathbf{n} = (n_1, n_2, \ldots, n_p), \qquad \sum_{\mathbf{n}=\mathbf{0}}^{\infty} = \sum_{n_1=0}^{\infty}\sum_{n_2=0}^{\infty} \cdots \sum_{n_p=0}^{\infty} .$$

In the foregoing, two functions $\psi_m^{(1)}(x_k)$ and $\psi_{\mathbf{n}}^{(2)}(\mathbf{x}_{k-1})$ are orthonormal polynomials satisfying the orthogonality conditions:

$$\int \psi_m^{(1)}(x_k)\psi_n^{(1)}(x_k)P_0(x_k)dx_k = \delta_{mn}, \tag{21}$$

$$\int \psi_{\mathbf{m}}^{(2)}(\mathbf{x}_{k-1})\psi_{\mathbf{n}}^{(2)}(\mathbf{x}_{k-1})P_0(\mathbf{x}_{k-1})d\mathbf{x}_{k-1} = \prod_{i=0}^{p} \delta_{m_i n_i}, \quad (\mathbf{m} = (m_1, m_2, \ldots, m_p)). \tag{22}$$

All linear and nonlinear correlation informations concerned with the regression between x_k and $\mathbf{x}_{k-1}$ are reflected hierarchically in the expansion coefficients $A_{m\mathbf{n}}$. Based on the well-known Bayes' theorem and using Eq.(20), the conditional probability density function $P(x_k|\,\mathbf{x}_{k-1})$, which is essential in extracting various type linear and nonlinear correlation informations, is given in the expanded form as

$$P(x_k|\,\mathbf{x}_{k-1}) = P(x_k, \mathbf{x}_{k-1})/P(\mathbf{x}_{k-1})$$

$$= \frac{P_0(x_k) \sum_{m=0}^{\infty} \sum_{\mathbf{n}=\mathbf{0}}^{\infty} A_{m\mathbf{n}} \psi_m^{(1)}(x_k)\psi_{\mathbf{n}}^{(2)}(\mathbf{x}_{k-1})}{\sum_{\mathbf{n}=\mathbf{0}}^{\infty} A_{0\mathbf{n}} \psi_{\mathbf{n}}^{(2)}(\mathbf{x}_{k-1})}. \tag{23}$$

Then, by using Eqs.(18)(19) and (23), the new autoregression model for the stochastic process x_k is derived as follows:

$$x_k = \frac{\sum_{m=0}^{1}\sum_{n=0}^{\infty} A_{mn} C_m \psi_n^{(2)}(\mathbf{x}_{k-1})}{\sum_{n=0}^{\infty} A_{0n} \psi_n^{(2)}(\mathbf{x}_{k-1})} + \varepsilon_k , \tag{24}$$

$$(x_k = \sum_{j=0}^{1} C_j \psi_j^{(1)}(x_k), \quad C_j\text{: expansion coefficients}).$$

We have confirmed that the proposed regression model agrees with the well-known AR model when the stochastic process is Gaussian [10].

On the other hand, by using the additive property of acoustic intensity, the observation value y_k (in intensity variable) under the existence of external noise can be expressed as follows:

$$y_k = x_k + v_k. \tag{25}$$

We assume that the statistics of external noise v_k (on an intensity scale) are known in advance. This Section is to find a digital filter for estimating a specific signal x_k based on successive observations of y_k. Hereupon, the multi-variate state $\mathbf{x}_k$ suitable for the recursive estimation is considered. Each expansion coefficient A_{mn} in the state transition law of objective stochastic process (of Eqs.(23) and (24)) essential to perform the recurrence estimation must be estimated based on the noisy observation y_k because the instantaneous values of $\mathbf{x}_k$ are unknown. After regarding the expansion coefficients A_{mn} ($m \leq M$; $\mathbf{n} \leq \mathbf{N}$) as unknown parameters, we define as $\mathbf{a} = (a_1, a_2, ..., a_I)^T = (\mathbf{a}_{(0)}^T, \mathbf{a}_{(1)}^T, ..., \mathbf{a}_{(M)}^T)^T$, $\mathbf{a}_{(m)} = (A_{m\mathbf{0}}, ..., A_{m\mathbf{N}})^T$ ($m= 0, 1, 2, ..., M$). For the simultaneous estimation of $\mathbf{x}_k$ and A_{mn}, the following transition model on the parameter is introduced.

$$\mathbf{a}_{k+1} = \mathbf{a}_k, \tag{26}$$

$$(\mathbf{a}_k = (a_{1,k}, a_{2,k}, ..., a_{I,k})^T = (\mathbf{a}_{(0),k}^T, \mathbf{a}_{(1),k}^T, ..., \mathbf{a}_{(M),k}^T)^T).$$

4.2.2 State Estimation Method for Specific Signal

In order to derive an estimation algorithm for the state variable $\mathbf{x}_k$ of an arbitrary distribution type, we focus our attention on the Bayes' theorem as the fundamental principle of estimation. Hereupon, since the parameter $\mathbf{a}_k$ is also unknown, the joint probability density function of $\mathbf{x}_k$ and $\mathbf{a}_k$ must be considered.

$$P(\mathbf{x}_k, \mathbf{a}_k | Y_k) = \frac{P(\mathbf{x}_k, \mathbf{a}_k, y_k | Y_{k-1})}{P(y_k | Y_{k-1})}, \tag{27}$$

where Y_k is a set of observation data $\{y_1, y_2, ..., y_k\}$ until a time k. In Eq.(27), the conditional joint probability density function $P(\mathbf{x}_k, \mathbf{a}_k, y_k | Y_{k-1})$ showing an arbitrary distribution form can be expressed in the expansion series by taking the product of fundamental probability density functions $P_0(\mathbf{x}_k | Y_{k-1})$, $P_0(\mathbf{a}_k | Y_{k-1})$ and $P_0(y_k | Y_{k-1})$ for $\mathbf{x}_k$, $\mathbf{a}_k$ and y_k.

$$P(\mathbf{x}_k, \mathbf{a}_k, y_k | Y_{k-1}) = P_0(\mathbf{x}_k | Y_{k-1}) P_0(\mathbf{a}_k | Y_{k-1}) P_0(y_k | Y_{k-1})$$
$$\sum_{\mathbf{r}=0}^{\infty}\sum_{\mathbf{m}=0}^{\infty}\sum_{n=0}^{\infty} B_{\mathbf{rm}n} \theta_{\mathbf{r}}^{(1)}(\mathbf{x}_k) \theta_{\mathbf{m}}^{(2)}(\mathbf{a}_k) \theta_n^{(3)}(y_k),$$

$$B_{\mathbf{rm}n} = \langle \theta_{\mathbf{r}}^{(1)}(\mathbf{x}_k) \theta_{\mathbf{m}}^{(2)}(\mathbf{a}_k) \theta_n^{(3)}(y_k) | Y_{k-1} \rangle, \tag{28}$$

where functions $\theta_{\mathbf{r}}^{(1)}(\mathbf{x}_k)$, $\theta_{\mathbf{m}}^{(2)}(\mathbf{a}_k)$ and $\theta_n^{(3)}(y_k)$ are the orthonormal polynomials taking the fundamental probability density functions as the weighting functions. After substituting

Eq.(28) into Eq.(27), trough the same calculation process as Sect. 4.1.2, the estimate of an arbitrary polynomial function $f_{\mathbf{i},\mathbf{j}}(\mathbf{x}_k, \mathbf{a}_k)$ ($\mathbf{i} = (i_1, i_2, \ldots, i_p)^T$, $\mathbf{j} = (j_1, j_2, \ldots, j_l)^T$) of $\mathbf{x}_k$ and $\mathbf{a}_k$ with the ($\mathbf{i}$, $\mathbf{j}$)-th order can be derived by taking the conditinal expectation of the function $f_{\mathbf{i},\mathbf{j}}(\mathbf{x}_k, \mathbf{a}_k)$ and using the orthonormal condition of functions $\theta_{\mathbf{r}}^{(1)}(\mathbf{x}_k)$ and $\theta_{\mathbf{m}}^{(2)}(\mathbf{a}_k)$, as follows:

$$\widehat{f_{\mathbf{i},\mathbf{j}}}(\mathbf{x}_k, \mathbf{a}_k) = \langle f_{\mathbf{i},\mathbf{j}}(\mathbf{x}_k, \mathbf{a}_k) | Y_k \rangle = \frac{\sum_{\mathbf{r}=\mathbf{0}}^{\mathbf{i}} \sum_{\mathbf{m}=\mathbf{0}}^{\mathbf{j}} \sum_{n=0}^{\infty} D_{\mathbf{rm}} B_{\mathbf{rm}n} \theta_n^{(3)}(y_k)}{\sum_{n=0}^{\infty} B_{\mathbf{00}n} \theta_n^{(3)}(y_k)}, \tag{29}$$

where coefficients $D_{\mathbf{rm}}$ are constants in the case when the function $f_{\mathbf{i},\mathbf{j}}(\mathbf{x}_k, \mathbf{a}_k)$ is expressed in the following orthonormal expansion form in advance.

$$f_{\mathbf{i},\mathbf{j}}(\mathbf{x}_k, \mathbf{a}_k) = \sum_{\mathbf{r}=\mathbf{0}}^{\mathbf{i}} \sum_{\mathbf{m}=\mathbf{0}}^{\mathbf{j}} D_{\mathbf{rm}} \theta_{\mathbf{r}}^{(1)}(\mathbf{x}_k) \theta_{\mathbf{m}}^{(2)}(\mathbf{a}_k). \tag{30}$$

4.2.3 Derivation of the Prediction Formula

Let the arbitrary polynomial function with arbitrary i-th order of x_{k+1} be $g_i(x_{k+1})$. In the case of using Eq.(24) as the state transition, the prediction at a discrete time k can be given by

$$g_i^*(x_{k+1}) = \langle g_i(x_{k+1}) | Y_k \rangle = \langle g_i\left(\frac{\sum_{m=0}^{1} C_m \mathbf{a}_{(m),k}^T \Psi(\mathbf{x}_k)}{\mathbf{a}_{(0),k}^T \Psi(\mathbf{x}_k)} + \varepsilon_{k+1}\right) | Y_k \rangle \tag{31}$$

with

$$\Psi(\mathbf{x}_k) = (\psi_0^{(2)}(\mathbf{x}_k), \ldots, \psi_N^{(2)}(\mathbf{x}_k))^T. \tag{32}$$

Furthermore, in the case of adopting the conditional probability density function of Eq.(23) as the state transition law, using a property on thr conditional expectation, and using the property that if the relationship between $\mathbf{x}_k$ and x_{k+1} can be sufficiently represented by the probability distribution of x_{k+1} conditioned by $\mathbf{x}_k$, the remaining fluctuation factor can be reasonably assumed as an accidental error and is independent of Y_k, the prediction of $g_i(x_{k+1})$ can be given as

$$\begin{aligned} g_i^*(x_{k+1}) &= \langle\langle g_i(x_{k+1}) | \mathbf{x}_k, Y_k \rangle | Y_k \rangle = \langle \int g_i(x_{k+1}) P(x_{k+1} | \mathbf{x}_k) dx_{k+1} | Y_k \rangle \\ &= \langle \sum_{m=0}^{\min\{M,i\}} d_m \mathbf{a}_{(m),k}^T \Psi(\mathbf{x}_k) / \mathbf{a}_{(0),k}^T \Psi(\mathbf{x}_k) | Y_k \rangle, \end{aligned} \tag{33}$$

$$\left(g_i(x_{k+1}) = \sum_{m=0}^{i} d_m \psi_m^{(1)}(x_{k+1}), \quad d_m\text{: expansion coefficients}\right).$$

Furthermore, defining $h_{\mathbf{j}}(\mathbf{a}_{k+1})$ for an arbitrary polynomial function with the $\mathbf{j}$-th order of $\mathbf{a}_{k+1}$, and using Eq.(26), the prediction for the function $h_{\mathbf{j}}(\mathbf{a}_{k+1})$ at a discrete time k can be given by

$$h_{\mathbf{j}}^*(\mathbf{a}_{k+1}) = \langle h_{\mathbf{j}}(\mathbf{a}_{k+1}) | Y_k \rangle = \langle h_{\mathbf{j}}(\mathbf{a}_k) | Y_k \rangle. \tag{34}$$

The right sides of Eqs.(31), (33) and (34) can be obtained from the estimates (for the polynomial functions) of $\mathbf{a}_k$ and $\mathbf{x}_k$, and the recurrence estimation of the state can be achieved.

5. APPLICATION TO ACOUSTIC ENVIRONMENT

In order to examine experimentally the usefulness of the proposed signal information processing methods, these are applied to the actual observation data in a specific acoustic environment.

5.1 Application to Room Acoustics

The state estimation method based on the observation with the amplitude saturation under the existence of the additional noise (background noise) is applied to the estimation problem for the reverberation time of a room. For the formulation, let us focus on the Sabine's relationship[11]:

$$e_k = e_0 \exp\{-s_k k\}, \quad s_k = s\Delta t, \quad s = 6\ln 10/T, \tag{35}$$

where e_k is an average acoustic energy at k-th discrete time, T is a reverberation time of the room, and Δt denotes the sampling interval. Therefore, in the realistic situation with the existence of a background noise, the observation z_k at the k-th discrete time in the case of measuring the sound level by a sound level meter with a finite amplitude dynamic range is given by

$$z_k = g(y_k), \quad y_k = 10\log_{10}\left(\frac{e_0}{E_0}\exp\{-s_k k\} + 10^{v_k/10}\right), \quad (E_0 = 10^{-12}\ \text{watt/m}^2), \tag{36}$$

where v_k is a background noise level at the k-th discrete time and its statistics can be easily obtained by measuring the sound level in the case when the signal e_k is absent, because the background noise usually shows a stational property in the fluctuation form. Because a reberveration time to be estimated is originally constant, the following dynamical algorithm can be obtained for a computer technique:

$$s_{k+1} = s_k, \quad (T_k = 6\ln 10/s_k). \tag{37}$$

By regarding Eqs.(36) and (37) as the observation and system equations respectively, the state estimation method proposed in Sect.4.1 can be concretely applied to the problem of estimating a reverberation time.

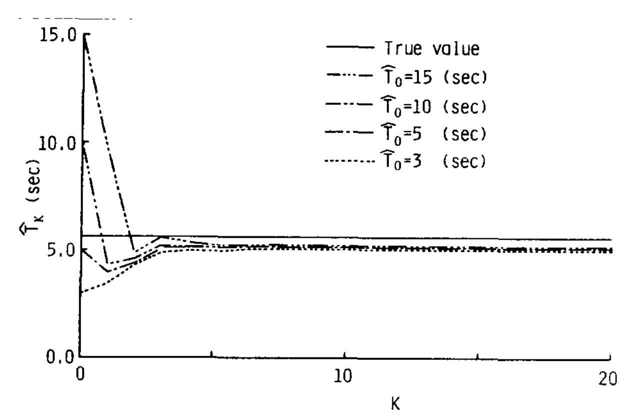

Fig.2 The results of estimating the reverberation time based on the noisy observations by use of Eq.(10) considering the expansion terms with the orders $m \leq 2$ and $n \leq 5$.

Figure 2 shows the estimated result in the case when the expansion terms with the orders $m \leq 2$ and $n \leq 5$ are utilized in Eq.(10). In this experiment, the amplitude restriction is taken a=90 [dB] and b=80 [dB] in Eq.(3), corresponding to one of several classes of dynamic range. The same estimation result on this parameter is obtained in spite of artificially empolying several kinds of

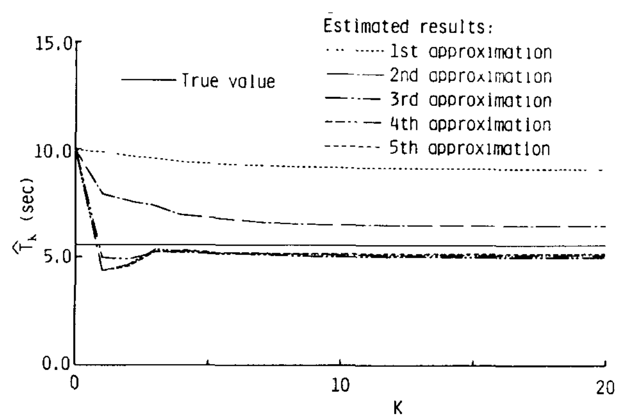

Fig.3 The results of estimating the reverberation time based on the noisy observations by use of Eq.(10) considering the expansion terms with the orders $m \leq 2$ and $n \leq n'$ for several values of n'.

arbitrary initial values. The estimated result is in good agreement with the true value obtained in advance in the idealized situation under the absence of a background noise. Furthermore, in order to investigate experimentally the relationship between the precision of estimates and the numbers of expansion terms considered in the estimation algorithm, the estimated results by use of Eq.(10) considering expansion terms with the orders $m \leq 2$ and $n \leq n'$ for several values of n' are shown in Fig.3. Here, let us define an estimate by use of the expansion expression from the first expansion term to a term containing the coefficient $F_{2n'}$ in Eq.(10) as the n'-th approximation of $\hat{T}_k$. It is obvious that the successive addition of higher expansion terms moves the estimates closer to the true value.

5.2 Application to Machine Noise

The machine noise is adopted as an example of the acoustic signal showing some complex time fluctuation forms, and the fluctuation wave form of a specific acoustic signal is estimated

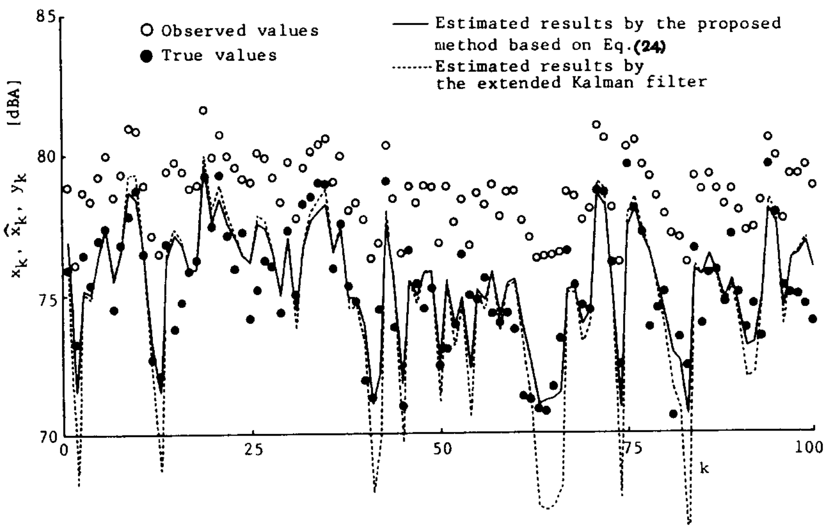

Fig.4 State estimation results for the machine noise contaminated by the background noise (σ_ε^2=1.0×10^{-9}).

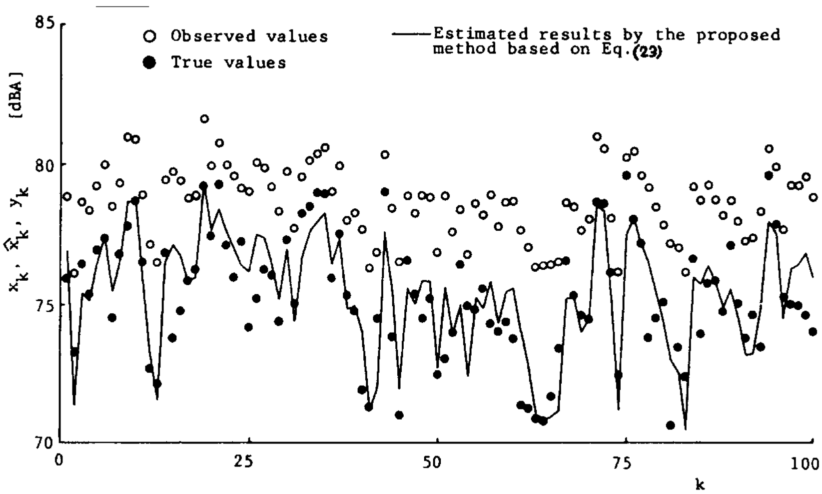

Fig.5 State estimation results for the machine noise based on the time transition model of probability function.

based on the observed data (with sampling interval of 0.2 [s]) contaminated by the background noise (i.e., white noise). First, by adopting Eq.(24) (with p=1) as the system equation, a specific signal is recursively estimated. For comparison, the estimation results calculated based on the same system equation linearlized by smoothing the statistical fluctuation of a specific signal as much as possible, by employing the standard state estimation method, are also shown.

$$x_{k+1} = G_k x_k + \varepsilon_{k+1}, \quad G_{k+1} = G_k, \tag{38}$$

where ε_{k+1} is a white noise with mean 0 and variance σ_ε^2 in the same way as Eq.(24). Because the parameter G_k in Eq.(38) is unknown (and then the system equation is nonlinear), x_k and G_k are simultaneously estimated by use of the extended Kalman filter [12]. The experimental result is shown in Fig.4 (in the case when $\sigma_\varepsilon^2=1.0\times10^{-9}$ [(W/m^2)^2]). The result by the proposed method shows a more accurate estimation for fluctuation wave form of the specific noise than the result based on the simple linear system model. Next, the estimation result in the case of adopting Eq.(23) (p=1) as the state transition law is shown in Fig.5. The estimates in the cases of adopting the smoothing model of Eq.(24) or Eq.(38) as the system equation are affected by the pre-established values of σ_ε^2. While the method adopting the conditional probability (i.e., Eq.(24)) reflecting many of the statistical information is estimated more precisely the true values as shown in Fig.5.

6. CONCLUSION

In this article, we have surveyed our past works on the acoustic signal information processing based on various types of the higher order nonlinear correlation. Concretely, we have discussed mainly about only several works related to the generalized Bayesian digital filters, which can estimate the true fluctuation of the acoustic signal from the observation data disturbed by an arbitrary stochastic type background noise.

In order to treat this kind of compound problem of a sound environmental system, it should be absolutely avoided to first simplify the methodology, in both sides of value and technology, by taking precedence of operational data processing (*The beauty of scenery is lost on the keen sportsman*).

That is, for this purpose, taking precedence of the real phenomenon, we have employed positively not only the usual linear correlation but also a variety of the higher order nonlinear correlations along with a real time axis as it is as possible. Concretely, we have found out some generalized Bayesian digital filters in a wide sense by applying some hierarchical and rational signal information processing on the observation data along with a real time.

At the start of analysis, the modeling of system and observation equations is important. In this modeling, it is rather necessary to use positively much of the variation information, which is often difficult to treat, without keeping away artificially unavoidable modeling error. That is, it might be difficult to overcome this compound problem of real type only by the computer treatment or only by just the handling of the Bayes' theorem. Our attention should be paid to the above point (*Too much medicine is harmful*).

The validity of the methods proposed here has been confirmed experimentally by applying them to some specific actual environmental sound data. However, we would like to get the reader's permission that a page limitation has to preclude an inclusion of the detailed algorithms and many other experimental results.

ACKNOWLEDGMENT

Many thanks are due to Prof. K. Hatakeyama, Messrs. M. Ueda, O. Murakami and K. Yamamoto for their helpful assistance and discussions.

REFERENCES

[1] Z. Maekawa; "Architectural Acoustics", Kyoritsu (1983).
[2] "Acoustic Measurements According to ISO Standards and Recommendations, Bruel & Kj er, N erum.
[3] R. E. Kalman: "A New Approach to Linear Filtering and Prediction Problems", Trans. ASME, Series D, J. Basic Engineering, 82, pp.35-45 (1960).
[4] R. E. Kalman and R. S. Buch: "New Results in Linear Filtering and Prediction Theory", Trans. ASME, Series D, J. Basic Engineering, 83, pp.95-108 (1961).
[5] M. Ohta, A. Ikuta and N. Takaki: "A Stochastic Signal Processing of Incomplete Observation Data with Amplitude Limitation and State Estimation under the Existence of Additional Noise", Trans. IEICE E71, pp.8-15 (1988).
[6] M. Ohta, K. Hatakeyama and M. Nishimura: "Some Unified Methods in Multivariable Linear Filtering and Prediction Problems by Use of the Expansion Form of Bayes' Theorem", Proc. of the 26th Japan National Congress for App. Mech., pp.383-398 (1976).
[7] M. Ohta and H. Yamada: "New Methodological Trials of Dynamical State Estimation for the Noise and Vibration Environmental System—Establishment of General Theory and Its Application to Urban Noise Problems", Acoustics, 55, pp.199-212 (1984).
[8] S. Moriguchi, H. Utagawa and S. Hitotumatu: "Mathematical Formulas III", Iwanami-shoten (1960).
[9] A. Ikuta and M. Ohta: "Estimation Problem of System State of Non-Gaussian Type Stochastic System Based on the Time Transitions of Moment and Probability Function—Establishment of Digital Filter and Experiment", Trans. Inst. Electr. Inf. Com. Eng., J76-A, 8, pp.1105-1111 (1993).
[10] M. Ohta and A. Ikuta: "Generalization of AR Time Series Model with Non-Gaussian and Nonlinear Type and Its Application to the Sound Environment", Trans. Inst. Electr. Inf. Com. Eng., J75-A, 4, pp.726-735 (1992).
[11] L. E. Kinsler, A. R. Frey, A. B. Coppens and J. V. Sanders: "Fundamentals of Acoustics", p.317, John Wiley & Sons, Ltd. (1980).
[12] H. J. Kushner: "Approximations to Optimal Nonlinear Filters", IEEE Trans. Automat., AC-12, 5, pp.546-556 (1967).